# Influence of a Longitudinal Magnetic Field on an Electrical Discharge in Mercury Vapor at Low Pressure

CHARLES S. CUMMINGS\* AND LEWI TONKS Research Laboratory, General Electric Company, Schenectady, New York (Received October 28, 1940)

The effect of a uniform longitudinal magnetic field on a low pressure arc has been investigated in an arc tube with a Hull type cathode and movable and fixed probes. The magnetic field collimated the current flow from the cathode into a luminous beam which disappeared rather abruptly 30 cm from the cathode into a tube-filling glow. This glow was more intense at the axis than in the absence of field. By making the field more intense at the cathode the beam was made to fill the tube and the subsequent glow was indistinguishable from that observed in the absence of field. It thus appeared that a uniform field exercised no constrictive effect on the arc. Distortion of the probe characteristics by the field limited the range within which the probes could be used to low field values

#### I. INTRODUCTION

 $\mathbf{T}^{N}$  a recent paper Tonks<sup>1</sup> developed a theory of the effect of a longitudinal magnetic field on a low pressure arc. The experiments to be described constitute a test of the adequacy of the theory.

Rokhlin<sup>2</sup> has investigated some aspects of the same problem. Otherwise practically no significant work has been reported in the literature. Some of the effects sought are small and are frequently obscured by complicating circumstances. In particular, the theory of probes in the presence of a magnetic field is not in a satisfactory state, although the attempt has been made to develop one,<sup>3</sup> so that probe methods fail to give interpretable results in interesting ranges.

This paper consists of three main divisions. In the first the large scale and visible effects of the field will be discussed. The second will be devoted to the experimental results obtained from probe characteristics and will involve such questions as the effect of the magnetic field on the plasma potential distribution, arc gradient, electron temperature, and electron current at which interesting magnetic effects were still weak. It was demonstrable that the field imparted a transverse stiffness to the arc which caused the transmission of a disturbed condition over large distances in the direction of electron flow. The measurements showed that there was no marked change in electron temperature, gradient, or electron concentration in the presence of a field, and indicated that the limiting cross-sectional distribution of charge as transmitted disturbances die out is the same as in the absence of field. The change of transverse potential distribution with field was found to be roughly in accordance with the modified Boltzmann distribution for the electrons which was previously derived theoretically.

density distribution. Part 3 will then be devoted to calculations on the basis of these data and a comparison of theory and experiment.

#### II. APPARATUS

The experiments were carried out with a cylindrical glass discharge tube pictured in Fig. 1. The tube had an eight-celled, cylindrical, unipotential oxide-coated cathode 2.5 cm in diameter. The axis of the tube was horizontal and lay in the north-south direction, the cathode being to the north.

The mercury vapor pressure in the tube was controlled by the temperature of a small pool of mercury in the appendix at the cathode end of the tube. In all of the work here reported the condensed mercury temperature was 38.6°C, corresponding to a mercury vapor pressure of 5.4×10<sup>-3</sup> mm.

To prevent mercury vapor from condensing anywhere but in the thermostated appendix. small heaters were wound around the side arms that supported the probes and also around the bend at the upper end of the appendix. At the arc current employed (4 amperes) it was not necessary to place heaters about the main part of the tube since the energy dissipated by the arc sufficed to keep the temperature of the walls above 70°C.

<sup>\*</sup> Now with Remington Arms Co., Bridgeport, Conn.
<sup>1</sup> Lewi Tonks, Phys. Rev. 56, 360 (1939).
<sup>2</sup> G. N. Rokhlin, J. Phys. USSR 1, 347 (1939).
<sup>3</sup> G. Spiwak and E. Reichrudel, Tech. Phys. USSR 5, U 715 (1938).

Of the four probes, which are shown in Fig. 1, Nos. 1 and 3 were fixed cylindrical probes of tungsten wire 0.00249 cm in diameter. Number 1 was 1 cm long, number 3 was 1.02 cm long. Probe number 4 was a flat wall probe of molybdenum 1.27 cm square. Number 2 was a tungsten wire probe 0.00250 cm in diameter and 9 mm long. By means of a small magnet this probe could be moved to any position along a horizontal diameter of the tube. Five standard positions were chosen for measurements, Nos. 1 and 5, 1.75 cm off the axis; Nos. 2 and 4, 0.87 cm off the axis; and No. 3 on the axis.

All of the probes were mounted perpendicular to the tube axis. Had the probes lain along the axis all of the current paths to them would have been perpendicular to the axial magnetic field which would, therefore, have offered maximum resistance to the electron flow. A transverse probe, however, offers one direction of approach which is parallel to the lines of force, so that its characteristics will be less affected by the field. The transverse probe was used as a compromise between this factor and the greater sampling accuracy of the axial probe.

That the transverse position of even the movable probe does not contribute markedly to the inaccuracy can be seen from the following considerations. When the probe lies at the center of the tube its center lies at a maximum of potential and electron concentration. Hence the average of these quantities in that neighborhood differs only slightly from the value at the axis. When the probe is near the tube wall the difference in radial distance of probe center and ends is small, so that the average is again not far off.

At first considerable difficulty was experienced with contamination of the probes by adsorbed oxygen which gave rise to irregularities in the characteristics. The same difficulty has been met with previously.<sup>4</sup> The remedy adopted was to sputter the probe for various lengths of time  $(\frac{1}{2}$  min. to 5 min.) at 20 volts with respect to space potential before taking each reading. Even after this treatment deviations of the characteristic curves from the ideal shape have been observed to occur. These are thought to



FIG. 1. Arc tube and magnetic field solenoid.

represent some complicating conditions in the arc not accounted for by the present arc or probe theory. The total sputtering during the course of the experiment was not sufficient to reduce the probe diameter appreciably.

The usual technique of taking and interpreting probe characteristics was employed. From a plot of the logarithm of the probe current against the probe voltage curves were obtained from which the various arc characteristics were determined either directly or by calculation. Space potentials were obtained from the position of the break in the upper part of the curves. The diameters of the cylindrical probes were too large to allow them to be calculated from an  $i^2$  vs. V curve.

The magnetic field was obtained from a solenoid consisting of five short sections each 15.2 cm long by 12.6 cm in diameter having 147 turns apiece and one section, coil "a" in Fig. 1, 7.6 cm long with 74 turns. The arrangement of these coils with respect to the tube is shown in the same figure. When run in series they gave a field of 12.1 oersteds per ampere. It was possible to excite coil "a" independently of the rest. This enabled us to change the configuration of the field in the neighborhood of the cathode.

# III. QUALITATIVE ASPECTS

When all the coils were connected in series and a current passed through them, a portion of the arc extending from the cathode toward the movable probe became concentrated into a definite beam. The arc here became much brighter than before, the brightness increasing toward the axis. A brightness pattern corresponding to the eight vanes of the cathode was observed to persist along the axis of the tube for

<sup>&</sup>lt;sup>4</sup>E. S. Lamar and K. T. Compton, Phys. Rev. 9, 1069 (1931); A. L. Reimann, Phil. Mag. 20, 594 (1935).



FIG. 2. Ion current anomaly due to transmission of cathode current redistribution.

the length of this beam, a distance of about 30 cm. Rather abruptly, within a space of 3 or 4 cm pattern and beam both disappeared into a tube-filling glow which persisted to the anode. The distribution of luminosity in this glow was less uniform than in the absence of magnetic field, the brightness being markedly more intense toward the axis.

Higher fields did not have any appreciable effect in extending the length of the beam although the sharpness of the pattern in it was improved by stronger fields. On the other hand, decreasing the pressure of the mercury vapor in the tube did increase the length of the beam.

At zero field a uniform glow filled the whole tube including the region behind the anode. At high fields ( $\sim$ 120 oersteds) the glow behind the anode disappeared.

A magnetic field tends to constrain electrons to drift along the lines of force by decreasing their mobility in directions perpendicular to the field. Thus the longitudinal magnetic field caused the initial distribution of electrons at the cathode to persist toward the anode for a distance dependent primarily on the pressure in the tube.

An irregularity in probe behavior was traced to this longitudinal transmission of electron distribution. Occasionally a positive ion currentcharacteristic taken with the wall probe showed a step which was not reproducible. Figure 2 is an example. The points were taken in the order shown by the arrows. This effect could be explained by a redistribution of emission among the cathode sectors (which is not unusual) coupled with the partial transmission of such a redistribution down the arc.

As a check on whether sufficient transmission

persisted beyond the well-defined beam, the following test was made. The wall probe was held so negative with respect to the plasma that only positive ions were reaching it. With no current in the solenoid, a permanent horseshoe magnet was held near the cathode. It caused a small increment (or decrement) in the probe current. The magnet was then held against the arc tube at an equal distance on the anode side of this probe. The effect of the magnet was approximately the same for both positions as well as for other positions which gave the same field at the probe. This showed that the field of the magnet was influencing the arc at the probe directly. On the other hand, in the presence of a longitudinal field of some 50 oersteds, the magnet near the cathode had a much greater effect than without the field, but away from the cathode it had about the same effect. This demonstrated the transmission of a disturbance at the cathode along the arc as far as the wall probe.

A real difficulty arises regarding the nature of the concentrated plasma in the beam. The thickness of the wall sheath precludes any but minute positive ion currents across it from plasma to wall. This points immediately to an essential difference between this arc plasma and a plasma that fills the arc tube. What happens to the ions formed, why they do not flow radially outward, whether there is an increasing portion of the arc current carried by the ions as the cathode is approached, and so on, are all questions which it is not possible to answer now.

It may be that the length of the beam portion of the arc is such that enough primary electrons penetrate to its end without inelastic collisions to maintain the necessary rate of ionization. There would then be little gradient in the beam, and only beyond its end would the gradient rise



FIG. 3. Schematic diagrams of field configurations at cathode for two arrangements of coil currents.

to the value necessary for maintaining the electron temperature. Thus, in the beam, the energy for ionization would come from the cathode drop, while beyond the beam this energy would be supplied by the gradient.

Until recently the idea has been prevalent that an arc constricts in a uniform longitudinal field. This belief has been questioned by Tonks on theoretical grounds<sup>1</sup> and has been refuted by Rokhlin's experiments.<sup>2</sup> The present experiments give additional proof that a uniform longitudinal magnetic field does not constrict the arc, for, as has been mentioned, the arc actually spreads from the narrow beam into the tube-filling arc in such a field. The constrictions that have been observed in the past undoubtedly arose from a converging magnetic field and persisted by virtue of the transmission effect which has been discussed above. In the present experiments, except for a slight convergence of the field near the cathode, the arc lies in an essentially uniform field throughout its length (Fig. 3(a)).

If, on the other hand, the field at the cathode is given the proper divergence from a strong field to a uniform field of the same intensity as before, as sketched in Fig. 3(b), the arc is observed to expand from the cathode and fill the tube; also a normal distribution<sup>5</sup> of electron current is obtained. This matter will be discussed more quantitatively in the next section under the heading of random electron current distribution and the "peaking" effect.



FIG. 4. Wall probe characteristic distorted by magnetic field.



FIG. 5. Probe characteristics along arc diameter—no field. Successive curves are displaced 1 v to separate them. Space potentials in volts are attached to the break in each curve.

### IV. EXPERIMENTAL RESULTS

#### a. General considerations

The arc was investigated under the following magnetic field conditions: (1) zero field, (2) 4 amp. in the coils (48.4 oersteds), (3) 6 amp. in the coils (72.6 oersteds), and (4) 20 amp. in coil "a" and 4 amp. in the rest of the coils. Figure 3(a) illustrates schematically the nature of the fields involved in (2) and (3) whereas Fig. 3(b) illustrates case (4). Many of the probe characteristics show deviations from the ideal form, and this requires some discussion.

At high magnetic fields (72.6 oersteds) the characteristics (log *i vs. V*) do not have a straight portion. They do not even consist of two straight sections, but are considerably rounded throughout their length. This is well illustrated in Fig. 4. At this field strength the ordinary probe theory is no longer adequate. Space potential cannot be estimated from these curves to better than  $\pm 0.5$  volt, and no reliable estimate of electron temperature can be made. In view of the inadequacy of probe theory no further calculations have been attempted at this field strength. Needless to say, it would be desirable to have a probe theory that would make interpretation of such results possible.

According to present theory the semilog probe characteristics taken at points across a diameter

<sup>&</sup>lt;sup>5</sup> The term "normal distribution" as used here refers to the distribution found in the zero field case and is not necessarily synonymous with the Bessel distribution predicted by the general arc theory.

of the arc should coincide in their straight portions. The zero magnetic field characteristics shown in Fig. 5 fulfill this requirement only partially. Their slopes are nearly the same so that they agree in electron temperature. If they were plotted on the same voltage scale, the straight portions of the curves for positions 3, 4 and 5 would coincide, but that for position 1 would lie 10 to 20 percent below the curve for position 2 and the latter 5 to 10 percent below the curve for position 3. The significance of this is not clear.

One of the most noticeable characteristics of the zero field curves is the apparent existence of a "break" in the curves. This shows that the electron velocity distribution deviates from the Maxwellian so that the concept of temperature becomes approximate. The value of  $T_e$ , which is required for calculations, has been taken from the lower portions of the curves where the difference in slopes is least.

The experimental points are not accurate enough to fix exactly the nature of the deviation from the ideal form, but they do show the existence of some anomaly. The deviations from the ideal form of the curves at 48.4 oersteds, as seen in Fig. 6, are still more marked but they are not so great that an estimate of the electron temperature cannot be made. Fortunately the irregularities in these curves do not seriously affect the determination of space potentials and random electron currents as they do in the 72.6-oersted case.



FIG. 6. Probe characteristics along arc diameter with field. Runs 159–163 with  $I_H=4$ , run 175 with  $I_H=4(20)$ .

#### b. Random electron current and peaking effect

Figure 7 is a plot of the random electron current to the probe as a function of the position of the probe along the tube diameter for the cases of zero field and 48.4 oersteds. It is seen that, in the presence of a magnetic field, there is a very considerable "peaking" with respect to the normal distribution curve. Furthermore, characteristics taken at the wall probe (No. 4) show a decrease in electron current to the wall when a magnetic field is applied. It is significant also that at probe No. 3 the increase in random current density was only about half as great as at probe No. 2. It seemed probable that the abnormal electron density was caused by transmission from the concentrated beam especially as it decreased with distance along the arc. It should then be possible by spreading the arc at the cathode with a suitable field configuration there to achieve a normal distribution of electrons in the arc which should persist down it.

To test this, a current of 4 amperes was maintained in all coils except coil "a." The current in this coil was raised to 20 amperes.



FIG. 7. Relative electron densities across arc without and with magnetic field.

This arrangement is designated by "4(20)." It gave rise to the type of field illustrated in Fig. 3(b). The immediately obvious effect was the filling of the tube near the cathode by the arc so that the cathode beam was absent. The glow was uniform and in general the external appearance of the arc throughout its whole length was quite indistinguishable from that in the zero field case. Probe measurements bore out the similarity. Runs on probe No. 4 showed



FIG. 8. Variation of axial electron concentration along arc with no field, with uniform field, and with field modified to give a normal distribution. The probe currents, corrected to the length of probe No. 3, are proportional to concentration.

no appreciable difference in wall current between zero field and the 4(20) field. Runs on probes No. 2 and No. 3 showed that at both places the random electron current density at the center of the arc had been reduced to its normal value. The effects of the fields zero, 4 and 4(20) are illustrated in Fig. 8.

Taken together, these facts are in accord with the theory that the normal distribution of electrons is characteristic of an arc in its steady state even in the presence of a magnetic field. Incidentally, the approach of an abnormal distribution toward a normal one is capable of theoretical treatment.<sup>6</sup>

It is unfortunate that time did not permit a more complete exploration of the 4(20) case, so that in what follows we must rely on the less satisfactory comparisons between the zero and 4-amp. field cases.

It will be seen in Fig. 7 that the random electron current distribution curve for the zero field case is unsymmetrical with respect to the tube axis. This is probably due to the vertical component of the earth's magnetic field which would tend to displace the arc toward the west, that is, to the right in the figure. On the other hand, the distribution for 48.4 oersteds is practically symmetrical. In this case the resultant magnetic field dips only about  $0.7^{\circ}$  with respect to the axis of the tube. Thus, in tending

to follow the lines of force, the electrons would build up an asymmetry in a vertical rather than a horizontal plane. But the effect is probably too small to be detected even had we been equipped to neutralize the earth's field at our apparatus.

## c. Total number of electrons in cross section

From the data of Fig. 7 it was possible to make a rough estimate of the total number  $N_e$  of electrons in unit length of cross section of the arc in the two cases zero and 4-amp. field. This was done by plotting  $n_e r$  against r and measuring the areas under the curves. The approximate symmetry of the curves made it permissible to average the data for positions 1 and 5 and for 2 and 4 and assume complete symmetry. This is theoretically justifiable to a first-order approximation.

The results are:

zero field  $16.2 \times 10^{11}$  electrons/cm<sup>-1</sup> 48.4 oersteds  $17.9 \times 10^{11}$ .

## d. Space potential distribution and arc gradient

In Fig. 9 are shown the space potential distributions across the diameter of the tube for the two cases of zero field and 48.4 oersteds. The smaller lateral mobility of the plasma electrons in a longitudinal magnetic field should decrease the plasma field necessary to maintain the balance between ion and electron currents to the wall. Consequently we should expect the curve for the magnetic field case to be flatter than for the zero field case. But a direct comparison of plasma fields can only be made for identical distributions of electron density. The peaking of the 48.4-oersted distribution would in itself require an increase of plasma field which would counteract any flattening tendency. It will appear below where the theoretical analysis is undertaken that a definite relative flattening



FIG. 9. Potential distribution across arc without and with magnetic field.

<sup>&</sup>lt;sup>6</sup> This is developed in the next paper.

of the potential distribution has occurred. Had time permitted, a cross-section plot for the 4(20) case would have been made, for which a direct comparison of plasma fields would have been possible.

At zero field the longitudinal gradient was  $0.33 \text{ v cm}^{-1}$ . At H=48.4 oersteds the gradient decreased to  $0.28 \text{ v cm}^{-1}$ . Although the direction of change was in accordance with the predictions of the theory, it should be remarked (1) that the effect was much larger than expected, and (2) the deviation of the electron distribution in the cross section from the normal was probably of primary importance. The gradient of  $0.32 \text{ v cm}^{-1}$  in the 4(20) case bears witness to this.

With increasing magnetic field the total arc drop was also observed to decrease. However, in the 4(20) case the total arc drop, like the gradient, had approximately its zero field value. As a matter of fact there is very little in these experiments to indicate a difference between the zero field and 4(20) cases. Table I summarizes these data.

#### e. Electron temperature

Theory indicates that as the loss of carriers to the walls decreases with increasing magnetic field the electron temperature should decrease. The experimental results were complicated by unexplained variations. A résumé is given in Table II. It shows that a magnetic field did in fact lower  $T_e$ , but why it should be lower at probe 3 than at 2 is not known. Also, the lower gradient in the 4 compared to the 4(20) case, and the fact that the peaking in the former should make the

TABLE I. Values of arc gradient for different magnetic fields.

$I_H$ (AMP.)	$X(v \text{ cm}^{-1})$	TOTAL DROP (VOLTS)	
0	0.33	$32.7 \pm 0.3$	
2	0.31	$30.9 \pm 0.6$	
4	0.28	$29.7 \pm 0.4$	
4 (20)	0.32	$32.9 \pm 0.1$	

TABLE II.	Values	of electron	temberature.
-----------	--------	-------------	--------------

		T.		
Probe	$I_H = 0$	$I_H = 4$	$I_H = 4(20)$	
No. 2 Position 3 No. 2 T <sub>e</sub> averaged over	19,200	18,700	18,200	
positions 1 to 5	18,750	18,300		
No. 3	18,200	17,400		

fractional loss of carriers less are not consistent with the low  $T_e$  in the latter case.

By using line intensities to determine electronic energy Rokhlin<sup>2</sup> was able to show that electron temperature decreased with increasing field. He worked with two vapor pressures of mercury,  $0.9 \times 10^{-3}$  and  $5.9 \times 10^{-3}$  min., but he does not state to which his electron energy results apply. His Fig. 14 shows a 15 to 20 percent drop in energy for 50 oersteds. As the present results are at complete variance with this, it is probable that his figure applies to the lower vapor pressure, which means that to obtain comparable results our field would have had to be about  $50 \times (5.4/0.9) = 300$  oersteds. This is not inconsistent with the small change found by us.

### V. Theory

In the presence of a longitudinal magnetic field, according to Tonks' theory, the Boltzmann equation  $n_e = n_0 \exp(-eV/kT)$  is to be replaced by the equation

$$= n_0 \exp\left(-eV/kT\tau\right),\tag{1}$$

where

$$r = \frac{\alpha D_e - \mu D_p}{\alpha D_e + \mu D_p T_e / T_p} < 1, \qquad (2)$$

in which  $\mu$  is the ratio of radial drift velocity of electrons and ions.

Equation (1) can be written

 $n_e$ 

$$\ln n_e = (-e/(kT\tau)) V + \ln n_0.$$
 (3)

A rough test of the theory is now possible inasmuch as a plot of  $\ln n_e vs. -V$  taken from Figs. 7 and 9 should give a straight line of slope  $e/kT\tau$ . From this a value of  $\tau$  can be obtained and compared with the value calculated from Eq. (2). Figure 10 is a replot of the data in Figs. 7 and 9. From the H=0 curve, using 19,000° as the reasonable average value for  $T_e$ ,  $\tau_0=0.99$  which agrees with the theoretical value unity. For H=48.4 and with  $T_e=18,300°$ ,  $\tau_{48.4}=0.75$  which is significantly less than unity and shows a relative flattening of the potential distribution.

Before a value of  $\tau$  can be computed from Eq. (2), a number of other quantities must be determined.

#### a. Electron diffusion coefficient

The electron diffusion coefficient  $D_e$  can be found from known quantities by means of the equation for the arc current  $i_x$ :

$$i_x = e^2 N_e X D_e / k T_e. \tag{4}$$

Solving for  $D_e$ ,

$$D_e = 5.39 \times 10^{14} i_x T_e / N_e X \text{ (practical units).}$$
 (5)

In the 48.4-oersted case

$$D_{eH} = \frac{5.39 \times 10^{14} \times 4 \times 18,300}{1.79 \times 10^{12} \times 0.28} = 7.87 \times 10^{7}.$$

In the zero field case

$$D_{e0} = \frac{5.39 \times 10^{14} \times 4 \times 19,000}{1.62 \times 10^{12} \times 0.33} = 7.66 \times 10^7.$$

The two calculations should give the same result, since the electronic motion to which the calculation applies is along the lines of force. The agreement is reasonably good.

#### b. Determination of $\alpha$

The constant  $\alpha$  that appears in Eq. (2) is defined by the following relations.<sup>7</sup>

$$\alpha = \gamma_{\alpha} \alpha_a, \tag{6}$$

$$\alpha_a = 1/(1+h'^2), \tag{7}$$

$$h' = 1.374 \times 10^{-4} (D_e H_z / T_e),$$
 (8)

where  $\gamma_{\alpha}$  is a correction factor which for H = 48.4 oersteds has the value 1.57.

Using  $D_{eH} = 7.87 \times 10^7$ ,  $H_Z = 48.4$  oersteds,

$$T_e = 18,300^{\circ}$$
, then  $h' = 0.591 H_Z = 28.6$ ,  
 $\alpha = 1.92 \times 10^{-3}$ 

At zero field the value of  $\alpha$  is, of course, unity.

### c. Positive ion diffusion coefficient $D_n$

The ion current density to the wall affords a means to determine  $D_p$ . In view of the inexact nature of the arc theory it is probably more accurate to use the ion current equation in a form which involves  $N_e$  rather than  $n_0$ , the axial electron density. In this form the ion current



FIG. 10. Electron current density (proportional to electron density) against space potential across arc, showing departure from Boltzmann distribution with magnetic field.

equation is<sup>8</sup>

$$I_{p} = 1.248 \frac{\alpha D_{e} D_{p} (T_{e} + T_{p}) e N_{e}}{(\alpha D_{e} T_{p}' + \mu D_{p} T_{e}) 0.432 \pi a^{3}}.$$
 (9)

This can be solved for  $D_p$ .

In the absence of magnetic field H=0,  $\alpha=1$ , the value of  $\mu$  is unimportant since the term in which it appears is negligible,  $N_e=1.62\times10^{12}$ , a=2.3,  $D_{e0}=7.66\times10^7$ ,  $T_e=19,000^\circ$ K,  $T_p\simeq T_{gas}$  $\simeq 350^\circ$ K,  $I_p=7.5\times10^{-4}$  amp. cm<sup>-2</sup> from measurement on probe No. 4. Then  $D_{p0}=690$ .

When H=48.4 oersteds,  $\alpha=1.92\times10^{-3}$ ,  $N_e$ = 1.79×10<sup>12</sup>,  $D_{eH}=7.87\times10^7$ ,  $T_e=18,500$ ,  $T_p \simeq$  $T_q \simeq 350$ ,  $I_p=6.54\times10^{-4}$ . There is some uncertainty about the value of  $\mu$ . The suppression of electron flow to the wall probe while the ion current was being measured made  $\mu=0$  locally. Elsewhere, however, since the tube walls were insulating,  $\mu=1$ . Thus for the effective  $\mu 0 < \mu < 1$ and it probably lies nearer unity. We find,

$$D_{pH} = 720$$
 for  $\mu = 1$ ,  
 $D_{pH} = 580$  for  $\mu = 0$ ,

in good agreement with  $D_{p0}$ .

For the calculation of  $\tau$  by Eq. (2) we have chosen  $D_{pH} = 690$  as a reasonable value and  $\mu = 1$ since the cylindrical probe measurements involved no unbalance between ion and electron currents to the walls. For the remaining quantities the 48.4-oersted case values were used. Then

$$\tau_{48.4} = \frac{1.92 \times 10^{-3} \times 7.87 \times 10^{7} - 690}{1.92 \times 10^{-3} \times 7.87 \times 10^{7} + 690 \times 18,300/350} = 0.80.$$

This is to be compared with the previously

 $<sup>^{7}</sup>$  Equations (6), (7) and (8) are Eqs. (6) and (7) of reference 1.

<sup>&</sup>lt;sup>8</sup> See Eq. (24.5) in reference 1.

obtained value 0.75. The agreement is as good as the accuracy of the experiments would lead us to expect.

#### VI. SUMMARY

The absence of a constricting effect in a *uniform* longitudinal magnetic field has been confirmed. The primary electrons from the cathode formed a collimated beam which bore the imprint of cathode structure and which disappeared rather abruptly into a general glow. "Abnormal" electron distributions were found to be transmitted with decreasing amplitude along the arc in the direction of the anode.

A magnetic field of 70 oersteds distorted probe characteristics at  $5.4 \times 10^{-3}$  mm pressure so badly as to make them uninterpretable. Therefore attention was confined to a 50-oersted field. The stable cross-sectional distribution of electron density was the same with this field as without field and a distribution which differed from the "normal" tended to become normal in the direction of the anode.

The abnormal distribution was found to obey a modified Boltzmann distribution in accordance with the theory of an arc in a longitudinal field.

Unfortunately the magnetic field distortion of the probe characteristics began in the range where the effects on the arc were just becoming large enough to exhibit significant differences from the zero field condition. As a consequence the experimental results did not exhibit the magnetic effects as vividly as could be wished and the test of the theory was a rather mild one.

MARCH 15, 1941

#### PHYSICAL REVIEW

VOLUME 59

# Transmission of an Electron Density Disturbance Along a Positive Column in a Longitudinal Field

LEWI TONKS

Research Laboratory, General Electric Company, Schenectady, New York (Received November 28, 1940)

The distribution of ions and electrons in the cross section of a uniform positive column is maintained by the radial motions of these particles. This distribution is designated as "normal." A disturbance of this distribution at some point in the column is followed on the anode side by an asymptotic approach to the normal. In the absence of a longitudinal magnetic field the recovery of a normal distribution occurs within a very short distance, but a longitudinal magnetic field slows down the readjustment by decreasing the radial mobility of the electrons. By making certain simplifying assumptions a theory for the approach of a disturbed column back to normal when the disturbance is cylindrically symmetrical has been worked out. The distribution is developed in a series of zero order Bessel functions, and it is found that the first term, which

## 1. INTRODUCTION

**I**N the uniform positive column of a low pressure discharge there is a *normal* distribution of electron (and ion) density in the cross section which maintains a balance, in each element of volume, between the new ions and electrons formed there and the net rate at which corresponds to the normal distribution, approaches a constant amplitude along the column whereas successive terms have successively greater space decrements. The decrement of the *i*th term approaches the constant value

$$\frac{(x_i^2-x_1^2)\alpha(T_e+T_p)}{(1-\alpha)T_ea^2(-\epsilon)},$$

where  $x_i$  is the *i*th root of  $J_0(x) = 0$ ,  $\alpha$  is the factor giving the reduction in transverse electron mobility,  $T_e$  and  $T_p$ are the electron and positive ion temperatures, a is the radius of the column and  $\epsilon$  is  $(e/kT_e)(\partial V/\partial z)$ . When compared with the experimental results of Cummings and Tonks, the theory calls for a reduction of the second term of the series expansion to 50 percent in 12.5 cm of arc length whereas experiment gave 33 percent.

these particles escape radially under the influence of the electric fields and concentration gradients. The net longitudinal outflow is zero because of the axial uniformity of the arc. Such a normal distribution exists whether the column be in a finite uniform magnetic field or none at all but the distribution will of course be different depending on the orientation and strength of the