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The Continuous X-Rays Excited by the Beta-Particles of $_{15}P^{32}$

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The internal and external x-rays (innere and äussere bremsstrahlung) excited by the beta-particles of $_{15}P^{32}$ were investigated by means of a Freon-filled ionization chamber connected to a sensitive d.c. amplifier. The intensity of the internal x-rays excited by the disintegration electrons when leaving the nucleus was found by the extrapolation method to be approximately equal to one-fourth of that of external x-rays excited by completely stopping the electrons in aluminum. From the experimental data the total energies of the two radiations were estimated. For internal x-rays this estimate leads to about 0.0020 mc^2 per disintegration electron, and for the external x-rays excited in aluminum to about 0.0082 mc^2. On the other hand, by making use of Lyman's magnetic beta-spectrum of $_{15}P^{32}$ the energy of internal x-rays calculated from the theory of Knipp and Uhlenbeck was 0.0020 mc^2 per disintegration electron and the energy of the external x-rays expected from the theory of Bethe and Heitler was 0.0089 mc^2. Therefore the experimental results are in good accord with the theoretical predictions. The relation between the intensity of the external x-rays excited by completely stopping electrons in Pb, W, Sn, Ag, Mo, Cu, Fe, Al, C and Be and the atomic number was investigated. It was found to be in fairly good agreement with the theoretical curve derived according to Bloch's formula for energy loss of electrons by inelastic collisions (F. Bloch, Ann. d. Physik **16**, 285 (1933); Zeits. f. Physik **81**, 363 (1933)) and the theory of Bethe and Heitler, namely that the intensity is proportional to the square of the atomic number. From the shape of the absorption curves of the total x-rays it will be seen that the spectral distribution of the external x-rays is independent of Z (atomic number) and is also approximately the same as that of the internal x-rays.

I. INTRODUCTION

THE disintegration of most beta-radioactive substances is accompanied by the emission of gamma-rays. The gamma-rays are highly monochromatic and may produce a secondary beta-ray emission due to a photoelectric effect of the gamma-quantum on an electron of the disintegrating atom itself. However, there are certain beta-emitting nuclei which not only give a normal continuous electron spectrum, but also emit no monochromatic gamma-rays. Nevertheless there are always fairly weak inhomogeneous gamma-rays accompanying this kind of disintegration. This was first observed by Aston[1] in his measurements on RaE and since then it has been repeatedly investigated in different laboratories.[2] The most probable interpretation of this inhomogeneous low intensity gamma-radiation is the one in which the radiation is considered as secondary in origin, emitted by the changing dipole moment of the atom when the electronic charge is suddenly shifted from the nucleus to a region outside, by the emission of a beta-particle. The whole theory of this process, known as

[1] G. H. Aston, Proc. Camb. Phil. Soc. **23**, 935 (1927).

[2] S. Bramson, J. de phys. et rad. **66**, 721 (1930); E. Stahel and D. J. Conmore, Physica **2**, 707 (1935); G. J. Sizoo and D. J. Conmore, Physica **3**, 921 (1936); E. McMillan, Phys. Rev. **47**, 801 (1935); G. V. Droste, Zeits. f. Physik **100**, 529 (1936); G. J. Sizoo, C. Eickman and P. Green, Physica **6**, 1057 (1939); E. Stahel and J. Guillessen, J. de phys. et rad. **1**, 12 (1940).

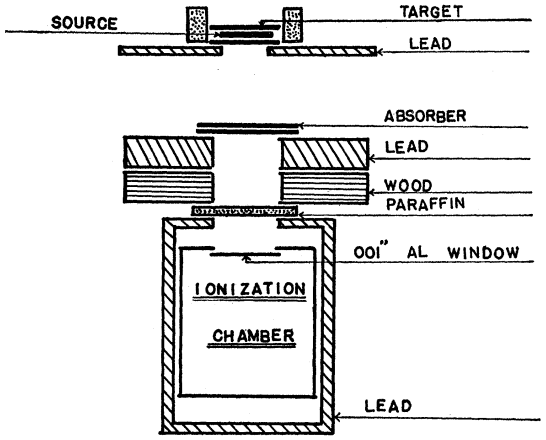

FIG. 1. Experimental arrangement.

internal x-rays (*innere* bremsstrahlung) has been worked out recently by Knipp and Uhlenbeck.[3]

On the other hand, when an electron passes through the electrostatic field of a nucleus, it is generally deflected. Since this deflection always produces a certain acceleration, the electron, according to the classical theory, must emit radiation. In the quantum theory, there will be a certain probability that a light quantum K is emitted, the electron making a transition to another state. This is usually called the external x-rays (*äussere* bremsstrahlung) to contrast with the internal x-rays (*innere* bremsstrahlung) and its rate of production has been calculated by Bethe and Heitler.[4]

The radiation loss becomes an appreciable amount at energies of several million volts. The availability of strong sources of artificial radioactive substances in this laboratory has made it possible to test their prediction. $_{15}P^{32}$ has been chosen for this purpose because it emits beta-particles with a maximum energy of 1.7 Mev and no gamma-rays. Its slow decay, of half-life 14.30 days, makes it particularly suited for this investigation. Moreover, the theoretical calculation by Knipp and Uhlenbeck is based on the assumption that the influence of the nuclear charge on the electron emitted can be neglected. $_{15}P^{32}$ has a rather small nuclear charge ($Z=15$); therefore it has another advantage so far as this requirement is concerned.

[3] J. K. Knipp and G. E. Uhlenbeck, Physica **3**, 425 (1936).

[4] H. Bethe and W. Heitler, Proc. Roy. Soc. **146**, 83 (1934).

The purpose of this investigation is to attempt to arrive at some conclusions as to whether this inhomogeneous gamma-radiation of low intensity from $_{15}P^{32}$ is entirely due to the secondary effect, namely, the radiation loss of the fast electrons coming out of the nuclei, or only partly due to it, the rest originating directly in the nucleus. It is also an intention of this work to make a comparison of the quantum yield between experimental results and theoretical predictions.

II. EXPERIMENTAL PROCEDURE

The radioactive $_{15}P^{32}$ (of an order of 5 millicuries) was activated by bombarding phosphorus with deuterons on an inner target[5] in the Berkeley cyclotron. Since phosophorus prepared in this way always contains some impurities, especially Mn, Fe and Co, in order to make sure that the gamma-rays from these impurities are no longer present, a very extensive chemical purification by repeated precipitations in alkaline and acid solutions was first carried out. The phosphate solution was warmed in a water bath and the sulfate molybdate reagent then added and shaken. The solution was allowed to stand not less than several hours. Then the precipitate was filtered on a calcium filter and carefully washed with 2 percent ammonium nitrate. After having been thoroughly washed, the precipitate was again dissolved by pouring ammonium hydroxide through the filter, and acidified with nitric acid. Since the phospho-molybdate precipitation is effective in recovering the phosphorus free of contaminants, the process was repeated several times. Finally, the phosphorus was precipitated down as magnesium ammonium phosphate by adding magnesia mixture to the ammonia solution. The magnesium ammonium phosphate thus obtained was again converted into magnesium pyrophosphate which was then evenly spread on extremely thin Cellophane (1.5 mg/cm^2) and framed in a light cardboard ring in order to avoid as much as possible any external x-rays excited from the mounting.

The measurements were carried out by an ionization chamber which was made of steel, lined on the inside with cardboard, and the inner

[5] R. R. Wilson and W. D. Kamen, Phys. Rev. **54**, 1031 (1938).

cylinder was a fine steel cage of 10-cm diameter and height. It was filled with Freon (CCl_2F_2), slightly above atmospheric pressure. The window was aluminum of 7 mg/cm² thickness. The ionization chamber was completely shielded with 3 cm of lead and connected to a sensitive d.c. amplifier of the type modified by DuBridge.[6]

III. Measurements

(a) The absorption of the beta-rays in aluminum

An absorption curve of the beta-rays from $_{15}P^{32}$ in aluminum has been carefully measured. The experimental arrangement is shown in a schematic diagram (Fig. 1). No paraffin is in front of the window for measuring the absorption of the beta-rays. The grid current in the circuit was always balanced by adjusting the background current which was in turn adjusted by varying the distance of a weak radium source from the ionization chamber. A glance at the absorption curve (Fig. 2) reveals that after the electrons have been completely absorbed, a feeble yet penetrating radiation still remains. Its intensity amounts to only one four-thousandth of that of electrons. The part of the absorption curve beyond a thickness of an absorber of 500 mg/cm² appears to follow an exponential law. This straight part may be extended to intersect the line representing gamma-ray intensity in order to give an end point, which reads 780 mg/cm². According to the empirical formula of Feather[7] this value corresponds to energy of 1.72 Mev, which is in good accord with Lyman's[8] accurate result (1.69 Mev) determined by the magnetic spectrograph method.

(b) The absorption of x-rays excited in different materials by beta-particles from $_{15}P^{32}$

The experimental arrangement is the same as in Fig. 1. The electron source which consists of an extremely thin layer of $_{15}P^{32}$ was placed between two pieces of metal which served as targets. Then a wall of paraffin, having a thickness of 1.5 cm, was used to encircle the source in order to make sure no beta-particle could come out. The thickness of the metal under investigation was so chosen that it was just enough to stop all the electrons. The geometrical condition was excellent. The ionization chamber and source were 12 cm apart, and the absorbers were always situated midway between them. However, in order to assure the reliability of the coefficient of absorption calculated under these conditions, auxiliary experiments were also performed. One was to check the absorption coefficient of annihilated gamma-radiation from Cu^{61} under exactly the same conditions. In this case it gave a value of $\mu_{Pb}=1.73$ cm^{-1}, which was in good agreement with the value $\mu_{Pb}=1.73$ cm^{-1} stated in Gentner's[9] paper. The second method was to increase the distance between the source and chamber to 18 cm. If any radiation scattered from the absorbers had gotten into the chamber, then by altering the geometrical condition, it would have shown a noticeable difference in these two absorption curves (Fig. 3). In fact, no appreciable difference was detected.

From the parallel parts of these absorption curves (Fig. 4) it may be concluded that the

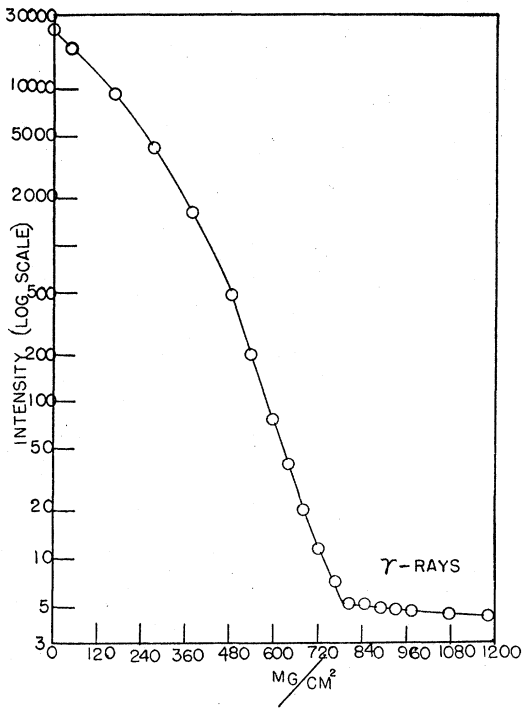

FIG. 2. Absorption in aluminum of the beta-particles from $_{15}P^{32}$.

[6] L. A. DuBridge and H. Brown, Rev. Sci. Inst. **4**, 532 (1933).
[7] N. Feather, Proc. Camb. Phil. Soc. **34**, 599 (1938).
[8] E. M. Lyman, Phys. Rev. **51**, 1 (1937).
[9] W. Gentner, Phys. Rev. **51**, 1 (1937).

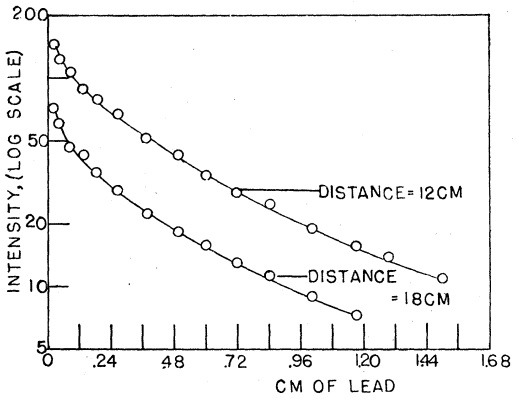

FIG. 3. Absorption in lead of the internal x-rays of $_{15}P^{32}$ plus the external x-rays excited in lead by beta-particles from $_{15}P^{32}$ under two different geometrical conditions.

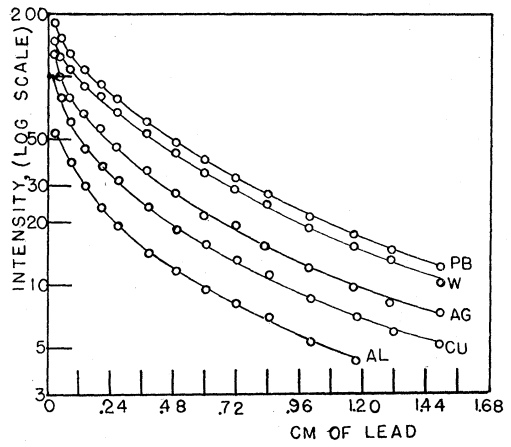

FIG. 4. Absorption in lead of the internal x-rays of $_{15}P^{32}$ plus the external x-rays excited in different metals by beta-particles from $_{15}P^{32}$.

energy distribution of the harder part of the x-rays excited in different metals by fast electrons are more or less the same. The irregularities shown at the beginning of each curve seem to be justified by interpreting them as due to the unequal self-absorption in different targets.

Although the absorption curve of the x-rays shows definitely that the distribution of energy is continuous, if one takes any one of them, for instance, the absorption curve of x-rays excited in Al (900 mg/cm^2), (Fig. 5) and proceeds by the ordinary method of analyzing an absorption curve, it can be shown that the curve is quite well represented by a resultant of three exponential curves *a*, *b*, *c* (Fig. 5) which have mass absorption coefficients, 0.0952, 0.278, and 2.30, respectively. In other words, the corresponding energies of these effective components are roughly 880, 370, and 160 kev. The apparent ratio of intensity among them is approximately 1 : 1.76 : 3.6. It is important to emphasize that the method of analyzing the absorption curve into three components is only a means by which the average energy per quantum may be estimated. Certainly it does not imply that there are only three components of x-rays existing. From these data, the average energy per quantum calculated is around 330 kev, but it must be borne in mind that this is only true when the sensitivity of the chamber responding to gamma-rays of different energies is the same. Otherwise the ratio must be subjected to correction. Unfortunately neither ionization chamber nor counter

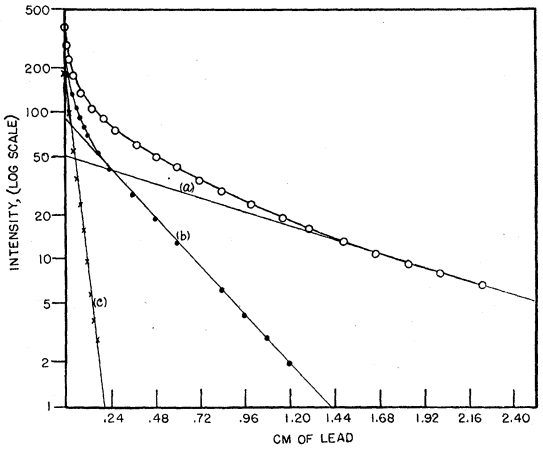

FIG. 5. Analyzed absorption curve in lead of the internal x-rays plus the external x-rays excited in aluminum. The mass absorption coefficient for curve (*a*) is 0.0952, for curve (*b*) is 0.278, for curve (*c*) is 2.30.

can ever fulfill this requirement; and, up to date, there is little known about the correlation between them.

(c) The relation between the intensity of x-rays excited and the atomic number

According to the theoretical calculation by Bethe and Heitler, the intensity of x-rays excited by the beta-particles is proportional to the square of the atomic number Z. However, the number of nuclei encountered by an electron in passing through a metal before it is completely stopped is roughly inversely proportional to the atomic number of the stopping material. (For details

see IV (b), theoretical calculation for external x-rays.) Therefore, the intensity of x-rays excited by completely stopping electrons in a metal would be expected to be approximately proportional to the atomic number. Owing to the unequal self-absorption in the target, as shown in the absorption curves, it was necessary to put a certain thickness of lead in front of the chamber to minimize this effect. A piece of lead of 8000 mg/cm² thickness was used. It gave a curve slightly convex towards the abscissa axis (curve C, Fig. 6). If the curve is extended to intersect the ordinate axis, then the intercept will give the value of the intensity of internal x-rays. However, it is difficult to get very strong artificial radioactivity in a mono-atomic layer of small area. So long as the sample has a finite thickness, obviously the intercept will not give the true value of the intensity of internal x-rays, but rather the sum of the internal x-rays and the external x-rays excited in the sample itself. Nevertheless, there are still ways to correct this small amount of external x-rays excited in the sample itself. The method employed here is based on the experimental fact that the external x-rays excited from thin foils is approximately proportional to its mass per square centimeter. In other words, the ascending curve of the intensity of the external x-rays plotted against the mass per square centimeter is a fairly straight line near the origin.

Therefore inactive phosphate was prepared and was carefully made into two thin pieces, each of which had a thickness of 20 mg/cm², exactly equal to half of that of the active sample. The pieces were then placed adjacent to each side of the sample to double its thickness. Another series of measurements of the intensity against the atomic number of the stopping material was made. The results are shown in curve D. The intensity of the x-rays excited in aluminum was not appreciably altered by increasing the thickness of the sample. This may be explained by the fact that the atomic number of aluminum is approximately equal to the equivalent atomic number of the sample.* But those excited from the materials whose atomic numbers are either greater or smaller than that of aluminum were decreased or increased, respectively, as the thickness of the sample was doubled. The difference between the two readings which were taken before and after increasing the thickness of the sample gives the difference of the intensity between that excited in the metal and that in the sample. If the differences thus obtained are added to the corresponding points for those elements having an atomic number larger than that of aluminum and subtracted from its corresponding point for those elements lying below aluminum in the periodic table, then those points will represent the true values of the intensity of the total x-rays excited in different metals by the electrons from a mono-atomic layer. The curve passing through these points is shown in Fig. 6 as curve A, which shows that the internal x-rays have a value only one-fourth that of the external x-rays excited in aluminum.

One would naturally wonder why such an indirect estimation of internal x-rays has been used instead of using a direct and straightforward measurement by bending all the electrons away

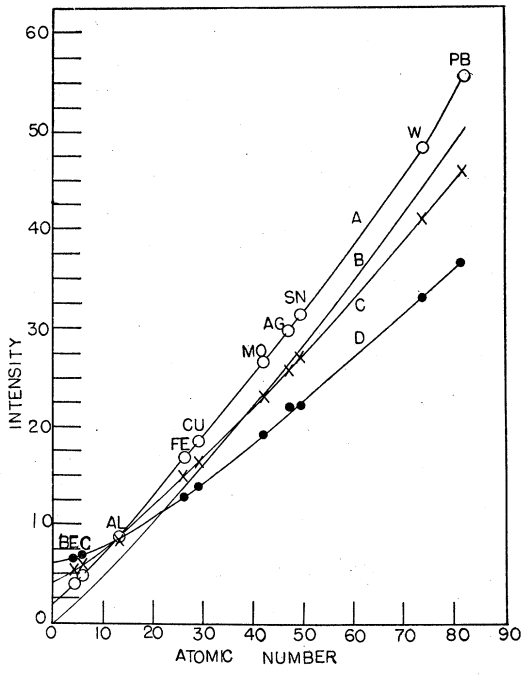

FIG. 6. The intensity of x-rays excited in different metals plotted against the atomic number. Curve A—Extrapolated curve for an infinitely thin sample; B—theoretically calculated curve; C—actually measured curve for a sample of a thickness of 40 mg/cm²; D—actually measured curve for a sample of a thickness of 80 mg/cm².

* The equivalent atomic number of the sample $Mg_2P_2O_7$ is 10.8.

in a strong magnetic field. The reason for doing this is that there are always unavoidable amounts of external x-rays excited on the pole faces and walls, part of which will be scattered into the chamber and therefore increase the uncertainty of the value of the internal x-rays. The direct method can perhaps be well carried out in case a very strong source is available, so that the conditions of proper collimating and shielding can be fulfilled.

(d) The ratio between the number of electrons and x-rays excited

In order to be able to compare the experimental results with the theoretical calculations, the ratio between the number of electrons and the x-ray quantum excited must be known. This was roughly estimated by measuring the ionization ratio between the electrons and x-rays excited in aluminum in the ionization chamber, and gave a value of 4050. For comparison, the ionization ratio between the positrons and the annihilation radiation of Cu^{61} was determined under exactly the same condition and the value 45 was found. Since there are two annihilation gamma-rays per positron which enter the chamber, the ratio between one electron and one gamma-quantum will be approximately 90. Since the beta-rays from $_{15}P^{32}$ are slightly more penetrating than those from Cu^{61}, it seems advisable to apply a correction factor of about 50/68 [10] to the efficiency obtained from Cu^{61}. Therefore the ratio is 66. The correction for gamma-rays of different energies is rather difficult to estimate because it involves the wall effect which complicates the matter. If it is assumed that the ionization produced by x-rays is comparable to that by annihilation radiation, the number of x-ray quanta produced per electron is $66/4050=0.016$. This value times the average energy per quantum gives the average energy radiated in aluminum and source $0.016 \times 330 = 5.28$ kev per electron. Since the intensity of the internal x-ray is only one-fourth of that excited in aluminum, the average energy of internal x-rays excited per electron is $1.05/510 = 0.0020\ mc^2$. The external x-ray excited in aluminum is about $0.0082\ mc^2$.

[10] W. Wilson, Proc. Roy. Soc. **A85**, 240 (1911).

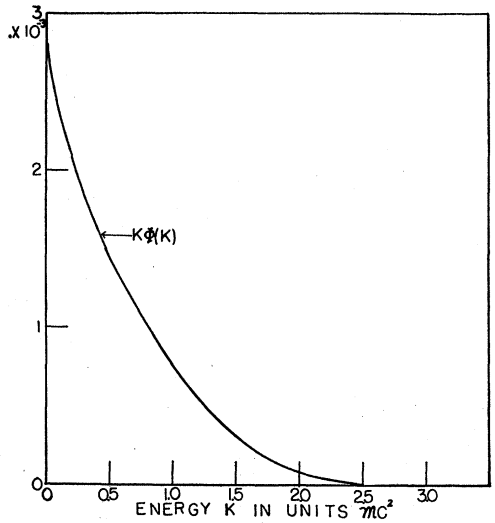

FIG. 7. $k\Phi(k)$ of the internal x-rays of $_{15}P^{32}$ as a function of the energy.

IV. THEORETICAL CALCULATIONS

(a) Internal x-rays (*innere* bremsstrahlung)

According to Knipp and Uhlenbeck's theory of internal x-rays the probability that an electron, created with energy W_e radiates a light quantum of energy k, is represented by

$$\phi=\frac{\alpha p}{\pi p_e k}\left[\frac{W_e^2+W^2}{W_e p}\log(W+p)-2\right] \qquad (1)$$

where W is the energy of the electron after having given to the radiation field a quantum of energy k or $W_e=W+k$. P_e and p are the momentum of the electron before and after the radiation occurred.

From Eq. (1) the distribution in energy of the x-ray radiation for a given initial electron energy can be plotted. The entire radiation spectrum $\Phi(k)$ for the whole distribution of electrons from $_{15}P^{32}$ can be obtained by multiplying the radiation probability ϕ for an electron of energy W_e by $P(W_e)dW$, the probability of the emission of an electron with that initial energy W_e, and integrating over all values of W_e from zero to the end point of the electron spectrum. The function $P(W_e)$ can be directly derived from Lyman's data on the momentum distribution of the electrons from $_{15}P^{32}$. For the present purpose the total x-ray intensity, $k\Phi(k)$, has been plotted in this way. It is shown in Fig. 7. The area under

the curve gives the energy of internal x-rays per electron as approximately 0.0020 mc^2.

The internal x-ray calculation used here involves two approximations. The first, neglect of the effect of the nuclear charge on the motion of the emitted electron, is justified in this case, where the parameter $Z\alpha$ is only ~ 0.1. The second is the assumption that the x-radiation and the electron emission are independent processes, justifying the calculation of the total x-radiation by the use of the observed electron spectrum. This assumption implies that the coupling between the heavy and light particles involved in beta-decay does not depend on the electron momentum.[11] Since the long lifetime of $_{15}P^{32}$ is interpreted as meaning that the transition involved is a "forbidden" one, involving the carrying away of one or more units of angular momentum by the emitted electron, the coupling here would be expected to be at least proportional to the electron wave-length. The close agreement with the simple theory is therefore rather unexpected, and may indicate that the coupling is not strongly dependent on electron wave-length, or merely that the integrated quantities measured are insensitive to changes in the spectrum of the radiation.

(b) External x-rays (*äussere* bremsstrahlung)

According to Bethe and Heitler's theory of external x-rays, the average energy lost in one collision may be obtained by integrating the intensity $k\phi_k$ over all frequencies from 0 to $W_e-\mu$ where W_e is the energy of the electron and μ is equal to mc^2, the rest energy of the electron. Therefore this is

$$\int_0^1 k\phi_k d\left(\frac{k}{W_e-\mu}\right), \tag{2}$$

where k is the energy of the quantum emitted, ϕ_k is the cross section for the emission of a quantum k.

ϕ_k is a rather complicated function of k; however, from the intensity distribution curve of the bremsstrahlung shown in Fig. 14 [12] (*The Quantum Theory of Radiation* by Heitler), one can see that for electrons of small energy, the distribution curve, except for the very soft radiation, can be roughly represented by a straight line. In this case, the average energy of the electrons coming from $_{15}P^{32}$ is around 680 kev. Therefore the function ϕ_k can be written as

$$k\phi_k \leqq 12\bar{\phi}W_e\left(1-\frac{k}{W_e-\mu}\right). \tag{3}$$

Substituting into Eq. (2)

$$\int_0^1 k\phi_k d\frac{k}{(W_e-\mu)} \leqq \frac{12\bar{\phi}W_e}{W_e-\mu}\int_0^{W_e-\mu}\frac{W_e-\mu-k}{W_e-\mu}dk \leqq 6\bar{\phi}W_e. \tag{4}$$

The average energy lost in the whole range can be obtained by integrating the value obtained in (4) over all collisions in the passage.

$$W_{\text{rad}} = 6N\bar{\phi}\int_0^{R_{max}} W_e dx. \tag{5}$$

But the radiation energy loss of an electron of 1 Mev in passing through a metal is small in comparison with the ionization energy loss, and therefore one can write

$$dx = \frac{dE}{(dE/dx)_{\text{ioniz}}}.$$

Assuming that the range-energy curve is nearly linear in this region, an average stopping power for an electron energy about one Mev is used.

Applying Eq. (5) on page 219 of reference 15 for $(dE/dx)_{\text{ioniz}}$ it gives

$$dx = \frac{dE}{13.2NZ\phi_0\mu(1+0.35\log_{10}(82/Z))} \tag{6}$$

for any atomic number Z.

Substituting (6) and (5) one gets

$$W_{\text{rad}} = 6N\frac{r_0^2Z^2}{137}\frac{3}{8\pi r_0^2}\times\frac{1}{13.2NZ\phi_0\mu(1+0.35\log_{10}(82/Z))}\int_\mu^W W_e dW_e = 0.19\times10^{-4}\frac{Z}{\mu(1+0.35\log(82/Z))}(W^2-\mu^2) = \frac{Z\times0.19\times10^{-4}}{\mu(1+0.35\log(82/Z))}(pc)^2. \tag{7}$$

[11] P. Morrison and L. I. Schiff, Phys. Rev. **58**, 24 (1940).
[12] W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, 1936).

For aluminum, $Z=13$:

$$W_{\text{rad}}=\frac{Z\times 0.148\times 10^{-4}}{\mu}(pc)^2.$$

The average value of $(pc)^2$ for the entire electron spectrum of $_{15}P^{32}$ is evaluated from Lyman's result. It is equal to $4.63\mu^2$.

Therefore the average energy loss in aluminum is:

$$W_{\text{rad}}(\text{Average})=13\times 0.148\times 10^{-4}\times 4.63\mu$$
$$=0.0089mc^2.$$

Moreover, Eq. (7) derived above also gives the variation of the intensity of the external x-rays excited in different metals with the atomic number. Since the equation contains the atomic number not only in its numerator, but also involves the logarithm of the atomic number in its denominator, the intensity is expected to increase slightly more rapidly as the atomic number increases. The curve representing this theoretical relation between the intensity and the atomic number is shown in Fig. 6, curve B.

Discussion

Although owing to the well-known difficulties encountered in the investigation of continuous energy spectra the results thereof always involve some uncertainties, quite a few satisfactory conclusions can be drawn from this information. The most interesting one is the intensity ratio between internal x-rays and external x-rays excited in different metals. According to the recent work of Sizoo and his collaborators,[13] the yield of internal x-rays is several times larger than expected from theoretical calculations. In the meantime they concluded that the yield of external x-rays excited by electrons in transversing metals is a few times smaller than predicted. The large discrepancies between their experimental results and theoretical calculation may be explained by the fact that part of the external x-rays excited on the pole faces and walls has gotten into the counter. Therefore the value of internal x-rays seems much too high.

A quantitative comparison between experimental and theoretical values is rather ambiguous because of the difficulties mentioned above. Nevertheless the energy distributions of these two radiations are analogous, so that the intensity ratio between them will not be influenced by the uncertainties introduced in assuming equal sensitivity of the chamber. In order to avoid complicating the experiment by the scattering from the pole faces of the magnet, an indirect method was used as stated above. This method gave a ratio of one to four between internal x-rays and external x-rays excited in aluminum, which is in excellent accord with the theoretical predictions.

When Bethe and Heitler worked out the theory of external x-rays in 1934, they had hoped that some day a strong artificial beta-emission radioactive substance which emits no gamma-rays might be available in order to test the theory. In the light of this investigation we may conclude that, first, the energy distribution of external x-rays excited is independent of the nature of the target used, but the intensity of the x-rays does vary with the atomic number of the target. It is proportional to the square of the atomic number. We may also say that the average quantum yield in this process is also of the predicted order of magnitude.

It is a great pleasure for the author to express her gratitude to Professor E. O. Lawrence for his suggestion of investigating this problem and his valuable guidance throughout the course of the work. She also wishes to acknowledge her special indebtedness to Professor E. McMillan for his encouragement and criticism which have made this investigation possible, and to Dr. M. Kamen for his help in preparing the strong samples.

[13] G. J. Sizoo, C. Eijkman and P. Green, Physica **6**, 1057 (1939).