On the Neutron-Proton Interaction*

WILLIAM RARITA† AND JULIAN SCHWINGER
Department of Physics, University of California, Berkeley, California

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Field theories of nuclear forces predict the existence of a spin dependent interaction similar in character to the coupling energy between two dipoles. The purpose of this paper is to study the influence of such spin-spin forces on the behavior of the neutron-proton system. A phenomenological theory is developed in which are adopted simplified rectangular well potentials whose constants are determined to fit the binding energy and quadripole moment of the deuteron, and the scattering of slow neutrons in hydrogen. The range of the forces is chosen to be that deduced from proton-proton scattering. The effects investigated include the magnetic moment of the deuteron, the scattering of neutrons in hydrogen, the radiative capture of slow neutrons, and the photo-disintegration of the deuteron. Most of the effects considered can be understood as a simple consequence of the reduced amount of the $^3S_1$ ground state of the deuteron occasioned by the admixture of a small percentage of a $^3D_1$ state. The phenomenological theory here employed adequately represents the experimental data, with the exception of the photomagnetic disintegration of the deuteron which would seem to require a detailed knowledge of the charge-bearing field.

INTRODUCTION

QUANTUM mechanical concepts have been remarkably successful in providing qualitative interpretations of nuclear phenomena. Attempts to obtain quantitative correlations of nuclear data have proceeded by the heuristic introduction of novel interactions with exchange properties. Several field theories have been proposed to supply a physical foundation for the particular forms of these empirical interaction energies. Although these theories have been beset by divergence difficulties, so that no reliance can be placed on the detailed form of the interactions they predict, the field theories do constitute suggestive models for the understanding of the general nature of nuclear forces.

Current nuclear theories postulate equal interaction energies between all pairs of nuclear particles. There are six such types of interactions, satisfying the physical requirement of invariance under the rotation-reflection group and not explicitly involving the momenta of the interacting particles.[1] In terms of the isotopic spin formalism, the exchange properties of these six interactions are symbolized by

$$1, \quad \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2, \quad S_{12}, \quad \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2, \quad \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2, \quad \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 S_{12}$$

$$S_{12}=\frac{3\boldsymbol{\sigma}_1\cdot\mathbf{r}_{12}\boldsymbol{\sigma}_2\cdot\mathbf{r}_{12}}{r_{12}^2}-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2. \qquad (1)$$

The interaction potential constructed from the four operators not containing S_{12} is a linear combination of the conventional Majorana, Heisenberg, Wigner and Barlett forces. The additional S_{12} interaction terms are not only possible *a priori*, but appear naturally in any field theory constructed to yield a spin dependence of nuclear forces. It is the purpose of this and a subsequent paper, to investigate the influence of interaction terms of the type S_{12} and $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 S_{12}$ on the properties of the two particle neutron-proton system.

GENERAL FORMULAE

The generalization of the interaction operator produces, essentially, a modification of the symmetry properties of the Hamiltonian, which in the usual theory is evidently invariant under rotations of the spin coordinates and space coordinates separately, corresponding to the existence of stationary states characterized by the quantum numbers L, S, m_L, m_S. Interaction terms of the type S_{12}, however, are invariant only under the coupled rotation of the spin and space coordinates. The rotational invariance of the Hamiltonian with the more general interaction yields, therefore, only the total angular momentum quantum numbers J and m for the description of stationary states.

Although spin and orbital angular momentum are not in general conserved individually, the total spin of a *two*-particle system is a constant of the motion. The possibility of describing stationary states by the quantum number S in

* A preliminary report of this work was presented at the November, 1938 meeting of the American Physical Society. Cf. J. Schwinger, Phys. Rev. **55**, 235 (1939).

† On sabbatical leave from Brooklyn College, Brooklyn, New York.

[1] E. Wigner, Phys. Rev. **51**, 106 (1937).

this case is a consequence of the symmetry of the Hamiltonian in the spins of the two particles. By virtue of this symmetry, stationary state wave functions may be chosen to be either symmetrical or anti-symmetrical in the spin coordinates of the particles. The well-known fact that symmetrical spin wave functions correspond to triplet states while anti-symmetrical spin wave functions correspond to singlet states demonstrates the validity of this theorem. The fourth quantum number, in addition to S, J and m, necessary to completely describe the stationary states of the neutron-proton system is provided by the parity, the eigenvalue of the reflection operator.

The singlet states of the neutron-proton system exhibit no novel features in consequence of the easily demonstrable fact that S_{12} has the eigenvalue zero in a singlet state. This is necessarily true since the conservation of total angular momentum becomes the conservation of orbital angular momentum in singlet states. In triplet states, however, new phenomena may be expected to originate in the nonconservation of orbital angular momentum.

A triplet state of definite total angular momentum J is conveniently regarded as a mixture of all possible states of orbital angular momentum L consistent with the rules for compounding angular momenta. Thus a state with $J=1$ is a mixture of ${}^3S_1$, ${}^3P_1$ and ${}^3D_1$ states. The existence of the parity quantum number permits a further classification into even and odd states based upon the theorem that a state of orbital angular momentum L has the parity quantum number $(-1)^L$. The state $J=1$, for example, decomposes into an even state which is a mixture of ${}^3S_1$ and ${}^3D_1$, and an odd ${}^3P_1$ state. We obtain in this way a complete classification of all triplet states, which is illustrated in Table I for the first few values of J.

The investigation of the properties of these states is facilitated by expressing the $SLJm$ eigenstates in terms of the eigenstates of SLm_Lm_S. It is not necessary, however, to utilize the general formulae, for only the properties of the state ${}^3S_1+{}^3D_1$ are of interest in this paper, and in this case special methods are available. That part of the wave function of the ${}^3S_1$ state with magnetic quantum number m which involves angular and spin coordinates is clearly $(4\pi)^{-\frac{1}{2}}\chi_1{}^m$, where $\chi_1{}^m$ denotes the spin wave function appropriate to a triplet state with magnetic quantum number m. The factor $(4\pi)^{-\frac{1}{2}}$ is introduced for normalization purposes. The corresponding part of the ${}^3D_1$ wave function is proportional to $S_{12}\chi_1{}^m$. That this represents a triplet state with $J=1$ is evident from the rotational invariance and spin symmetry of S_{12}. To prove that it represents a D state, we need merely note that $\nabla^2(r^2S_{12}\chi_1{}^m)=0$. The normalized part of the ${}^3D_1$ wave function involving angular and spin coordinates is thus $\frac{1}{4}(2\pi)^{-\frac{1}{2}}S_{12}\chi_1{}^m$. Introducing the symbols $u(r)/r$ and $w(r)/r$ for the radial wave functions of the S and D states, respectively, we then obtain for the wave function of the ${}^3S_1+{}^3D_1$ state the expression

$$\psi=(4\pi)^{-\frac{1}{2}}\left\{\frac{u}{r}+2^{-\frac{3}{2}}\left(\frac{3\boldsymbol{\sigma}_1\cdot\mathbf{r}\boldsymbol{\sigma}_2\cdot\mathbf{r}}{r^2}-1\right)\frac{w}{r}\right\}\chi_1{}^m. \quad (2)$$

In stating that only the ${}^3S_1+{}^3D_1$ state need be considered in detail for the interpretation of the experimental data relevant to the triplet state, we have restricted consideration to energies so small that only S states, and therefore the ${}^3D_1$ state through its coupling with the ${}^3S_1$ state, will be influenced by the neutron-proton interaction. This restriction will be removed in the sequel to this paper. Since these states are symmetrical in the space coordinates of the neutron and the proton, it is sufficient for our purposes to regard the interaction operator as a mixture of ordinary interactions, spin exchange interactions and interactions of the type S_{12}. The inclusion of space exchange operators would be a luxury of no consequence. These remarks find their mathematical expression in the following formula for the interaction operator:

$$V=-\{1-\tfrac{1}{2}g+\tfrac{1}{2}g\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+\gamma S_{12}\}J(r). \quad (3)$$

TABLE I. *Classification of triplet states.*

J	PARITY EVEN	PARITY ODD
0		${}^3P_0$
1	${}^3S_1+{}^3D_1$	${}^3P_1$
2	${}^3D_2$	${}^3P_2+{}^3F_2$
3	${}^3D_3+{}^3G_3$	${}^3F_3$

The quantities g and γ will in general involve the distance between the two particles.

In the singlet state the interaction assumes a particularly simple form,

$$V_{\text{singlet}} = -(1-2g)J(r),$$

which is readily obtained by replacing $\boldsymbol{\sigma}_2$ with $-\boldsymbol{\sigma}_1$. The corresponding wave equation for the relative motion of the neutron and the proton

$$E\psi = (\hbar^2/M)\nabla^2\psi - (1-2g)J(r)\psi, \tag{4}$$

is of standard Schrödinger type and deserves no further consideration.

Upon insertion of the wave function (2) into the triplet state wave equation,

$$E\psi = \frac{\hbar^2}{M}\nabla^2\psi - \left[1+\gamma\left(\frac{3\boldsymbol{\sigma}_1\cdot\mathbf{r}\boldsymbol{\sigma}_2\cdot\mathbf{r}}{r^2}-1\right)\right]J(r)\psi, \tag{5}$$

we obtain the following differential equations for the $^3S_1$ and $^3D_1$ radial wave functions:

$$\begin{aligned}\frac{d^2u}{dr^2}+\frac{M}{\hbar^2}[E+J]u &= -2^{\frac{3}{2}}\gamma\frac{M}{\hbar^2}Jw,\\ \frac{d^2w}{dr^2}-\frac{6w}{r^2}+\frac{M}{\hbar^2}[E+(1-2\gamma)J]w &= -2^{\frac{3}{2}}\gamma\frac{M}{\hbar^2}Ju.\end{aligned} \tag{6}$$

The discussion, with the aid of these equations, of the ground state of the deuteron, the scattering of neutrons by protons, the radiative capture of slow neutrons, and the photo-disintegration of the deuteron forms the content of the remainder of this paper.

The Ground State of the Deuteron

Of the radial dependence of the neutron-proton interaction, little is known but that it is of short range. To facilitate the determination of the effects of the postulated interaction, we shall make the simplifying assumption that the quantities g and γ be constants and that $J(r)$ be a rectangular potential well of depth V_0 and range r_0.

The wave function of the deuteron ground state satisfies the simultaneous differential equations (6), with $-E=|E_0|=2.17$ Mev. Outside the range of interaction these equations are readily integrable, yielding

$$\begin{aligned}u(r>r_0) &= Ae^{-\alpha(r-r_0)}, \quad \alpha=(M|E_0|/\hbar^2)^{\frac{1}{2}},\\ w(r>r_0) &= Be^{-\alpha(r-r_0)}(1+3/\alpha r+3/(\alpha r)^2).\end{aligned} \tag{7}$$

The differential equations descriptive of the ground state wave function at distances less than r_0 are:

$$\begin{aligned}(d^2/dr^2+\kappa^2)u(r) &= -\lambda^2 w(r),\\ (d^2/dr^2-6/r^2+\kappa'^2)w(r) &= -\lambda^2 u(r),\end{aligned} \tag{8}$$

with the abbreviations:

$$\begin{aligned}\kappa^2 &= (M/\hbar^2)(V_0-|E_0|),\\ \kappa'^2 &= (M/\hbar^2)((1-2\gamma)V_0-|E_0|),\\ \lambda^2 &= 2^{\frac{3}{2}}\gamma(M/\hbar^2)V_0.\end{aligned} \tag{9}$$

Although more elegant methods of treating these equations undoubtedly exist, the procedure adopted was the expansion of u and w in infinite power series,

$$\begin{aligned}u(r) &= \sum_0^\infty A_n x^{n+1} + \ln x \sum_0^\infty C_n x^{n+2},\\ w(r) &= \sum_0^\infty B_n x^{n+3} + \ln x \sum_0^\infty D_n x^{n+3},\\ x &= r/r_0,\end{aligned} \tag{10}$$

which provide solutions of the differential equations if the constants satisfy the recursion formulae:

$$\begin{aligned}&(n+1)(n+2)A_{n+1}\\ &\quad +(2n+3)C_n+(\kappa r_0)^2A_{n-1} = -(\lambda r_0)^2 B_{n-3},\\ &(n+1)(n+2)C_n+(\kappa r_0)^2C_{n-2} = -(\lambda r_0)^2D_{n-3},\\ &n(n+5)B_n+(2n+5)D_n\\ &\quad +(\kappa' r_0)^2B_{n-2} = -(\lambda r_0)^2A_n,\\ &n(n+5)D_n+(\kappa' r_0)^2D_{n-2} = -(\lambda r_0)^2C_{n-1}.\end{aligned} \tag{11}$$

Table II. *Quadrupole moments.*

V_0/E_0	$\gamma V_0/\|E_0\|$	$Q(10^{-27}\text{ cm}^2)$
9.779	0	0
6.60	4.79	2.67
6.40	4.96	2.73
6.00	5.30	2.84
5.50	5.72	2.95
3.57	7.14	3.34
0	9.42	3.71
−4.00	12.0	4.05
−6.81	13.6	4.26

The solutions of these recursion relations may be expressed linearly in terms of the two arbitrary constants A_0 and B_0. The reader will be spared the sight of these solutions, for numerical values of the constants are more conveniently obtained by successive solution of the recursion relations than by numerical substitution into an explicit formula.

The continuity of the logarithmic derivatives of u and w provides two equations,

$$\left(\frac{r_0}{u}\frac{du}{dr}\right)_{r=r_0}=-\alpha r_0,$$
$$\left(\frac{r_0}{w}\frac{dw}{dr}\right)_{r=r_0}=-\left(2+\frac{(\alpha r_0)^2(1+\alpha r_0)}{(\alpha r_0)^2+3\alpha r_0+3}\right), \quad (12)$$

which suffice to determine B_0/A_0 and V_0 for a given choice of r_0 and γ. The continuity of u and w then permit the constants A and B of (7) to be expressed in terms of A_0, which in turn may be derived from the normalization condition:

$$1=\int_0^\infty (u^2+w^2)dr$$
$$=\int_0^{r_0}(u^2+w^2)dr+\frac{A^2}{2\alpha}+\frac{B^2}{2\alpha}\left(1+6\frac{(1+\alpha r_0)^2}{(\alpha r_0)^3}\right), \quad (13)$$

thus completing the solution of the ground state problem.

This procedure provides a relation between V_0 and γ for a fixed r_0. To obtain a unique set of constants γ and V_0, it is necessary to employ some additional property of the neutron-proton system, sensitive to the magnitude of the spin-spin interaction term (S_{12}). Such a property is embodied in the recently discovered electric quadripole moment of the deuteron. A nonspherical distribution of charge is a consequence of the $^3D_1$ state adjoined to the $^3S_1$ state by the spin-spin forces. Conversely, the experimental existence of an unsymmetrical charge distribution in the ground state demands the introduction of a noncentral force, for otherwise the ground state would be pure S in character. The quadripole moment Q is defined as the value of $\frac{1}{4}(3z^2-r^2)$ averaged over the asymmetrical charge distribution obtained from the wave function (2), in the magnetic sub-state $m=1$.

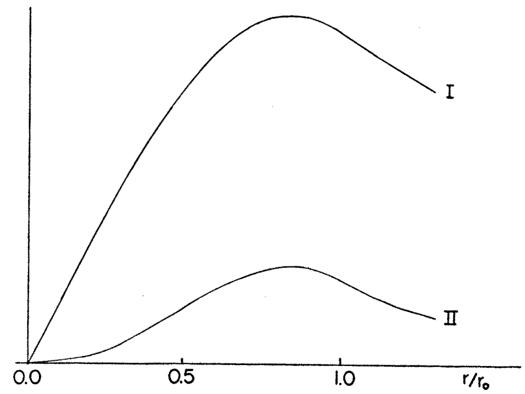

FIG. 1. S and D radial wave functions. I: $u(r)$; II: $w(r)$

$$Q=\frac{2^{\frac{1}{2}}}{10}\int_0^\infty r^2(uw-2^{-\frac{3}{2}}w^2)dr. \quad (14)$$

Nordsieck's evaluation[2] of the experiments of Rabi[3] and his co-workers gives for the value of the quadripole moment of the deuteron: $Q=+(2.73\pm0.05)\times10^{-27}$ cm^2. The positive sign indicates that the charge distribution is prolate with respect to the direction of the deuteron spin. Inspection of the interaction operator (3) shows that the condition necessary for the potential energy to be a minimum with the relative position vector $\mathbf{r}$ aligned parallel to the spin is that γV_0 be positive. Some sets of values for γ and V_0, consistent with this condition, were determined to fit the observed binding energy; and the quadripole moment was computed for each set. The results of these calculations for $r_0=2.80\times10^{-13}$ cm are summarized in Table II. The final set of constants, consistent with both binding energy and quadripole moment, is thus:

$$V_0/|E_0|=6.40, \quad \gamma=0.775, \quad r_0=2.80\times10^{-13}\text{ cm}.$$

The S and D radial functions appropriate to these constants are plotted in Fig. 1. The uncertainty in these values, arising from the 2-percent limit of error in the interpretation of the quadripole moment measurements, is estimated as 3 percent for V_0 and 6 percent for γ.

[2] A. Nordsieck, Phys. Rev. 58, 310 (1940).
[3] J. M. B. Kellogg, I. I. Rabi, N. F. Ramsey and J. R. Zacharias, Phys. Rev. 57, 677 (1940).

In principle, a test of this assignment of constants is provided by the measured magnetic moment of the deuteron. The magnetic moment associated with the ${}^3S_1+{}^3D_1$ state is no longer the sum of the intrinsic neutron and proton moments, for the ${}^3D_1$ state introduces an orbital contribution proportional to the probability of observing the deuteron in the ${}^3D_1$ state, i.e., $\int_0^\infty w^2dr$. To justify this statement let us consider the magnetic moment operator of the neutron-proton system, expressed in units of the nuclear magneton, $e\hbar/2Mc$; *viz.*,

$$\mathbf{M}=\mu_n\boldsymbol{\sigma}_n+\mu_p\boldsymbol{\sigma}_p+\tfrac{1}{2}\mathbf{L}, \tag{15}$$

where μ_n and μ_p denote the magnetic moments of the neutron and proton in magneton units, and $\mathbf{L}$ represents the internal orbital angular momentum of the system in units of $\hbar$. The factor of $\frac{1}{2}$ occurring in (15) stems from the fact that, although two particles contribute to the orbital angular momentum $\mathbf{L}$, only one possesses a charge. The expression for $\mathbf{M}$ is conveniently rewritten as follows:

$$\mathbf{M}=\tfrac{1}{2}\mathbf{J}+(\mu_n+\mu_p-\tfrac{1}{2})\mathbf{S}+\tfrac{1}{2}(\mu_n-\mu_p)(\boldsymbol{\sigma}_n-\boldsymbol{\sigma}_p), \tag{16}$$

where to $\mathbf{S}$ and $\mathbf{J}$ we attribute their usual significance. The magnetic moment of the deuteron is then obtained by calculating the average value of M_z in the magnetic sub-state of the deuteron with unit magnetic quantum number. This evaluation may be simplified by noting that all triplet state matrix elements of $(\boldsymbol{\sigma}_n-\boldsymbol{\sigma}_p)$ vanish, and that $\mathbf{S}$ may be replaced by its component in the direction of $\mathbf{J}$, i.e.,

$$\mathbf{S}\rightarrow\mathbf{J}\frac{\mathbf{S}\cdot\mathbf{J}}{\mathbf{J}^2}=\mathbf{J}\frac{\mathbf{J}^2+\mathbf{S}^2-\mathbf{L}^2}{2\mathbf{J}^2}=\mathbf{J}\frac{4-\mathbf{L}^2}{4},$$

substituting for $\mathbf{J}^2$ and $\mathbf{S}^2$ their eigenvalue 2. Hence, for the purpose of determining μ_D, the magnetic moment of the deuteron, the magnetic moment operator may be replaced by

$$\mathbf{J}[\mu_n+\mu_p-\tfrac{1}{4}(\mu_n+\mu_p-\tfrac{1}{2})\mathbf{L}^2]. \tag{17}$$

From the evident fact that the average value of $\mathbf{L}^2$ is $6\int_0^\infty w^2dr$, we obtain immediately the desired formula for the magnetic moment of the deuteron; *viz.*,

$$\mu_D=\mu_n+\mu_p-\tfrac{3}{2}(\mu_n+\mu_p-\tfrac{1}{2})\int_0^\infty w^2dr, \tag{18}$$

or

$$\mu_n=\frac{\mu_D-\frac{3}{4}\int_0^\infty w^2dr}{1-\frac{3}{2}\int_0^\infty w^2dr}-\mu_p. \tag{19}$$

The wave functions of Fig. 1 imply a D state probability

$$\int_0^\infty w^2dr=0.039.$$

The neutron magnetic moment inferred from Eq. (19) combined with the measurements of μ_p and μ_D, $\mu_p=2.785\pm0.02$, $\mu_D=0.855\pm0.006$, namely, $\mu_n=-1.908\pm0.02$, differs but slightly from that obtained by simple subtraction: -1.930 ± 0.02. The direct measurement of the moment of a free neutron by Bloch and Alvarez[4] gives $\mu_n=-1.935\pm0.02$. Presumably the discrepancy between these two results is not significant, in view of the overlapping errors of the direct measurements. Should increased accuracy in the measurements reveal a definite divergence, the concept of unperturbed intrinsic moments implicit in these calculations, would have to be abandoned. However, it should be remembered that the possibility still exists that a different radial dependence of the interactions would yield a smaller D state probability. The only information available on this question is furnished by Bethe's calculations[5] on the "cut-off" neutral mesotron potential which predicts

$$\int_0^\infty w^2dr=0.06.$$

Further evidence for the nonconservation of orbital angular momentum, manifest in the orbital contribution to the deuteron magnetic moment, is provided by the small, but real, difference between the magnetic moments of the deuteron and ${}_3\mathrm{Li}^6$. The slight difference between the ground states of the two nuclei contemplated in the usual theory is quite insufficient to account for more than a negligible fraction of the difference between $\mu({}_1\mathrm{H}^2)=0.854$ and $\mu({}_3\mathrm{Li}^6)$

[4] L. W. Alvarez and F. Bloch, Phys. Rev. **57**, 111 (1940).
[5] H. A. Bethe, Phys. Rev. **57**, 390 (1940).

$=0.820$. Indeed, according to current theory, the magnetic moments of all the odd-odd nuclei should be equal; whereas, in actuality, they form a monotonically decreasing sequence;[6] *viz.*, $\mu({}_1H^2)=0.854$, $\mu({}_3Li^6)=0.820$, $\mu({}_5B^{10})=0.597$, $\mu({}_7N^{14})=0.402$. The explanation of this anomalous behavior is to be sought in the increasing admixture of states with higher orbital angular momentum to the fundamental state $^3S_1$. To admit the validity of this interpretation, one need merely note that the magnetic moment of a member of this nuclear class is represented by

$$\mu=(\mu_n+\mu_p)-(\mu_n+\mu_p-\tfrac{1}{2})\times[(\mathbf{L}^2)_{Av}-(\mathbf{S}^2)_{Av}+2]/4. \quad (18')$$

Further, with increasing nuclear complexity, one may anticipate a progressive relaxation of the prohibition of singlet-triplet mixing, so rigidly enforced in the deuteron, thus introducing an additional nuclear moment variation with atomic weight of the character demanded by experiment.

Neutron-Proton Scattering

The scattering of neutrons by protons has proved a fruitful source of information concerning the neutron-proton interaction. Slow neutron scattering experiments in paraffin, and ortho- and parahydrogen have demonstrated the existence of spin dependent interactions which do not yield a bound singlet state of the deuteron. An exacting test of any interaction designed to produce these results is provided by the experimental values of scattering cross sections at higher energies. The most accurate measurements available at present are those of Zinn, Seely and Cohen,[7] and Aoki[8] who obtain a scattering cross section of $(2.40\pm0.10)\times10^{-24}$ cm^2 for a neutron energy of 2.8 Mev. The current theoretical value depends upon the range of interaction in the triplet state, which is not known with certainty. Calculations on the binding energies of light nuclei indicate an interaction range somewhat greater than 2×10^{-13} cm. Similar results have been obtained from experiments on proton-proton scattering, although these experiments, in reality, are pertinent only to the interaction of the ^1S state. The theoretical value of the cross section at 2.8 Mev, calculated from the interaction ranges thus obtained, is apparently larger than the experimental value. The first question which presents itself, therefore, is whether the inclusion of the S_{12} interaction term will serve to decrease the theoretical value of the scattering cross section.

The interdependence of spin and orbital motion expressed by the interaction operator (3) complicates the calculation of scattering cross sections. The orbital part of the incident wave corresponds to a state with zero orbital magnetic quantum number in the direction of wave propagation. The total component of angular momentum in this direction is consequently equal to the spin magnetic quantum number of the initial beam. Although total angular momentum is conserved, spin angular momentum alone is not, and one will therefore find scattered waves with altered values of the spin magnetic quantum number associated with corresponding nonvanishing values of the orbital magnetic quantum number. A natural consequence of this nonconservation of spin angular momentum is a dependence of the scattering cross section upon the spin magnetic quantum number of the incident beam.

Confining our attention to the triplet state, which alone exhibits these phenomena, we may write for the wave function of an incident wave propagated in the $\mathbf{k}$ direction with magnetic quantum number $m=1, 0, -1$:

$$\exp[i\mathbf{k}\cdot\mathbf{r}]\chi_1{}^m=\left\{\sum_{L=0}^{\infty}(2L+1)i^L\frac{g_L(kr)}{kr}P_L(\cos\vartheta)\right\}\chi_1{}^m, \quad (20)$$

where

$$g_L(\rho)=(\pi\rho/2)^{\frac{1}{2}}J_{L+\frac{1}{2}}(\rho)\sim\sin(\rho-\tfrac{1}{2}\pi L), \quad (21)$$

[6] S. Millman, P. Kusch and I. I. Rabi, Phys. Rev. **56**, 165 (1939); M. Phillips, Phys. Rev. **57**, 160 (1940).
[7] W. H. Zinn, S. Seely and V. W. Cohen, Phys. Rev. **56**, 260 (1939).
[8] H. Aoki, Proc. Phys.-Math. Soc. Jap. **21**, 232 (1939).

and k is related to E, the energy of relative motion, by

$$k=(ME)^{\frac{1}{2}}/h. \tag{22}$$

To simplify the calculations, we shall assume that only the even state in the continuum with $J=1$ is perturbed by the neutron-proton interaction. The justification of this approximation has already been presented. The wave function of this state is of the general form (2) with the S and D radial functions determined by the simultaneous differential equations (6). Within the range of interaction these equations assume the form (8) encountered in the solution of the ground state problem, save that $-|E_0|$ must be replaced by E in the definitions of κ^2 and κ'^2. Methods identical with those presented in the previous section may be employed for their solution. In the region $r>r_0$, the differential equations (6) may be integrated explicitly, *viz*:

$$\begin{aligned} u(r)&=\sin(kr+\delta_0),\\ w(r)&=\eta\{-\sin(kr+\delta_2)-(3/kr)\cos(kr+\delta_2)+[3/(kr)^2]\sin(kr+\delta_2)\}. \end{aligned} \tag{23}$$

Two relations between δ_0, δ_2 and B_0/A_0 are provided by the continuity of the logarithmic derivatives of u and w. The third necessary relation can only be obtained from an examination of the conditions necessary for the existence of a solution to the scattering problem.

In consequence of the hypothesis that only the even state with unit angular momentum is perturbed by the neutron-proton interaction, the wave function representing the scattering of the two particles will be obtained by replacing that part of the incident wave (20) which is of unit angular momentum and even parity by a suitable multiple of the ${}^3S_1+{}^3D_1$ wave function we have just discussed. The requisite part of the incident wave is evidently of the form (2) with u and w proportional to $g_0(kr)$ and $g_2(kr)$, respectively. Inasmuch as the entire S part of the incident wave must be included therein, we write the desired wave function as

$$\left[\frac{g_0(kr)}{kr}+\beta\left(\frac{3\boldsymbol{\sigma}_1\cdot\mathbf{r}\boldsymbol{\sigma}_2\cdot\mathbf{r}}{r^2}-1\right)\frac{g_2(kr)}{kr}\right]\chi_1{}^m. \tag{24}$$

The constant β may be determined by demanding that (18) be orthogonal to the remainder of the incident wave. Values of β thus obtained depend upon the manner in which the total angular momentum is quantized with respect to the direction of propagation of the incident beam. It is easily verified that

$$\beta=-\frac{1}{8}\left(m\left|\frac{3\boldsymbol{\sigma}_1\cdot\mathbf{k}\boldsymbol{\sigma}_2\cdot\mathbf{k}}{k^2}-1\right|m\right) \tag{25}$$

and thus $\beta=-\frac{1}{4}$ for $m=\pm1$ and $\beta=\frac{1}{2}$ for $m=0$.

Consider first those scattering processes for which $m=\pm1$. In accordance with our previous remarks, the wave function describing this type of scattering is:

$$\exp[i\mathbf{k}\cdot\mathbf{r}]\chi_1{}^{\pm1}-\left[\frac{g_0(kr)}{kr}-\frac{1}{4}S_{12}\frac{g_2(kr)}{kr}\right]\chi_1{}^{\pm1}+A^{(1)}\left[\frac{u^{(1)}(r)}{kr}+2^{-\frac{1}{2}}S_{12}\frac{w^{(1)}(r)}{kr}\right]\chi_1{}^{\pm1}, \tag{26}$$

where the superscript (1) is employed to distinguish quantities from the analogous, but different, quantities encountered in the scattering process with $m=0$. The constants $A^{(1)}$, and $\eta^{(1)}$ of (23) are specified by requiring that (26) satisfy the boundary conditions of containing only diverging spherical waves at infinity in addition to the incident wave. Utilizing the asymptotic formulae:

$$\begin{aligned} &g_0(kr)\sim\sin kr,\quad g_2(kr)\sim-\sin kr,\\ &u^{(1)}(r)\sim\sin(kr+\delta_0{}^{(1)}),\quad w^{(1)}(r)\sim-\eta^{(1)}\sin(kr+\delta_2{}^{(1)}), \end{aligned} \tag{27}$$

the desired asymptotic form of (26) is realized with the choice of constants:

$$A^{(1)}=e^{i\delta_0^{(1)}},\quad \eta^{(1)}=-2^{-\frac{1}{2}}e^{i(\delta_2^{(1)}-\delta_0^{(1)})}. \tag{28}$$

This value of $\eta^{(1)}$ provides the third equation of continuity necessary to completely specify the wave function of the ${}^3S_1+{}^3D_1$ state, for it essentially determines the value of $w^{(1)}(r)/u^{(1)}(r)$ outside the potential well. It is important to realize that the occurrence of the imaginary i in the third condition will result in complex values for B_0/A_0, and the phases $\delta_0^{(1)}$ and $\delta_2^{(1)}$. The complex nature of the phases may be interpreted as an expression of the "damping" of the S and D states arising from the interconversion caused by their mutual coupling.

The asymptotic form of the scattering wave function, obtained upon insertion of the constants (28) into formula (26), is:

$$\exp[i\mathbf{k}\cdot\mathbf{r}]\chi_1^{\pm 1}+\frac{e^{ikr}}{r}\left\{\frac{e^{2i\delta_0^{(1)}}-1}{2ik}+\frac{1}{4}S_{12}\frac{e^{2i\delta_2^{(1)}}-1}{2ik}\right\}\chi_1^{\pm 1}. \tag{29}$$

The square of the absolute value of the coefficient of e^{ikr}/r represents the scattering cross section per unit solid angle, which, in terms of the real and imaginary parts of the phases,

$$\delta_0^{(1)}=\kappa_0^{(1)}+i\zeta_0^{(1)},\quad \delta_2^{(1)}=\kappa_2^{(1)}+i\zeta_2^{(1)}, \tag{30}$$

may be written:

$$\begin{aligned}\sigma_{|m|=1}(\vartheta)=\frac{1}{k^2}\{&e^{-2\zeta_0^{(1)}}(\sin^2\kappa_0^{(1)}+\sinh^2\zeta_0^{(1)})+\tfrac{1}{8}(5-3\cos^2\vartheta)e^{-2\zeta_2^{(1)}}(\sin^2\kappa_2^{(1)}+\sinh^2\zeta_2^{(1)})\\ &+\tfrac{1}{2}(3\cos^2\vartheta-1)e^{-\zeta_0^{(1)}-\zeta_2^{(1)}}[\sinh\zeta_0^{(1)}\sinh\zeta_2^{(1)}+\sinh\zeta_2^{(1)}e^{-\zeta_0^{(1)}}\sin^2\kappa_0^{(1)}\\ &+\sinh\zeta_0^{(1)}e^{-\zeta_2^{(1)}}\sin^2\kappa_2^{(1)}+e^{-\zeta_0^{(1)}-\zeta_2^{(1)}}\sin\kappa_0^{(1)}\sin\kappa_2^{(1)}\cos(\kappa_0^{(1)}-\kappa_2^{(1)})]\}.\end{aligned} \tag{31}$$

By integration of this expression over all solid angles, we obtain the total cross section for neutron-proton scattering with magnetic quantum number $m=\pm 1$:

$$\sigma_{|m|=1}=\frac{4\pi}{k^2}\{e^{-2\zeta_0^{(1)}}(\sin^2\kappa_0^{(1)}+\sinh^2\zeta_0^{(1)})+\tfrac{1}{2}e^{-2\zeta_2^{(1)}}(\sin^2\kappa_2^{(1)}+\sinh^2\zeta_2^{(1)})\}. \tag{32}$$

The neutron-proton scattering with $m=0$ may be treated in a completely analogous fashion. In place of (28) we must employ the constants

$$A^{(0)}=e^{i\delta_0^{(0)}},\quad \eta^{(0)}=2^{\frac{1}{2}}e^{i(\delta_2^{(0)}-\delta_0^{(0)})}, \tag{33}$$

in order to obtain the correct asymptotic form of the scattering wave function; namely,

$$\exp[i\mathbf{k}\cdot\mathbf{r}]\chi_1^{0}+\frac{e^{ikr}}{r}\left\{\frac{e^{2i\delta_0^{(0)}}-1}{2ik}-\frac{1}{2}S_{12}\frac{e^{2i\delta_2^{(0)}}-1}{2ik}\right\}\chi_1^{0}. \tag{34}$$

In deriving the scattering cross section from this asymptotic formula, it is necessary to evaluate the diagonal matrix element of S_{12} with respect to the spins of the two particles. This may be done most easily by noting that the diagonal sum of S_{12} vanishes. Inasmuch as S_{12} has the eigenvalue zero in a singlet state, it follows that the diagonal sum over the triplet states vanishes. Further, it is obvious that the diagonal matrix elements of S_{12} in the states $m=\pm 1$ have the value $(3\cos^2\vartheta-1)$, a fact which has been employed in the derivation of the scattering formulae for $m=\pm 1$. Therefore, the diagonal matrix element of S_{12} in the $m=0$ state, which is the desired quantity, has the value $-2(3\cos^2\vartheta-1)$.

With the aid of the result, we obtain the following formula for the cross section of the process in which the particles are scattered through an angle ϑ into unit solid angle:

$$\sigma_{m=0}(\vartheta)=\frac{1}{k^2}\{e^{-2\zeta_0^{(0)}}(\sin^2\kappa_0^{(0)}+\sinh^2\zeta_0^{(0)})+(3\cos^2\vartheta+1)e^{-2\zeta_2^{(0)}}(\sin^2\kappa_2^{(0)}+\sinh^2\zeta_2^{(0)})$$
$$+2(3\cos^2\vartheta-1)e^{-\zeta_0^{(0)}-\zeta_2^{(0)}}[\sinh\zeta_0^{(0)}\sinh\zeta_2^{(0)}+\sinh\zeta_2^{(0)}e^{-\zeta_0^{(0)}}\sin^2\kappa_0^{(0)}$$
$$+\sinh\zeta_0^{(0)}e^{-\zeta_2^{(0)}}\sin^2\kappa_2^{(0)}+e^{-\zeta_0^{(0)}-\zeta_2^{(0)}}\sin\kappa_0^{(0)}\sin\kappa_2^{(0)}\cos(\kappa_0^{(0)}-\kappa_2^{(0)})]\}. \quad (35)$$

The total cross section for scattering in the $m=0$ state is then

$$\sigma_{m=0}=\frac{4\pi}{k^2}\{e^{-2\zeta_0^{(0)}}(\sin^2\kappa_0^{(0)}+\sinh^2\zeta_0^{(0)})+2e^{-2\zeta_2^{(0)}}(\sin^2\kappa_2^{(0)}+\sinh^2\zeta_2^{(0)})\}. \quad (36)$$

The cross section actually observed in the triplet scattering of unpolarized neutron beams is related to the cross sections with definite magnetic quantum number by

$$\sigma_{\text{triplet}}=\tfrac{2}{3}\sigma_{|m|=1}+\tfrac{1}{3}\sigma_{m=0}. \quad (37)$$

Although this cross section would appear to involve eight constants, general conservation theorems provide several connecting relations which serve to reduce to three the number of constants required to describe this type of scattering process. The expression of the stationary state character of a wave function of type (2) is:

$$\left(u^*\frac{du}{dr}-\frac{du^*}{dr}u\right)+\left(w^*\frac{dw}{dr}-\frac{dw^*}{dr}w\right)=0, \quad (38)$$

describing zero flux across the surface of a sphere of radius r. This equation may also be recognized as the generalized Wronskian condition for regular solutions of our system of simultaneous second-order linear differential equations. When applied to the asymptotic forms of the S and D radial functions for the two kinds of magnetic sub-states: $m=0,\ \pm1$:

$$u^{(m)}(r)\sim e^{i\delta_0^{(m)}}\sin(kr+\delta_0^{(m)}),$$
$$w^{(m)}(r)\sim C^{(m)}e^{i\delta_2^{(m)}}\sin(kr+\delta_2^{(m)}):\quad C^{(1)}=2^{-\frac{1}{2}},\quad C^{(0)}=-2^{\frac{1}{2}} \quad (39)$$

this Wronskian condition yields:

$$e^{-2\zeta_0^{(m)}}\sinh 2\zeta_0^{(m)}+(C^{(m)})^2e^{-2\zeta_2^{(m)}}\sinh 2\zeta_2^{(m)}=0$$

or

$$(1-e^{-4\zeta_0^{(0)}})+2(1-e^{-4\zeta_2^{(0)}})=0,\quad (1-e^{-4\zeta_0^{(1)}})+\tfrac{1}{2}(1-e^{-4\zeta_2^{(1)}})=0. \quad (40)$$

If there exist two linearly independent regular solutions for a given magnetic sub-state, i.e., u_α, w_α; u_β, w_β, they must be subject to the Wronskian restriction:

$$\left(u_\alpha^*\frac{du_\beta}{dr}-\frac{du_\alpha^*}{dr}u_\beta\right)+\left(w_\alpha^*\frac{dw_\beta}{dr}-\frac{dw_\alpha^*}{dr}w_\beta\right)=0, \quad (41)$$

for an arbitrary linear combination of α and β must satisfy (38). The application of the rotation operator J_x+iJ_y to the $m=0$ wave function produces a regular solution for the $m=1$ magnetic sub-state which differs only in its asymptotic form from that represented in (39). Thus for the $m=1$ state, two linearly independent solutions exist, represented by

$$u_\alpha=u^{(1)},\quad u_\beta=u^{(0)},$$
$$w_\alpha=w^{(1)},\quad w_\beta=w^{(0)}.$$

The phases describing the two magnetic sublevels are thereby subject to the restriction

$$\delta_0{}^{(0)*}-\delta_0{}^{(1)}=\delta_2{}^{(0)*}-\delta_2{}^{(1)}$$

or

$$\kappa_0{}^{(0)}-\kappa_2{}^{(0)}=\kappa_0{}^{(1)}-\kappa_2{}^{(1)},$$
$$\zeta_0{}^{(0)}-\zeta_2{}^{(0)}=\zeta_2{}^{(1)}-\zeta_0{}^{(1)}. \qquad (42)$$

Further, the complex conjugate of a regular solution is again a regular solution, for the fundamental differential equations (6) involve only real coefficients. Hence an equally permissible pair of regular solutions is

$$u_\alpha=u^{(1)}, \quad u_\beta=u^{(0)*},$$
$$w_\alpha=w^{(1)}; \quad w_\beta=w^{(0)*},$$

which, when inserted in the Wronskian condition (41), introduces the additional equality:

$$e^{2i\delta_0{}^{(0)}}-e^{2i\delta_2{}^{(0)}}=e^{2i\delta_0{}^{(1)}}-e^{2i\delta_2{}^{(1)}}. \qquad (43)$$

These relations, (40), (42) and (43), when suitably combined, yield five independent restrictions; *viz.*,

$$\zeta_0{}^{(0)}=\zeta_2{}^{(1)}, \quad \zeta_0{}^{(1)}=\zeta_2{}^{(0)}$$
$$e^{-2\zeta_0{}^{(0)}}\cos(\kappa_0{}^{(0)}-\kappa_2{}^{(1)}) = e^{-2\zeta_0{}^{(1)}}\cos(\kappa_0{}^{(1)}-\kappa_2{}^{(0)}), \qquad (44)$$

the first of (42) and either of the Eqs. (40). When full advantage is taken of these relations, the total triplet scattering cross section is reduced to dependence on only three phase constants; *viz.*,

$$\sigma_{\text{triplet}}=(4\pi/k^2)\{e^{-2\zeta_0{}^{(0)}}(\sin^2\kappa_0{}^{(0)}+\sin^2\kappa_2{}^{(1)}) +(1-e^{-2\zeta_0{}^{(0)}})\}. \qquad (45)$$

Although the cross section for the scattering of neutrons with zero energy ($E=0$) may be obtained from the previous results by suitable limiting processes, it is more conveniently treated anew. At zero energy only the $^3S_1+{}^3D_1$ state is extant. Concerning the form of the wave function of this state within the range of interaction, nothing need be added to what has already been said. Outside the range of interaction, the S and D radial functions obtained by integration of (6) has the form:

$$u(r>r_0)=r+a\,; \quad w(r>r_0)=b/r^2. \qquad (46)$$

The equations of continuity,

$$\left(\frac{r_0}{u}\frac{du}{dr}\right)_{r=r_0}=\frac{r_0}{r_0+a}; \quad \left(\frac{r_0}{w}\frac{dw}{dr}\right)_{r=r_0}=-2, \qquad (47)$$

suffice to determine the constants B_0/A_0 and a. With the aid of these quantities, the constants A_0 and b may then be determined from the equations expressing the continuity of the S and D radial wave functions. The scattering is obviously isotropic, with the total cross section $4\pi a^2$.

We shall prove at this point that the S_{12} spin forces do serve to decrease the neutron-proton scattering cross section. For simplicity, we shall consider the case of zero energy. It will be convenient to append the subscript (1) to all quantities associated with triplet states in the continuum; quantities associated with the ground state will remain unmarked. With this notation, the equation expressing the orthogonality of the triplet wave function of zero energy and the ground state wave function reads

$$\int_0^\infty (uu_1+ww_1)dr=0. \qquad (48)$$

Introducing the known forms of the S radial functions outside the range of interaction (Eqs. (7) and (46)), the orthogonality relation becomes

$$0=\int_0^{r_0}uu_1dr+\int_0^\infty ww_1dr+(uu_1)_{r=r_0}\times\frac{1}{\alpha}\left[1+\frac{1}{\alpha(r_0+a_1)}\right]. \qquad (49)$$

Within the range of interaction, the ground state and continuum wave functions differ but little. In addition, the behavior of the two functions at distances greater than r_0 is almost identical in the regions which contribute appreciably to the integral $\int_0^\infty ww_1dr$. It is therefore permissible to replace u_1 and w_1 in Eq. (41) by u and w. In terms of the quantity ϵ, defined by

$$\int_0^{r_0}u^2dr+\int_0^\infty w^2dr=\tfrac{1}{2}\epsilon r_0(u^2)_{r=r_0}, \qquad (50)$$

the formula obtained from (41) for the total scattering cross section, $4\pi a_1{}^2$, may be written

$$\frac{4\pi\hbar^2}{M|E_0|}\left(\frac{1}{1+\frac{1}{2}\epsilon\alpha r_0}+\alpha r_0\right)^2. \qquad (51)$$

In the absence of the spin forces under discussion, the value of ϵ is close to unity, for the S wave function u has the form of a sine wave which reaches its maximum amplitude at approximately r_0, and the D wave function w is identically zero. Upon including these forces in the neutron-proton interaction, w assumes finite values while u, as a numerical investigation shows, still preserves the general form of a sine wave. As a consequence, ϵ will become larger than one, resulting in a decrease of the scattering cross section since (43) is a monotonically decreasing function of ϵ. This demonstrates that the spin forces of type S_{12} are capable of at least a partial explanation of the experimental data.

The result thus obtained is supported by numerical calculations of the scattering cross section utilizing the simplified potential we have adopted for analytical convenience. The calculated cross section at zero energy: $\sigma_{\text{triplet}}(E=0) = 4.21\times 10^{-24}$ cm², differs from that obtained by the usual potential well calculation, 4.30×10^{-24} cm², by only two percent. This small decrease is, of course, attributable to the quite small magnitude of $\int_0^\infty w^2 dr$. The depth of the singlet potential well, and therefore the quantity g, is obtained by requiring that the cross section for scattering of slow neutrons in hydrogen

$$\sigma_0 = \tfrac{3}{4}\sigma_{\text{triplet}}(E=0) + \tfrac{1}{4}\sigma_{\text{singlet}}(E=0) \qquad (52)$$

assume[9] the value 20×10^{-24} cm². The measure of the relative strength of the spin exchange interaction, g, thus calculated, is $g=0.0715$. It is of some interest to note that had the range been chosen four percent smaller, i.e., $r_0 = 2.7\times 10^{-13}$ cm, the value of g would have been zero. That is, with this choice of range, the data may be represented by an interaction operator composed of an ordinary and a spin-spin interaction term, with no spin-exchange interaction. The difference in singlet and triplet potential energies then arises entirely from the spin dependence embodied in S_{12}. A neutron-proton force of this character bears a suggestive resemblance to that of the symmetrical pseudo-scalar mesotron theory.

Neutron-proton scattering experiments employing the mono-energetic neutrons available from the D–D reaction have been performed in the energy interval 2.5–3.0 Mev. The most accurate experiments in this region, those of Aoki[8] and Zinn, Seely and Cohen,[7] are in fair accord and give $(2.40\pm 0.10)\times 10^{-24}$ cm² at the energy 2.82 Mev, adopted for theoretical calculation. The theoretical cross section is sensitive to the range of forces which, consistent with the fundamental concept of charge independence of the forces, has been chosen as $r_0 = 2.80\times 10^{-13}$ cm, the value derived from the analysis of proton-proton scattering experiments.[10] The ordinary potential well model predicts a total cross section $\sigma(2.82 \text{ Mev}) = 2.56\times 10^{-24}$ cm². The computations with the spin-spin interaction, employing the methods outlined earlier in this section, result in the following values for the phases:

$$\kappa_0^{(0)} = -0.9454, \quad \kappa_2^{(1)} = 0.00812, \quad \zeta_0^{(0)} = -0.01144. \qquad (53)$$

In the actual calculations, no use was made of the connecting relations between the phases, which were reserved for use as checks of the final numbers, with completely satisfactory results. Inserted in the formula (45), these phase constants imply a triplet scattering cross section $\sigma_{\text{triplet}} = 2.403\times 10^{-24}$ cm². Suitably averaged with the singlet cross section, $\sigma_{\text{singlet}} = 2.910\times 10^{-24}$ cm², there ensues the total neutron-proton cross section, $\sigma(2.82 \text{ Mev}) = 2.53\times 10^{-24}$ cm². Again we have succeeded in obtaining only a two-percent reduction in the triplet cross section.

It is difficult to decide whether a definite discrepancy exists. Although the theoretical value lies but barely higher than the upper limit of the experimental result, the experimental observations have been invariably less than the theoretical prediction appropriate to the range derived from proton scattering. It must be remembered, however, that successive experiments have shown a tendency to yield progressively higher values. If the present experimental magnitude of the cross section be accepted, the range in the triplet state must be reduced to 2.3×10^{-13} cm, thoroughly violating the postulate of charge independence of the forces.

[9] V. W. Cohen, H. H. Goldsmith, J. Schwinger, Phys. Rev. **55**, 106 (1939); H. B. Hanstein, Phys. Rev. **57**, 1045 (1940).

[10] G. Breit, H. M. Thaxton and L. Eisenbud, Phys. Rev. **55**, 1018 (1939).

It is not without interest to examine the nonisotropic angular distribution of the scattering demanded by the S_{12} interaction. The angular distributions in the separate magnetic sub-states are appreciably nonspherically symmetrical:

$$\sigma_{|m|=1}(\vartheta)=(1-0.0263\cos^2\vartheta)\times(2.381/4\pi)\times10^{-24}\text{ cm}^2,$$
$$\sigma_{m=0}(\vartheta)=(1+0.0559\cos^2\vartheta)\times(2.444/4\pi)\times10^{-24}\text{ cm}^2, \tag{54}$$

although the triplet angular cross section for unpolarized beams:

$$\sigma_{\text{triplet}}(\vartheta)=(1+0.0017\cos^2\vartheta)\times(2.402/4\pi)\times10^{-24}\text{ cm}^2 \tag{55}$$

exhibits but negligible deviations from isotropic scattering.

Radiative Capture of Slow Neutrons

The theory of radiative transitions between the discrete and continuum states of the neutron-proton system is of added interest in view of the multiplicity of processes which become possible when the general interaction (3) is employed. The simplest situation realized is that of the radiative capture of slow neutrons in hydrogen, for only continuum S states are involved. In addition to the usual magnetic dipole capture from the singlet state, ${}^1S_0\rightarrow{}^3S_1$, we now have the possibilities of magnetic dipole transitions from the triplet state; i.e., ${}^3S_1\rightarrow{}^3S_1$, ${}^3D_1\rightarrow{}^3D_1$, and electric quadripole transitions from the triplet state; ${}^3S_1\rightarrow{}^3D_1$, ${}^3D_1\rightarrow{}^3S_1$, ${}^3D_1\rightarrow{}^3D_1$. Moreover the conventional treatment of the magnetic dipole capture from the singlet state as modified for the ${}^3S_1$ wave function now constitutes only part of the ground state wave function, thus reducing the capture probability.

The probability per unit time of a magnetic dipole transition from an initial state Ψ_i to a final state Ψ_f, with emission of a light quantum of energy $\hbar\omega$ in the solid angle $d\Omega$ about the direction of the unit vector $\boldsymbol{\kappa}$, is given by the well-known formula:

$$(e^2/\hbar c)(\hbar\omega/Mc^2)^2\omega\times|(\Psi_f,\boldsymbol{\kappa}\times\mathbf{e}\cdot\mathbf{M}\Psi_i)|^2(d\Omega/8\pi). \tag{56}$$

Here $\mathbf{M}$ denotes the operator of the magnetic moment in nuclear magnetons, and $\mathbf{e}$ represents a unit vector in the direction of polarization of the light quantum. To calculate the total transition probability between degenerate states, this expression must be averaged with respect to the magnetic quantum number m of the initial state and summed over all magnetic quantum numbers m' of the final state.

Slow neutron capture from the singlet state occurs between the nondegenerate zero energy state:

$$\Psi_i=[u_0(r)/r]\chi_0{}^0,\quad u_0(r)\sim r+a_0 \tag{57}$$

and the triply degenerate deuteron ground state:

$$\Psi_f=(4\pi)^{-\frac{1}{2}}\{(u/r)+2^{-\frac{3}{2}}S_{12}(w/r)\}\chi_1{}^{m'}. \tag{2'}$$

To evaluate the required matrix elements we note that, of the three terms contained in the expression (16) for $\mathbf{M}$, only the third contributes to transitions between states of different total spin. Therefore,

$$(\Psi_f,\boldsymbol{\kappa}\times\mathbf{e}\cdot\mathbf{M}\Psi_i)=(\mu_n-\mu_p)(4\pi)^{\frac{1}{2}}\int_0^\infty uu_0dr(\boldsymbol{\kappa}\times\mathbf{e})\cdot(\chi_1{}^{m'},\tfrac{1}{2}(\boldsymbol{\sigma}_n-\boldsymbol{\sigma}_p)\chi_0{}^0). \tag{58}$$

The summation over the three final triplet states may be extended over all states, since the diagonal matrix element of $\boldsymbol{\sigma}_n-\boldsymbol{\sigma}_p$ vanishes in the singlet state. Hence

$$\sum_{m'}|\boldsymbol{\kappa}\times\mathbf{e}\cdot(\chi_1{}^{m'},\tfrac{1}{2}(\boldsymbol{\sigma}_n-\boldsymbol{\sigma}_p)\chi_0{}^0)|^2=(\chi_0{}^0,(\boldsymbol{\kappa}\times\mathbf{e}\cdot\tfrac{1}{2}(\boldsymbol{\sigma}_n-\boldsymbol{\sigma}_p))^2\chi_0{}^0)=(\chi_0{}^0,(\boldsymbol{\kappa}\times\mathbf{e}\cdot\boldsymbol{\sigma}_n)^2\chi_0{}^0)=1 \tag{59}$$

by the completeness relation and the fact that $\boldsymbol{\kappa}\times\mathbf{e}$ is a unit vector. This is an explicit demonstration that the emitted radiation is isotropic and unpolarized. By summing over the two independent polarization directions, and integrating (56) over all emission directions, we obtain the total capture probability:

$$w_{\text{mag dip}}^{(0\rightarrow1)}=4\pi\frac{e^2}{\hbar c}(\mu_n-\mu_p)^2\left(\frac{|E_0|}{Mc^2}\right)^2\frac{|E_0|}{\hbar}\left(\int_0^\infty u_0udr\right)^2, \tag{60}$$

introducing $|E_0|$, the binding energy of the deuteron, for the energy of the light quantum. The superscript, denoting the spin transition, distinguishes this type of magnetic dipole capture from that involved in capture from the triplet state. The cross section for the process is obtained from the transition probability through division by the incident neutron current density. Inasmuch as the wave function (57) of the initial state is normalized to unit particle density, the current density in the singlet state is numerically equal to the neutron velocity $(2E/M)^{\frac{1}{2}}$, where E denotes the neutron energy. Therefore the total neutron current density equals $4(2E/M)^{\frac{1}{2}}$. The capture cross section thereby obtained may be written

$$\sigma_{\text{mag dip}}^{(0\to 1)}=\pi\frac{e^2}{\hbar c}(\mu_n-\mu_p)^2\left(\frac{|E_0|}{2E}\right)^{\frac{1}{2}}\left(\frac{|E_0|}{Mc^2}\right)^2\alpha\left(\int_0^\infty u_0u\,dr\right)^2, \tag{61}$$

where α is defined in (7).

The matrix element of the magnetic moment operator between the initial state:

$$\Psi_i=\left\{\frac{u_1}{r}+2^{-\frac{3}{2}}S_{12}\frac{w_1}{r}\right\}\chi_1^m,\quad u_1(r)\sim r+a_1 \tag{62}$$

and the final state (2′) determines the probability of magnetic dipole capture from the triplet state. In computing this matrix element the operator $\mathbf{M}$ may be replaced by (17), the form utilized in the evaluation of the deuteron moment. It should be noted that the term $(\mu_n+\mu_p)\mathbf{J}$ contained therein gives no contribution to nondiagonal matrix elements. Therefore

$$(\Psi_f,\,\boldsymbol{\kappa}\times\mathbf{e}\cdot\mathbf{M}\Psi_i)=-\tfrac{3}{2}(4\pi)^{-\frac{1}{2}}(\mu_n+\mu_p-\tfrac{1}{2})\int_0^\infty ww_1dr\,\boldsymbol{\kappa}\times\mathbf{e}\cdot(\chi_1^{m'},\,\tfrac{1}{2}(\boldsymbol{\sigma}_n+\boldsymbol{\sigma}_p)\chi_1^m). \tag{63}$$

As before, the summation over the three final triplet states may be extended over all states. The justification in this situation proceeds from the diagonal nature of $\frac{1}{2}(\boldsymbol{\sigma}_n+\boldsymbol{\sigma}_p)\equiv\mathbf{S}$. Hence,

$$\sum_{m'}|(\boldsymbol{\kappa}\times\mathbf{e})\cdot(\chi_1^{m'},\,\tfrac{1}{2}(\boldsymbol{\sigma}_n+\boldsymbol{\sigma}_p)\chi_1^m)|^2=(\chi_1^m,\,(\boldsymbol{\kappa}\times\mathbf{e}\cdot\tfrac{1}{2}(\boldsymbol{\sigma}_n+\boldsymbol{\sigma}_p))^2\chi_1^m),$$

which has still to be averaged over the three initial triplet states. We may again use the artifice of extending the summation over all states. The result is a diagonal sum of $(\boldsymbol{\kappa}\times\mathbf{e}\cdot\mathbf{S})^2$; i.e.,

$$\tfrac{1}{3}\sum_{mm'}|(\boldsymbol{\kappa}\times\mathbf{e})\cdot(\chi_1^{m'},\,\tfrac{1}{2}(\boldsymbol{\sigma}_n+\boldsymbol{\sigma}_p)\chi_1^m)|^2=\tfrac{1}{3}Sp(\boldsymbol{\kappa}\times\mathbf{e}\cdot\tfrac{1}{2}(\boldsymbol{\sigma}_n+\boldsymbol{\sigma}_p))^2=\tfrac{2}{3}. \tag{64}$$

We obtain the total capture probability by the same discussion presented for singlet state capture:

$$w_{\text{mag dip}}^{(1\to 1)}=6\pi\frac{e^2}{\hbar c}(\mu_n+\mu_p-\tfrac{1}{2})^2\left(\frac{|E_0|}{Mc^2}\right)^2\frac{|E_0|}{\hbar}\left(\int_0^\infty ww_1dr\right)^2. \tag{65}$$

The total neutron current density corresponding to the triplet wave function (62) is obviously $(4/3)(2E/M)^{\frac{1}{2}}$. Upon performing the required division, we obtain the following formula for the cross section corresponding to magnetic dipole capture from the triplet state:

$$\sigma_{\text{mag dip}}^{(1\to 1)}=\frac{9\pi}{2}\frac{e^2}{\hbar c}(\mu_n+\mu_p-\tfrac{1}{2})^2\left(\frac{|E_0|}{2E}\right)^{\frac{1}{2}}\left(\frac{|E_0|}{Mc^2}\right)^2\alpha\left(\int_0^\infty ww_1dr\right)^2. \tag{66}$$

The third mechanism effective in the radiative capture of slow neutrons is the electric quadripole capture from the triplet state. The probability of an electric quadripole transition between the initial state (62) and the final state (2′) is represented by

$$(e^2/\hbar c)(\hbar\omega/Mc^2)^4\omega(\hbar/Mc)^{-4}|(\Psi_f,\,\tfrac{1}{4}\mathbf{e}\cdot\mathbf{r}\boldsymbol{\kappa}\cdot\mathbf{r}\Psi_i)|^2d\Omega/8\pi. \tag{67}$$

The required matrix element is found, by integrating over angles, to be

$$(\Psi_f,\ \tfrac{1}{4}\mathbf{e}\cdot\mathbf{r}\boldsymbol{\kappa}\cdot\mathbf{r}\Psi_i)=\frac{(2\pi)^{\frac{1}{2}}}{40}\int_0^\infty r^2(uw_1+wu_1-2^{-\frac{1}{2}}ww_1)dr(\chi_1^{m'},\ (\mathbf{e}\cdot\boldsymbol{\sigma}_1\boldsymbol{\kappa}\cdot\boldsymbol{\sigma}_2+\boldsymbol{\kappa}\cdot\boldsymbol{\sigma}_1\mathbf{e}\cdot\boldsymbol{\sigma}_2)\chi_1^{m}). \quad (68)$$

By proceeding as before, we obtain the total quadripole capture probability:

$$w_{\mathrm{el\ quad}}^{(1\to 1)}=\frac{\pi}{300}\frac{e^2}{\hbar c}\left(\frac{|E_0|}{Mc^2}\right)^2\frac{|E_0|}{\hbar}\alpha^4\left(\int_0^\infty r^2(uw_1+wu_1-2^{-\frac{1}{2}}ww_1)dr\right)^2. \quad (69)$$

The cross section for electric quadripole capture from the triplet state is therefore

$$\sigma_{\mathrm{el\ quad}}^{(1\to 1)}=\frac{\pi}{400}\frac{e^2}{\hbar c}\left(\frac{|E_0|}{2E}\right)^{\frac{1}{2}}\left(\frac{|E_0|}{Mc^2}\right)^2\alpha^5\left(\int_0^\infty r^2(uw_1+wu_1-2^{-\frac{1}{2}}ww_1)dr\right)^2. \quad (70)$$

The theoretical cross section for the magnetic dipole capture of thermal neutrons ($E=kT=0.025$ ev) from the singlet state, predicted by the customary theory, is $\sigma_c=0.312\times 10^{-24}$ cm^2. The modification embodied in our formula (61) reduces this value to $\sigma_c=0.302\times 10^{-24}$ cm^2, which is approximately the reduction to be expected from the four-percent probability of the $^3D_1$ state. The two additional modes of radiative capture discussed in this section, magnetic dipole and electric quadripole capture from the triplet states, have but a negligible influence on the total capture cross section. Indeed, even if the very existence of these transitions did not depend on the small fraction of $^3D_1$ state adjoined to the $^3S_1$ state, transitions from the triplet state could hardly be expected to compete successfully with a transition from the singlet state, for a neutron-proton system in the singlet state is in approximate resonance at small energies.

The radiative capture of slow neutrons by protons has been extensively investigated.[11] Although the results of the various experiments are not completely harmonious, those of apparently greater accuracy agree in obtaining a cross section equal to $\sigma_c=(0.27\pm 0.02)\times 10^{-24}$ cm^2. This value supposedly refers to a thermal neutron energy $E=kT$, but in actuality, the principal error in these measurements arises from the necessity of defining an effective energy by averaging over the uncertain energy spectrum of the thermal neutrons. The experiments are therefore insufficiently accurate to make the comparison with the theoretical value (0.30×10^{-24} cm^2) unsatisfactory. It should be stressed that even this measure of accord has been achieved only by employing the value of 20×10^{-24} cm^2 for the neutron-proton scattering cross section. The capture cross section is quite accurately proportional to the scattering cross section, and is effectively independent of the range of the forces. Thus, had the scattering cross section obtained by some investigators[12] ($\sim 14\times 10^{-24}$ cm^2) been used the capture cross section would be $\sigma_c\sim 0.22\times 10^{-24}$ cm^2 which is in apparent disagreement with the experimental measurement, particularly since the assumption of thermal equilibrium, used in defining the effective neutron energy, tends to underestimate the experimental capture cross section.

Photo-Disintegration of the Deuteron

Transitions from the ground state of the deuteron to the dissociated continuum states, induced by γ-ray absorption, may proceed by essentially four types of radiative processes: electric dipole, magnetic dipole and electric quadripole transitions to triplet states, and magnetic dipole transitions to singlet states. We shall confine our attention to low energy γ-rays, and therefore we shall neglect the magnetic dipole and electric quadripole transitions to the triplet state. Further, we shall adopt the

[11] E. Amaldi and E. Fermi, Phys. Rev. **50**, 899 (1936); C. H. Westcott, Proc. Camb. Phil. Soc. **33**, 122 (1937); O. R. Frisch, H. von Halban, Jr. and J. Koch, Kgl. Danske Vid. Sels. Math.-fys. Medd. **15**, 10 (1938); A. H. Spees, W. F. Colby and S. Goudsmit, Phys. Rev. **53**, 326 (1938).

[12] L. Simons, Kgl. Danske Vid. Sels. Math.-fys. Medd. **17**, 7 (1940); E. Amaldi, D. Bocciarelli, G. C. Trabacchi, Ricerca Scient. **11**, 121 (1940).

usual approximation which disregards the effect of nuclear forces on the ^3P states arising from electric dipole absorption.

Photoelectric absorption corresponds to a transition from the initial state described by the wave function (2), Ψ_i, to the P part of the plane wave $\exp[i\mathbf{k}\cdot\mathbf{r}]\chi_1^{m'}$, i.e.,

$$\Psi_f = 3i\left(\frac{\mathbf{k}\cdot\mathbf{r}}{kr}\right)\frac{g_1(kr)}{kr}\chi_1^{m'}, \quad \left(\frac{\hbar^2k^2}{M} = \hbar\omega - |E_0|\right). \tag{71}$$

The cross section for the absorption of a light quantum of energy $\hbar\omega$ described by the unit vectors $\mathbf{e}$, $\boldsymbol{\kappa}$, with the emission of the disintegration products into the solid angle $d\Omega$ about the propagation vector $\mathbf{k}$, is represented by:

$$\frac{e^2}{\hbar c}\frac{Mk}{\hbar^2}\hbar\omega\tfrac{1}{3}\sum_{m,m'}|(\Psi_f, \tfrac{1}{2}\mathbf{e}\cdot\mathbf{r}\Psi_i)|^2\frac{d\Omega}{4\pi}. \tag{72}$$

The required matrix element, simplified by performing the angle integrations, is composed of the terms describing the transitions $^3S_1 \rightarrow {}^3P$, $^3D_1 \rightarrow {}^3P$:

$$(\Psi_f, \tfrac{1}{2}\mathbf{e}\cdot\mathbf{r}\Psi_i) = -i\frac{\pi^{\frac{1}{2}}}{k}\left(\chi_1^{m'}, \left\{\frac{\mathbf{e}\cdot\mathbf{k}}{k}\int_0^\infty rg_1udr + \frac{2^{-\frac{1}{2}}}{5}\left(\frac{3}{2}\boldsymbol{\sigma}_1\cdot\frac{\mathbf{k}}{k}\boldsymbol{\sigma}_2\cdot\mathbf{e} + \frac{3}{2}\boldsymbol{\sigma}_1\cdot\mathbf{e}\boldsymbol{\sigma}_2\cdot\frac{\mathbf{k}}{k} - \frac{\mathbf{e}\cdot\mathbf{k}}{k}\right)\int_0^\infty rg_1wdr\right\}\chi_1^m\right). \tag{73}$$

The technique of averaging over the magnetic sub-states, described in the previous section, permits a simple derivation of the result:

$$\tfrac{1}{3}\sum_{m,m'}|(\Psi_f, \tfrac{1}{2}\mathbf{e}\cdot\mathbf{r}\Psi_i)|^2 = \frac{\pi}{k^2}\left\{\cos^2\Theta\left(\int_0^\infty rg_1udr\right)^2 + \frac{1}{25}(3+\cos^2\Theta)\left(\int_0^\infty rg_1wdr\right)^2\right\}. \tag{74}$$

Here $\cos\Theta = \mathbf{e}\cdot\mathbf{k}/k$, with Θ thus representing the angle at which the emitted particles emerge with respect to the polarization vector of the light quantum. The experimental situation pertains to an unpolarized beam of γ-rays, which necessitates the replacement of $\cos^2\Theta$ by its average over all directions of $\mathbf{e}$ perpendicular to $\boldsymbol{\kappa}$; *viz.*, $\frac{1}{2}\sin^2\vartheta$, where ϑ denotes the angle of emergence of the disintegration products with respect to the direction $\boldsymbol{\kappa}$ of the γ-ray. The cross section for electric dipole absorption, with emission into the solid angle $d\Omega$ is thus:

$$\frac{\pi}{2}\frac{e^2}{\hbar c}\frac{M\omega}{\hbar k}\left\{\sin^2\vartheta\left(\int_0^\infty rg_1udr\right)^2 + \frac{1}{25}(6+\sin^2\vartheta)\left(\int_0^\infty rg_1wdr\right)^2\frac{d\Omega}{4\pi}.\right. \tag{75}$$

It is noteworthy that the spin-spin forces demand a spherically symmetric term in addition to the usual $\sin^2\vartheta$ photoelectric angular distribution. The total photoelectric cross section is then

$$\sigma_{\text{el dip}} = \frac{\pi}{3}\frac{e^2}{\hbar c}\frac{M\omega}{\hbar k}\left\{\left(\int_0^\infty rg_1udr\right)^2 + \frac{2}{5}\left(\int_0^\infty rg_1wdr\right)^2\right\}. \tag{76}$$

The permissible transitions induced by magnetic dipole absorption to the singlet state are: $^3S_1 \rightarrow {}^1S_0$, $^3D_1 \rightarrow {}^1D_2$. The latter transition would be of little interest were it not that it interferes with the transition to the $^1S_0$ state and therefore modifies the angular distribution of the particles. The cross section for magnetic dipole absorption is obtained from (72) by replacing the component of the electric moment in the direction of the electric polarization vector, i.e., $(e/2)\mathbf{e}\cdot\mathbf{r}$ by its magnetic analog, *viz.*, $(e\hbar/2Mc)\boldsymbol{\kappa}\times\mathbf{e}\cdot\mathbf{M}$. Therefore,

$$\tfrac{1}{4}(e^2/\hbar c)(\hbar\omega/Mc^2)k\tfrac{1}{3}\sum_{m'}|(\Psi_f, \boldsymbol{\kappa}\times\mathbf{e}\cdot\mathbf{M}\Psi_i)|^2(d\Omega/4\pi) \tag{77}$$

describes the cross section for a magnetic dipole transition to the nondegenerate final state,

$$\Psi_f=\left\{e^{-i\delta_0}\frac{u_0(r)}{kr}-\frac{5}{2}\left(3\left(\frac{\mathbf{k}\cdot\mathbf{r}}{kr}\right)^2-1\right)\frac{g_2(kr)}{kr}\right\}\chi_0{}^0,\quad u_0(r)\sim\sin\,(kr+\delta_0). \tag{78}$$

Here we have written only the perturbed $^1S_0$ continuum state[13] and the ^1D portion of the plane wave $\exp\,[i\mathbf{k}\cdot\mathbf{r}]\chi_0{}^0$. Upon performing the summation over the magnetic quantum number m of the initial state, and averaging the result with respect to the polarization vector of the light quantum, one obtains the formulae:

$$\frac{\pi}{3}\frac{e^2}{\hbar c}(\mu_n-\mu_p)^2\frac{\hbar\omega}{Mc^2}\frac{1}{k}\left\{\left(\int_0^\infty uu_0dr\right)^2-2^{-\frac{1}{2}}\cos\delta_0\left(\int_0^\infty uu_0dr\right)\left(\int_0^\infty g_2wdr\right)(3\cos^2\vartheta-1)\right.$$
$$\left.+\frac{1}{4}(5-3\cos^2\vartheta)\left(\int_0^\infty g_2wdr\right)^2\right\}\frac{d\Omega}{4\pi}, \tag{79}$$

$$\sigma_{\text{mag dip}}=\frac{\pi}{3}\frac{e^2}{\hbar c}(\mu_n-\mu_p)^2\frac{\hbar\omega}{Mc^2}\frac{1}{k}\left\{\left(\int_0^\infty u_0udr\right)^2+\left(\int_0^\infty g_2wdr\right)^2\right\}, \tag{80}$$

which represent, respectively, the differential and total cross sections for the photomagnetic transition to the singlet state.

Numerical calculations have been performed for $\hbar\omega=2.62$ Mev, the strong γ-ray line of Th C″. The kinetic energy of the disintegration products is therefore 0.45 Mev. At this small energy the functions g_1 and g_2 are minute within the region in which w is appreciably different from zero, and thus no significant contribution is to be expected from the transitions initiating in the $^3D_1$ state. Indeed, the principal modification is to reduce the cross sections for these two processes proportionately to the reduction in the integral $\int_0^\infty u^2dr$, the S state probability. The total cross section for the photoelectric process is found to be $\sigma_{\text{el dip}}=11.99\times 10^{-28}$ cm^2, which is to be compared with 12.31×10^{-28} cm^2, computed from a simple rectangular well. The angular distribution represented in (75) differs but slightly from a $\sin^2\vartheta$ distribution: $\sin^2\vartheta+0.0007$. However, we shall show in the sequel that consideration of the interaction in P states, combined with large γ-ray energies, produces an appreciable relative intensity in the forward direction. The total photomagnetic cross section is $\sigma_{\text{mag dip}}=3.28\times 10^{-28}$ cm^2, while the angular distribution: $1-0.0035\cos^2\vartheta$ is slightly altered from spherical symmetry by the interference with the ^1D state. The latter effect increases rapidly with energy and at the higher energies considered in the subsequent paper produces a significant reduction in the forward intensity of the photomagnetic particles.

The quantities to be compared with experiment are the total cross section: $\sigma_{\text{photo}}=15.27\times 10^{-28}$ cm^2, and the net angular distribution: $(\sin^2\vartheta+0.182)$. The experiments of von Halban[14] reveal a most disturbing contradiction with this theory. The measured total cross section is only $(10\pm 0.8)\times 10^{-28}$ cm^2, while the intensity of the photo-neutrons ejected in the forward direction is at most 5 percent of that at right angles, in contrast with the theoretical expectation of 15 percent. Experiments of Chadwick, Feather and Bretscher[15] have provided confirmatory evidence by showing that a similar situation exists for the angular distribution of the photo-protons. These discrepancies could be removed if the photomagnetic cross section were much smaller than current theories predict. One point which should be mentioned is the weak 3.2-Mev γ-ray line which according to Ellis,[16] accompanies the main 2.6-Mev line of Th C″. This fact, however,

[13] The phase factor $e^{-i\delta_0}$ is introduced to make the asymptotic form of Ψ_f correspond to a plane wave and a *converging* spherical wave. Cf. N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Oxford University Press, 1933), p. 258.

[14] H. von Halban, Jr., Comptes rendus **206**, 1170 (1938).

[15] J. Chadwick, N. Feather and E. Bretscher, Proc. Roy. Soc. **A163**, 366 (1937).

[16] C. D. Ellis, Proc. Phys. Soc. **50**, 213 (1938).

provides but little solace, for to explain the angular distribution the 3.2-Mev line would have to be the principal component of the spectrum. This is a rather serious situation since the photo-disintegration process is simply the inverse of the magnetic capture process, which is in good accord with experiment. The only evident explanation is that there exists a further contribution to the magnetic moment operator arising from mesotron exchange currents. However, it is not clear why the small energy difference between the two continuum states involved in capture and photo-disintegration (450 kev) should have such a marked effect.

The general conclusion to be drawn from the preceding sections is that a satisfactory phenomenological theory of the neutron-proton system can be developed, with the exception of the magnetic photo-disintegration process, where for the first time we meet a phenomenon whose explanation apparently demands a detailed application of a field theory.

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X-Ray Diffraction Maxima at Other Than Bragg Angles

PAUL KIRKPATRICK
Stanford University, California

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Photographic observations of molybdenum $K\alpha$ lines internally reflected from (111) planes of diamond show that the angle of deviation is not constant with angle of incidence but varies in qualitative agreement with Zachariasen's theory of diffuse scattering and with observations of Raman and Nilakantan. Observations of the variation of line intensity and width with angle of incidence do not agree well with the theory in the cases observed. Spectrometer observations of reflections from calcite with slightly ground and with untreated cleavage surfaces upon which monochromatic radiation was incident at angles differing from the Bragg angle show detectable reflection over an 11° range of angles of incidence in the former case but only 2° in the latter. The large difference between these ranges indicates that disordered crystal particles are the principal source of reflected intensity at other than Bragg angles with the predominant crystal planes in the former case. The presumption is strong that this source has operated as a partial or complete explanation of some of the reflections hitherto reported as anomalous.

THE fact that x-rays may diffract from crystals in directions not precisely assignable to Bragg planes has been repeatedly discovered during the past thirty years. A frequent experience has been the observation of unexpected spots or radial streaks on Laue-spot photographs located close to, and presumably causally associated with, the ordinary and more readily explicable ones.[1]

The effects are usually such as to suggest that the reflecting planes, or some of them, possess in addition to the usual reflecting or diffracting characteristics the ability to reflect incident radiation in directions such that the angles of incidence and reflection are not precisely equal or that the latter angle is not limited to a single well-defined value. Such characteristics might result, it has been or may be suggested, from wavy "planes" having reflecting areas not sharing the general inclination, from mosaic blocks slightly askew, or from transient disorder accompanying thermal vibration. The latter suggestion, advanced in general terms of Faxén,[2]

[1] Many references are given by I. E. Knaggs, K. Lonsdale, A. Muller, A. R. Ubbelohde, Nature **145**, 820 (1940).

[2] H. Faxén, Zeits. f. Physik **17**, 226 (1923).