Radiative X-Ray Transitions Within the L Shell

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Whereas N–N and M–M lines have been observed in the x-ray spectra of several elements, L–L lines have not been identified. However, isolated lines of sodium (375A), magnesium (317A) and aluminum (290A) reported by Skinner are here designated as L_I–$L_{II,III}$. Thus the L_I term in these elements may now be evaluated, and reliable predictions of L_I may be made for the elements silicon to chromium.

THE x-ray lines most easily observed are those for which the initial and final states differ in the quantum number n. Many transitions have been observed, however, in which $\Delta n=0$. Thus Magnusson[1] has identified as N–N transitions the lines of several elements in the ultra-soft region, and Siegbahn and Magnusson[2] cite several M–M lines. Transitions within the L shell have apparently not been reported, although two are allowed: L_I–L_{II} and L_I–L_{III}.

Among the competing processes whereby a missing L_I electron may be replaced, the radiative L–L transitions should be very weak in heavy elements, since the energy released by L_I–M is so much greater than that released by L_I–$L_{II,III}$, that an L_I ionization is almost always followed by the former process instead of the latter. In light elements, however, this inequality is less emphatic. We are therefore led to seek the L–L radiation in the spectra of the lightest elements having completed L shells.

The wave-length of such transitions can be predicted with the aid of the screening-doublet law. This law, as relating the term values T of L_I and L_{II}, may be written either in the form

$$(T_{LI}/R)^{\frac{1}{2}}-(T_{LII}/R)^{\frac{1}{2}}=\text{constant} \qquad (1)$$

or in the form

$$(T_{LI}/R)-(T_{LII}/R)=\text{linear function of } Z. \qquad (2)$$

Table I illustrates the application of (1) and (2) to the prediction of the transition L_I–L_{II}. Here the known terms for elements $Z\geqslant 26$ are as evaluated by Siegbahn;[3] the remaining term values are based on the observations of Skinner.[4] Column 2 of Table I gives in parentheses predictions of L_I, made with the aid of column 4 (cf. Eq. (1)). Similarly, predictions in column 5 are based on column 7, where $\Delta T/R$ is estimated by graphical extrapolation (cf. Eq. (2)). Column 8 presents the final prediction of L_I, in which column 2 has been weighted twice as heavily as column 5. In column 9 is listed the wave-length of the L_I–L_{II} transition, predicted from columns 6 and 8.

Skinner,[4] in his exhaustive description of the L spectra of Na, Mg and Al, lists as unidentified one line for each of these elements. These lines are listed in the last column of Table I. The wave-length agreement is excellent, and strongly points to the identification of Skinner's lines as the L–L transitions. One of these lines appears on two of our own sodium spectrograms;[5] our value for the wave-length is 376.5A, the half-width at half-maximum being some 3A.

The line L_I–L_{III} should lie very close to L_I–L_{II}, and should be the brighter of the two. Indeed Skinner's lines are more correctly designated L_I–L_{III}.

With this identification one can now for the first time evaluate the level L_I of Na, Mg and Al. This evaluation is presented in Table II. The values of $\Delta(T/R)^{\frac{1}{2}}$ are also given, the parentheses indicating interpolated values for the elements $14\leqslant Z\leqslant 25$. The last column gives the L_I–L_{III} line as observed and rough predictions for the elements in which it has not been observed.

The width of the Na L_I–L_{III} transition implies

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[1] T. Magnusson, Zeits. f. Physik **79**, 161 (1932).

[2] M. Siegbahn and T. Magnusson, Zeits. f. Physik **88**, 559 (1934).

[3] M. Siegbahn, *Spektroskopie d. Röntgenstrahlen* (Springer, Berlin, 1931).

[4] H. W. B. Skinner, Phil. Trans. Roy. Soc. **239**, 95 (1940).

[5] W. M. Cady and D. H. Tomboulian, Phys. Rev. **59**, 381 (1941).

TABLE I. *Prediction of the line* L_I-L_{II} *for* Na, Mg *and* Al.

1	2	3	4	5	6	7	8	9	10
Z	$\left(\frac{T}{R}\right)^{\frac{1}{2}}(L_I)$	$\left(\frac{T}{R}\right)^{\frac{1}{2}}(L_{II})$	$\Delta\left(\frac{T}{R}\right)^{\frac{1}{2}}$	$\frac{T}{R}(L_I)$	$\frac{T}{R}(L_{II})$	$\Delta\frac{T}{R}$	$\frac{T}{R}(L_I)$	$\lambda(L_I-L_{II})$ CALC. (A)	λ OBS. (A)
50	18.09	17.46	0.63	327.4	304.9	22.5			
45	15.83	15.21	0.62	250.7	231.2	19.5			
40	13.66	13.04	0.62	186.6	170.0	16.6			
33	10.61	10.00	0.61	112.6	100.0	12.6			
30	9.40	8.78	0.62	88.4	77.1	11.3			
28	8.65	8.04	0.61	74.8	64.6	10.2			
26	7.91	7.29	0.62	62.5	53.2	9.3			
13	(2.92)	2.318	(0.60)	(9.0)	5.372	(3.6)	(8.68)	(275)	290
12	(2.51)	1.913	(0.60)	(6.9)	3.658	(3.2)	(6.50)	(321)	317
11	(2.10)	1.504*	(0.60)	(5.0)	2.263*	(2.7)	(4.61)	(388)	375

* Estimated from L_{III} by the spin doublet law.

TABLE II. *The L terms and L−L lines of light elements.*

Z	$\left(\frac{T}{R}\right)^{\frac{1}{2}}(L_I)$	$\left(\frac{T}{R}\right)^{\frac{1}{2}}(L_{II})$	$\Delta\left(\frac{T}{R}\right)^{\frac{1}{2}}$	$\frac{T}{R}(L_I)$	$\frac{T}{R}(L_{II})$	$\frac{T}{R}(L_{III})$	(A)
26 Fe	7.91	7.29	0.62	62.5	53.2	52.2	88†
25 Mn	(7.56)	6.95	(0.61)	(57.2)	48.3	47.4	(93)
24 Cr	(7.17)	6.56	(0.61)	(51.4)	43.0	42.3	(100)
23 V	(6.81)	6.20	(0.61)	(46.4)	38.5	37.9	(107)
22 Ti	(6.44)	5.83	(0.61)	(41.5)	34.0	33.6	(115)
21 Sc	(6.10)	5.50	(0.60)	(37.2)	30.3	30.0	(126)
20 Ca	(5.68)	5.08	(0.60)	(32.3)	25.8	25.5	(134)
19 K	(5.26)	4.66	(0.60)	(27.7)	21.7	21.5	(147)
17 Cl	(4.45)	3.86	(0.59)	(19.8)	14.9	14.8	(182)
16 S	(4.06)	3.480	(0.58)	(16.5)	12.11	12.02	(203)
15 P	(3.69)	3.112	(0.58)	(13.6)	9.68	9.60	(228)
14 Si	(3.31)	2.716	(0.59)	(11.0)	7.378	7.325	(248)
13 Al	2.913	2.318	0.595	8.485	5.372	5.343	290
12 Mg	2.552	1.913	0.639	6.513	3.658	3.638	317
11 Na	2.163	1.504*	0.659	4.678	2.263*	2.248	375

* Estimated from $LIII$ by the spin doublet law.
† Not observed.

a voltage spread of nearly one-half electron volt. Since L_{III} is known to be sharp, this width must inhere almost wholly in L_I.

An objection to the proposed identification of L_I–L_{III} may be raised: the value of $\Delta(T/R)^{\frac{1}{2}}$ rises suddenly at low Z if the identification is correct. This is, however, not surprising, for $\Delta(T/R)^{\frac{1}{2}}$ is proportional to the difference between two screening constants, and it is known that screening constants vary irregularly with Z in the region of low energies; the value of $\Delta(T/R)^{\frac{1}{2}}$ for the M_IM_{II} screening doublet, calculated for the elements[6] K to Co shows a somewhat similar behavior.

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[6] M. Siegbahn and T. Magnusson, Zeits. f. Physik **95**, 133 (1935).