the theory,[7] a lower limit to σ can be assigned by cutting off the frequency integral at $k_{0\,\max}=A$, some constant less than $2\hbar c/e^2$. There results

$$\sigma=\alpha\left(\frac{e^2}{\mu c^2}\right)^2 Z^2 d\epsilon\left\{\left(A+A\ln\frac{\pi E}{5A\mu c^2 Z^{\frac{1}{3}}}\right)\right.$$
$$\times\frac{(2-2\epsilon+7\epsilon^2)}{12}+\frac{\epsilon(34-34\epsilon+7\epsilon^2)}{24(1-\epsilon)}$$
$$\times\left[\ln^2\frac{2\pi E(1-\epsilon)}{5\mu c^2 Z^{\frac{1}{3}}\epsilon}-\ln^2\frac{\pi E}{5A\mu c^2 Z^{\frac{1}{3}}}\right]$$
$$+\left[\frac{16(1-\epsilon)}{3\epsilon}+\frac{13\epsilon}{12}-\frac{5\epsilon^3}{24(1-\epsilon)}\right]\ln\frac{2\pi E(1-\epsilon)}{5\mu c^2 Z^{\frac{1}{3}}\epsilon}$$
$$\left.-\frac{\epsilon(10-10\epsilon+3\epsilon^2)}{8(1-\epsilon)}-\frac{52}{9}\frac{(1-\epsilon)}{\epsilon}\right\} \quad (28)$$

for $E>(5/\pi)\mu c^2 Z^{\frac{1}{3}}A$. For $E<(5/\pi)\mu c^2 Z^{\frac{1}{3}}A$, we get (27) above, as with no cut-off.

A consideration of cosmic-ray bursts based on these and other calculations is given in another paper.

In conclusion, the authors wish to express their appreciation to Professor J. R. Oppenheimer for continued advice and encouragement.

[7] A rigorous treatment might show that the presence of high Fourier components diminishes the contribution from the low frequencies. We are here ignoring this possibility. See J. R. Oppenheimer, Phys. Rev. **47**, 44 (1935).

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Burst Production by Mesotrons

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Assuming that, under great absorbing thicknesses, cosmic-ray bursts are cascade showers from high energy soft secondaries produced in the shielding matter by mesotron-electron collisions and by mesotron bremsstrahlung, we have calculated the frequency of burst production as a function of burst size. For the mesotron of spin 1 and moment $e\hbar/2\mu c$, we have used the previously calculated knock-on formulae, supplemented by our own calculations of the bremsstrahlung; for the latter, the cross section has terms, significant for our work, in E, $\ln^2 E$, and $\ln E$. Up to energies close to 10^{11} ev, only slight modifications are introduced by omitting altogether those processes which cannot be treated by the Born approximation, and the *minimum* cross sections we used differ little from those given directly by the Born approximation. Using these cross sections, the cascade theory of showers, and a modified form of the Furry model to take into account the fluctuations, the frequency of burst production was calculated. The sea-level data of Schein and Gill give for the number of bursts of size greater than S, $N_S \backsim S^{-\gamma}$, with $\gamma=1.8$. Our calculations give for spin 1, $\gamma\sim 1.5$ and numerically too many by a factor of 20. Similar calculations for the mesotron of spin 0 give $\gamma\sim 1.8$ and the same in number as the observations within an uncertainty of about a factor 1.5. For spin $\frac{1}{2}$ and moment $e\hbar/2\mu c$, the bursts are approximately twice as numerous as for spin 0. This evidence thus favors spin 0, or possibly spin $\frac{1}{2}$, but tends to exclude spin 1.

I

Cosmic-ray bursts, insofar as they involve high energies of order 10^9–10^{11} ev, provide a feasible test of relativistic mesotron theory. Experiments have shown that the ionization in bursts does not show the characteristic high initial recombination of that due to slow heavy particles. Furthermore, bursts frequently appear simultaneously in ionization chambers one of which is above the other, and sometimes are larger in the lower chamber.[1] This appears to be conclusive evidence that at least the majority of bursts are not due to several slow heavy particles resulting from a nuclear explosion or evaporation but, rather, are due to many fast electrons resulting from the cascade multiplication of a high energy soft ray in the material above the chamber. Now the transition curves of Nie, and Steinke and Schmidt[2] for bursts in lead show a maximum at ~ 4 cm but no apparent decrease for thicknesses greater than 10 cm; bursts have also been observed at great depths underground.

[1] H. Nie, Zeits. f. Physik **99**, 776 (1936); H. Euler, Zeits. f. Physik **116**, 73 (1940).

[2] H. Nie, Zeits. f. Physik **99**, 453 (1936); E. C. Steinke and H. Schmidt, Zeits. f. Physik **115**, 740 (1940).

Since at these great thicknesses of absorber, and at great depths, the primary soft radiation is entirely absorbed out, the energetic soft rays which initiate bursts must be secondary to the mesotronic or penetrating component. Experiments on the size and frequency of large bursts (>100 particles) under thick absorbers can thus measure the cross sections for the production of energetic soft secondaries by mesotrons. Since these cross sections at high energies $\sim 10^{10}$ ev are markedly spin dependent, we see that accurate burst experiments can be used to decide between the various possible relativistic theories of the mesotron.

For these purposes the experiment must have a good (calculable) geometry, the absorber must be sufficiently thick to absorb out all primary soft radiation, and it must have statistically reliable data involving the energy range 10^{10}–10^{11} ev where spin effects become really prominent. In lead this means bursts of size 100 to 1000 particles. An examination of the experimental material on bursts shows that the sea-level data of Schein and Gill[3] alone satisfy these requirements.

In limiting the data to those experiments in which the cascade soft radiation from the air is ineffective in producing large bursts, we seriously restrict the useful experiments. This seems necessary since that part of the initial soft radiation which can be effective in making large bursts must arise from the degradation of primary rays with energy of order 10^{15}–10^{18} ev. Now the absorption coefficient in the cascade theory is critically dependent on the exact shape of the energy distribution curve: In this effective energy range both the number and energy distribution of the primaries are practically unknown. Thus it appears impossible at present to calculate to better than an order of magnitude the number of large bursts due to this source, and, in consequence, we must restrict ourselves to those experiments where it is clear that only mesotron secondaries are responsible for the bursts. At sea level, the necessary shielding matter must be the equivalent in shower units of 11 cm of lead; at high altitudes the absorber should be increased by the equivalent of 1.5 or 2 cm of lead per 1000 m elevation because of the rapid increase in the soft component with altitude.

[3] M. Schein and P. S. Gill, Rev. Mod. Phys. **11**, 267 (1939).

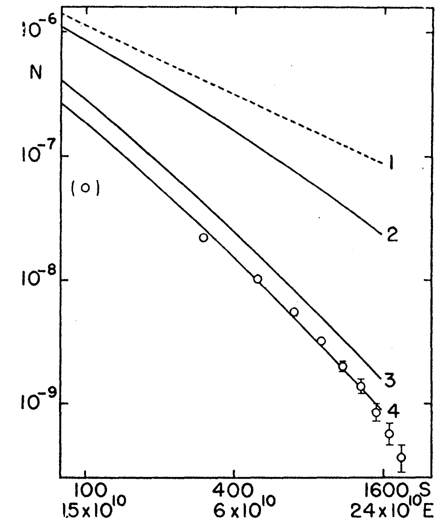

FIG. 1. Plot of burst frequency per sec. per cm² against minimum burst size given in terms of both the number of particles, S, and the energy, E, of the burst. Curve 1 is for spin 1 without cut-off, 2 for spin 1 with cut-off, 3 for spin $\frac{1}{2}$, and 4 for spin 0. The circles indicate the experimental points of Schein and Gill.

The apparatus of Schein and Gill, in the experiment to which we refer, consisted of a Carnegie model C cosmic-ray ionization meter. The spherical steel bomb forming the ionization chamber had walls 1.25 cm thick,[4] volume 19.3 liters, radius 18 cm, and contained argon at 50 atmospheres. Surrounding the ionization chamber was lead shot, the equivalent of 10.7 cm solid lead. A plot of their data, the frequency of bursts N_S containing more than S ionizing particles as a function of S from $S=100$ to 2000 appears in Fig. 1.[5]

[4] We are indebted to Dr. Schein for correcting this value.

[5] In plotting the experimental data we have departed slightly from the quoted burst sizes by considering the first group of size 1 actually to refer to size between 0.5 and 1.5 as stated. The experiments give the number of bursts between 100 and 300, then 300–500, etc., particles and in totaling to obtain the number of bursts of more than S particles we then have the points for $S=100$, 300, 500, etc., rather than for 200, 400, 600, etc., as given by Schein and Gill. This change is significant only for small S and results in flattening the curves near $S=100$. There is some reason to believe that some small bursts were missed because of the difficulty of distinguishing small bursts from background, and that the point for $S=100$ is somewhat low. In both the sea-level and high altitude results the point $S=100$ falls out of line with the curve from the remaining points.

II

Two distinct processes are important in the production of soft secondaries by mesotrons: the elastic collisions of mesotrons with atomic electrons (knock on), and the emission of γ-rays by mesotrons in the electric field of the nucleus (bremsstrahlung). The cross sections[6] for the former process are, in units of the *radiation* cross section $\bar{\varphi}=(e^2/\mu c^2)^2\alpha Z^2$, which is convenient for this work:

spin 0, moment 0

$$\sigma(E_0,\epsilon)=\frac{2\pi}{\alpha Z}\frac{\mu}{m}\frac{\mu c^2}{E_0}\frac{d\epsilon}{\epsilon^2}\left(1-\frac{\epsilon}{\epsilon_m}\right),$$

spin $\frac{1}{2}$, moment $e\hbar/2\mu c$

$$\sigma(E_0,\epsilon)=\frac{2\pi}{\alpha Z}\frac{\mu}{m}\frac{\mu c^2}{E_0}\frac{d\epsilon}{\epsilon^2}\left(1-\frac{\epsilon}{\epsilon_m}+\frac{\epsilon^2}{2}\right),$$

spin 1, moment $e\hbar/2\mu c$

$$\sigma(E_0,\epsilon)=\frac{2\pi}{\alpha Z}\frac{\mu}{m}\frac{\mu c^2}{E_0}\frac{d\epsilon}{\epsilon^2}\times\left[1-\frac{\epsilon}{\epsilon_m}+\frac{\epsilon^2}{3}+\frac{mE_0\epsilon}{3\mu^2c^2}\left(1-\frac{\epsilon}{\epsilon_m}+\frac{\epsilon^2}{2}\right)\right],$$

where $\epsilon_m\approx[1+(\mu^2c^2/2mE_0)]^{-1}$ is the maximum fractional energy transfer from a mesotron of energy E_0 to an electron, and μ is the mesotron mass. The bremsstrahlung cross sections,[7] in units $\bar{\varphi}$, for spin 0 and spin $\frac{1}{2}$ are

spin 0, moment 0

$$\sigma(E_0,\epsilon)=\frac{16}{3}\frac{(1-\epsilon)}{\epsilon}\left[\ln\frac{2(1-\epsilon)E_0}{(5/6)\mu c^2 Z^{\frac{1}{3}}\epsilon}-\frac{1}{2}\right],$$

spin $\frac{1}{2}$, moment $e\hbar/2\mu c$

$$\sigma(E_0,\epsilon)=\frac{16}{3}\left(\frac{3\epsilon}{4}+\frac{1-\epsilon}{\epsilon}\right)\left[\ln\frac{2(1-\epsilon)E_0}{(5/6)\mu c^2 Z^{\frac{1}{3}}\epsilon}-\frac{1}{2}\right].$$

The bremsstrahlung cross section for spin 1, moment $e\hbar/2\mu c$ has been evaluated by the authors in the previous paper. It is important to note, as was pointed out by Corben and Schwinger[6] that for the spins discussed above, the magnetic moment we associate with each is just that magnetic moment which gives *minimum* electromagnetic effects.

The bremsstrahlung cross sections depend on the mesotron mass as $1/\mu^2$ whereas, except for the high energy term for spin 1, the knock-on cross sections are essentially independent of μ. The appearance of $(5/6)\mu c^2 Z^{\frac{1}{3}}$ in the logarithm is an approximate expression for the lower limit of the impact parameter, the nuclear radius, and is really independent of μ. Experiments seem to indicate that μ lies between 150 and 200 electron masses; in our calculations we have used $\mu/m=177$ but we must expect some uncertainty, $\sim$20 percent in $1/\mu^2$ and in our results, from this source.

The bremsstrahlung cross sections for spins 0 and $\frac{1}{2}$ are probably correct up to energies such that $E_0\hbar/\mu^2c^3\approx\hbar^2Z^{-\frac{1}{3}}/me^2$ or $E_0\approx 5\times 10^{11}$ ev in lead, where the screening of the nuclear field by the atomic electrons becomes important. Above this energy the cross sections are essentially constant.

In contrast, the bremsstrahlung cross section for spin 1 is but little affected by the atomic screening since it derives its most important contribution from small impact parameters even at high energies. Oppenheimer, Snyder and Serber[8] have shown, however, that the perturbation treatment leading to (2) of the previous paper is open to serious doubt for $E_0>2\times 10^{10}$ ev because of the large coupling energies associated with the dominant high frequencies. Neglecting the possibility[9] that the presence of the very high frequencies might materially reduce the contribution from lower frequencies, we have obtained a *minimum* estimate of the cross section by eliminating the contribution of frequencies which cannot be correctly treated by perturbation methods. We have accordingly used the cross section (28) of the previous paper in which frequencies $>137\mu c^2$ have been eliminated and which includes additional terms, important

[6] H. S. W. Massey and H. C. Corben, Proc. Camb. Phil. Soc. **35**, 463 (1939); H. C. Corben and J. Schwinger, Phys. Rev. **58**, 953 (1940); H. J. Bhabha, Proc. Roy. Soc. **A164**, 257 (1938).

[7] The cross section for spin 0 was calculated by us. That for spin $\frac{1}{2}$ was obtained from W. Heitler, *The Quantum Theory of Radiation* (Oxford, 1936), p. 168, with the appropriate modification made for the finite size of the nucleus.

[8] J. R. Oppenheimer, H. Snyder and R. Serber, Phys. Rev. **57**, 75 (1940).

[9] J. R. Oppenheimer, Phys. Rev. **47**, 44 (1935).

for $E_0 \sim 10^{10}$ ev, in $\ln E_0$. The effect of this frequency cut off at $137\mu c^2$, which is one-half the limiting frequency suggested by Oppenheimer, Snyder and Serber, is to diminish the cross section for $E_0 > 8\times 10^{10}$ ev; the decrease is only of order 10 percent at 10^{11} ev. The use of this cross section diminishes the burst probability by ~ 20 percent for $S=100$ and by ~ 80 percent for $S=1000$.

III

The number of mesotrons at these high energies has been determined, for energies up to $\sim 10^{11}$ ev, by the observations of Clay and his co-workers, and Wilson[10] on the cosmic-ray intensity at great depths below sea level. They find the number of mesotrons at depth h below the top of the atmosphere decreases as $h^{-\gamma}$ with $\gamma=1.8$ or 1.9 up to 300 m water equivalent and $\gamma=2.4$ for $300\text{ m} < h < 1000$ m water equivalent. Observations at ~ 1500 m depth indicate an even more rapid decrease in the intensity. If ionization is the most important mechanism of energy loss up to $E_0 \sim 10^{12}$ ev these measurements can be immediately translated into an integral energy spectrum at sea level. There results that the number of mesotrons at sea level of energy greater than E_0 is approximately $(E_0+1.8\times 10^9)^{-\gamma}$ with $\gamma=1.9$ up to $E_0=6\times 10^{10}$ ev and $\gamma=2.4$ for $6\times 10^{10} < E_0 < 2\times 10^{11}$. The corresponding differential spectrum would begin to deviate from a power law of -2.9 at $E_0 \sim 10^{11}$ ev. If this is the case, the cross section for fractional energy loss per atom must be less than 2×10^{-28} cm^2 in earth. If the actual spectrum at sea level were represented by $\gamma=1.9$ up to $E_0=2\times 10^{11}$ ev, the cross section 2×10^{-28} cm^2 could account for the additional absorption at high energies represented by the increase in γ with depth. Now the cross section for radiative energy loss in earth of mesotrons of spin 0 or $\frac{1}{2}$ is considerably less than 2×10^{-28} cm^2 and in these cases we may assume, neglecting specifically nuclear energy loss, that γ increases for high energies at sea level. On the other hand, for mesotrons of spin 1 the cross section for fractional energy loss by bremsstrahlung is $\sim 2\times 10^{-28}$ cm^2 so that we may take $\gamma=1.9$ up to the highest measured energies ($\sim 2\times 10^{11}$ ev) in the appropriate sea level spectrum. For $E_0 > 2\times 10^{11}$ ev, there is no direct evidence on the mesotron spectrum.

We have taken the total mesotron intensity at sea level to be 0.01 mesotron per unit solid angle near the vertical, per cm^2 per sec. as suggested by Nordheim and Hebb, and Johnson.[11] Wilson's readings with a slanted counter telescope and the observations on the angular distribution of mesotrons at sea level are both in agreement with the hypothesis that where produced, near the top of the atmosphere, the mesotrons are hemispherically isotropic, the angular distribution at and below sea level being the result of the increased absorption thickness at angles from the vertical. Thus we take for the differential mesotron spectrum at sea level

$$N(E_0)dE_0d\Omega = \frac{0.02(E_c)^{1.9}dE_0d\Omega}{(E_0+1.8\times 10^9 \sec\theta)^{2.9}},$$

where the exponent in the denominator increases to 3.4, with the appropriate change in normalization, at $E_0=10^{11}$ ev in the cases of spin 0 and $\frac{1}{2}$. The effect of mesotron decay is to reduce the number of low energy mesotrons which are ineffective in burst production; there results a deviation from the power law at low energies. In order to give correctly the total number of mesotrons with a power law spectrum which is correct at high energies, the spectrum must be cut off at $E_0 \sim 0.6\ 10^9$ ev. This is effected by setting $E_c=(1.8+0.6)\times 10^9 = 2.4\times 10^9$ in the differential spectrum. This spectrum gives a total intensity $\sim \cos^2\theta$ as observed but for $E_0 \gg 1.8\times 10^9$ ev the distribution is hemispherically isotropic.

Experimental data on bursts are not concerned with the average ionization under thick absorbers or even with the average ionization due to the cascade multiplication of a single soft ray. On the contrary, they are a body of information on a series of *individual* showers. Thus, in discussing the cascade multiplication of a high energy soft secondary in the shielding material, we cannot content ourselves with calculating the average number of ionizing particles in the shower but

[10] P. H. Clay, A. van Gemert and J. Clay, Physica **6**, 184 (1939); J. Clay and A. van Gemert, Physica **6**, 497 (1939); V. C. Wilson, Phys. Rev. **53**, 337 (1938).

[11] L. W. Nordheim and M. H. Hebb, Phys. Rev. **56**, 494 (1939); T. H. Johnson, Rev. Mod. Phys. **10**, 208 (1938).

must investigate individual cases or fluctuations around this average and the probability of finding a particular number of particles as observed. This situation is accentuated by the fact that the number of high energy secondaries will, like the number of mesotrons, decrease rapidly with increasing energy. This means that fluctuations in which more than the average number of particles are found will be heavily weighted, whereas the reverse fluctuations, producing fewer than average, will be relatively unimportant. Not merely the breadth of the probability distribution but also its asymmetry is thus of some importance. We do not know the actual probability law $P(E, S, x)$ that there will be S ionizing particles at a distance x from the beginning of a shower initiated by a soft ray of energy E. For the approximate Furry[12] model which neglects ionization (and is thus independent of E) and treats electrons and γ-rays in the same way, $P=e^{-x}(1-e^{-x})^{S-1}$, $S_{Av}(x)=e^{x}$, and $\langle S^2\rangle_{Av}-(S_{Av})^2=(S_{Av})^2-S_{Av}$ or $\sigma=S_{Av}(1-1/S_{Av})^{\frac{1}{2}}$. Although we cannot use this explicit form for P and S_{Av}, this model gives formally an expression for the dispersion $\sigma\sim S_{Av}$ which is of the right order of magnitude, and near the beginning of the shower, is certainly correct. Near the maximum of the shower, where the largest contributions arise, the detailed calculations of Nordsieck, Lamb and Uhlenbeck[13] show that $\sigma\sim\frac{1}{2}S_{Av}$. In this region the actual fluctuations are certainly much better approximated by the Furry model than those, $\sigma\sim(S_{Av})^{\frac{1}{2}}$ of the Poisson formula. We have, in accordance, used a modification of the Furry formula which gives the same expression for the fluctuations and can be written

$$P(E, S, x)=\frac{1}{S_{Av}(E, x)}\left[1-\frac{1}{S_{Av}(E, x)}\right]^{S-1}$$

where $S_{Av}(E, x)$ is chosen to fit the calculated[14] average multiplication of a soft ray of energy E. Actually since P is asymmetric in the direction of emphasizing small S, its effect is smaller than would be expected from the large value of σ: it gives about twice as many bursts as calculations based on the assumption of no fluctuations. To express the fact that the actual fluctuations are intermediate between S_{Av} and 0, we have so corrected P as to reduce the burst probability by $\sim\sqrt{2}$.

In addition to the above general features, the number of mesotrons, the cross sections for production of soft secondaries, and the multiplication of these secondaries, which are common to any similar description of bursts, there are some features peculiar to the experimental arrangement. The spherically symmetric geometry of Schein and Gill is certainly the most desirable but their steel ionization chamber inside the lead absorber was of sufficient thickness ~ 1.3 cm to introduce important transition effects in the passage of the shower from the lead to the interior of the chamber.[15] The effective thickness, in view of the effects of scattering and oblique incidence can be taken to be ~ 1.5 cm or 0.8 shower units of iron. The essential effect of the iron is to reduce the number of particles in the shower to a value intermediate between the numbers normally expected in lead and in iron. The number of particles in a shower is measured by the number at maximum which is $\sim E/9\beta$ for $E\sim 10^{10}$ ev where β is the energy, characteristic of the material, where multiplication stops. For iron $\beta=22.4$ Mev, whereas for lead, refinements[16] of the usual theory give $\beta\sim 8$ Mev; in general β varies roughly as $1/Z$. The transition effects can be represented by using an effective β between 8 and 22 Mev. Different estimates of this indicate that about $\frac{1}{3}$ of the shower particles in lead are removed by the iron so that $\beta\approx 12$ Mev. We have chosen $\beta=13$ Mev and do not think this value is in error by more than 15 percent.

In some burst experiments the scattering of the shower particles in the absorber, which results in their angular divergence, is an important consideration; we will see, however, that in this experiment scattering is not serious and the resulting cross-sectional area of a burst emerging from the absorber is small compared to the size of the ionization chamber for all burst sizes. The angles involved in the multiplicative acts themselves are always small, except

[12] W. H. Furry, Phys. Rev. 52, 569 (1937).
[13] A. Nordsieck, W. E. Lamb, Jr. and G. E. Uhlenbeck, Physica 4, 344 (1940).
[14] R. Serber, Phys. Rev. 54, 317 (1938).
[15] We are indebted to Professor H. A. Bethe for calling our attention to the effect of the thick steel wall.
[16] Dr. Corben has kindly communicated to us the results of his refinements of the cascade theory for heavy elements.

at the lowest energies, and can be neglected here. The theory of multiple scattering gives $\langle\theta^2\rangle_{Av}\sim tZ^2/E^2$ so that the scattering is important only at low energies which occur near the end of the path of the burst in the absorber. Large and small bursts are alike in the energies involved in this region, differing only in the number of particles. The only essential difference between large and small bursts is that the large one starts earlier in the absorber and undergoes additional high energy multiplicative processes which involve extremely small angles. Thus the mean scattering angle is independent of burst size. We can in fact confine our attention to the scattering in the last few cm of absorber which is, here, iron. The mean scattering angle $\langle\theta^2\rangle_{Av}^{\frac{1}{2}}\sim 20°$ for $E\sim\beta=13$ Mev which, occurring in the last cm of material, corresponds to a burst about 1 cm in diameter having half its particles confined to a cone of vertical angle $\sim 40°$. Tending to counteract the decrease in burst size due to the increased path length in iron is the scattering back into the ionization chamber of particles which have traversed it once. Thus, with a spherical ionization chamber immediately surrounded by dense absorber, the corrections due to scattering appear to be small and can be treated as slightly augmenting the transition effects. This is not the case for those experiments which are performed with only a flat plate of absorber above the chamber, especially if it is separated any considerable distance from the chamber. Then the scattering can result only in some of the burst particles missing the chamber. This may be a considerable fraction of the burst if the absorber is lead. In particular, there may then be a differentiation against large bursts and a distortion of the burst curve. These considerations will be important in discussing the experiments of Nie, Steinke and Schmidt.

IV

In calculating the number of bursts, it is simplest to find the number of bursts of size greater than S. On the modified Furry model the probability formula for this is then

$$P'(E,x,S)=\sum_{r=S+1}^{\infty}P(E,x,r)=\left(1-\frac{1}{S_{Av}(E,x)}\right)^S.$$

In approximating to $S_{Av}(E,x)$ we have taken

$$S_{Av}(E,x)=\left\{1-\left(1-\frac{9\beta}{E}\right)\left(\frac{x}{7}\right)^{58\beta/E}\times\exp\left[\frac{58\beta}{E}\left(1-\frac{x}{7}\right)\right]\right\}^{-1},$$

where x is measured in shower units of length

$$x_0=\left[4\alpha Z^2\left(\frac{e^2}{mc^2}\right)^2 N\ln\frac{191}{Z^{\frac{1}{3}}}\right]^{-1}$$

and the numerical constants were chosen to approximate to Serber's calculation of cascade multiplication. This form for S_{Av} is good for E in the range $10^{10}-10^{11}$ ev for our purposes which merely require a good approximation to the height and width of the maximum. The expression has compensating errors in that for small bursts it underestimates the height of the maximum and overestimates its breadth. The total probability P'' of getting a burst greater than S from an initial ray of energy E is then obtained by integrating P' over the thickness of the absorber. This integral may be extended to ∞ since the major contribution comes near the maximum of S_{Av}. Then

$$P''(E,S)=x_0\int_0^\infty P'(E,x,S)dx$$

$$=x_0\int_0^\infty\left(1-\frac{9\beta}{E}\right)^S\left(\frac{x}{7}\right)^{58\beta S/E}\times\exp\left[\frac{58\beta S}{E}\left(1-\frac{x}{7}\right)\right]dx$$

or

$$P''(E,S)=7x_0(E/9\beta S)^{\frac{1}{2}}e^{-9\beta S/E}.$$

We can estimate how to correct the error introduced by the somewhat large fluctuations of the Furry model by calculating $I=\int_0^\infty P''(E,S)\times(dE/E^3)$ which is closely connected with the number of bursts of size greater than S. With the above formula, $I=0.077x_0/(\beta S)^2$, the maximum of the integrand occurring at $E=3.6\beta S$ and the effective distance in the absorber or burst range being $\sim x_0$. If we assume no fluctuations and integrate the effective length, weighted as $1/E^3$, where there are more than S particles, there results $I=0.035x_0/(\beta S)^2$. Here the most probable energy is $10\beta S$ and the effective burst range $3.5x_0$.

For $\sigma \sim \frac{1}{2}S_{\text{Av}}$ we take the most probable energy and the effective range to be the harmonic means of the respective quantities for $\sigma = S_{\text{Av}}$ and $\sigma = 0$; thus we take the most probable energy to be $6\beta S$ and the effective range $1.9x_0$. The modified probability law is

$$P'''(E, s) = 13.5x_0(E/15\beta S)^{\frac{1}{2}}e^{-15\beta S/E}$$

which we use in the remaining calculations. For this law, $I = 0.053x_0/(\beta S)^2$.

We must now integrate this probability over all secondary energies with the appropriate weights given by the number of mesotrons and the cross section for production of secondaries. The number of bursts of size greater than S per cm² per sec. due to any particular cross section for secondary production is then

$$N_i(S) = \bar{\varphi}N\int_0^\infty dE\int_0^{\epsilon_m}\frac{d\epsilon}{\epsilon}\int d\Omega \times F\left(\frac{E}{\epsilon}, \theta\right)\sigma_i\left(\frac{E}{\epsilon}, \epsilon\right)P'''(E, S),$$

where E is the energy of the secondary and ϵ the fractional energy transfer, the energy of the primary mesotron being E/ϵ; $F(E/\epsilon, \theta)$ is the number of mesotrons at sea level per unit solid angle per unit energy per cm² per sec. of energy E/ϵ at an angle θ with the vertical; and σ_i is a cross section, measured in units $\bar{\varphi}$, for the creation of a secondary of energy E by a primary of energy E/ϵ in the absorber. If we introduce P''', x_0, and F explicitly and express E in units $15\beta S$ there results

$$N_i(S) = \frac{13.5}{4\ln(191/Z^{\frac{1}{3}})}\frac{m^2}{\mu^2}\times 0.02\times\left(\frac{2.4\times 10^9}{15\beta S}\right)^{1.9} \times\int_0^\infty\frac{e^{-1/E}dE}{E^{2.4}}\int_0^{\epsilon_m(15\beta SE)}\epsilon^{1.9}d\epsilon \times\int\frac{d\Omega\sigma_i(15\beta SE/\epsilon, \epsilon)}{(1+1.8\times 10^9\epsilon\sec\theta/15\beta S)^{2.9}},$$

where it is understood that the spectrum and integrals are to be appropriately modified at high energies in the cases spin 0 and $\frac{1}{2}$. For the knock-on cross sections, which are proportional to $1/Z$ in these units, the last two integrals were carried out exactly with the exponent 2.9 being replaced by 3 in the θ integral. This is unimportant since $1.8\times 10^9\epsilon\sec\theta/(15\beta S)$ is small for most angles for $S > 100$. The resulting quantity, which may be called a modified burst production cross section, was plotted, the corrections to the mesotron spectrum above $E_0 = 10^{11}$ ev introduced in the graph, and the remaining integral carried out by approximate methods. The corresponding burst frequencies are tabulated in Table I for different values of βS for lead. To make estimates for other elements, these burst probabilities due to *knock on* must be multiplied by $82/Z$.

TABLE I. *Number of bursts per cm² per sec. with more than S particles produced in great thicknesses of lead by different processes.*

	Knock on			Bremsstrahlung		
βS	Spin 0	Spin $\frac{1}{2}$	Spin 1	Spin 0	Spin $\frac{1}{2}$	Spin 1
10^9	$6.8\cdot 10^{-8}$	$8.7\cdot 10^{-8}$	$1.2\cdot 10^{-7}$	$2.0\cdot 10^{-7}$	$3.2\cdot 10^{-7}$	$1.0\cdot 10^{-6}$
2.10^9	$1.3\cdot 10^{-8}$	$1.8\cdot 10^{-8}$	$3.2\cdot 10^{-8}$	$7.3\cdot 10^{-8}$	$1.2\cdot 10^{-7}$	$4.8\cdot 10^{-7}$
4.10^9	$2.3\cdot 10^{-9}$	$3.3\cdot 10^{-9}$	$8.1\cdot 10^{-9}$	$2.2\cdot 10^{-8}$	$3.8\cdot 10^{-8}$	$2.1\cdot 10^{-7}$
8.10^9	$3.5\cdot 10^{-10}$	$5.4\cdot 10^{-10}$	$2.1\cdot 10^{-9}$	$6.0\cdot 10^{-9}$	$1.0\cdot 10^{-8}$	$8.7\cdot 10^{-8}$
16.10^9	$5.1\cdot 10^{-11}$	$8.1\cdot 10^{-11}$	$5.5\cdot 10^{-10}$	$1.4\cdot 10^{-9}$	$2.5\cdot 10^{-9}$	$3.3\cdot 10^{-8}$
32.10^9	$7.1\cdot 10^{-12}$	$1.2\cdot 10^{-11}$	$1.5\cdot 10^{-10}$	$2.9\cdot 10^{-10}$	$5.1\cdot 10^{-10}$	$1.2\cdot 10^{-8}$
64.10^9	$9.7\cdot 10^{-13}$	$1.6\cdot 10^{-12}$	$3.9\cdot 10^{-11}$	$5.0\cdot 10^{-11}$	$9.0\cdot 10^{-11}$	$3.7\cdot 10^{-9}$

The bremsstrahlung cross sections are more complicated functions of ϵ and it was necessary to approximate slightly by calculating

$$\int_0^{\epsilon_m}\epsilon^2\sigma d\epsilon,$$

the burst production cross section, and correcting it at low energies by multiplying by

$$\int\frac{d\Omega}{(1+1.8\times 10^9\bar{\epsilon}\sec\theta/15\beta S)^3}$$

with $\bar{\epsilon} = \frac{3}{4}$. This modified burst production cross section was then treated as before and tabulated. These burst frequencies depend on Z only through β.

The total burst frequency for any mesotron spin and any element for βS in the range given, can be found by adding $82/Z$ N(knock on) to N(bremsstrahlung) and substituting an appropriate value of β which varies roughly as $1/Z$. These calculations cannot with any certainty be extended beyond $\beta S = 16\times 10^9$ ev because of the lack of *a priori* knowledge of the mesotron spectrum above $E_0 \sim 2\times 10^{11}$ ev. In Fig. 1 we have plotted $N(S)$ against (S) on a double logarithmic scale for spins 0, $\frac{1}{2}$, and 1, together with the experimental results of Schein and Gill.

V

The principal sources of error or uncertainty in these calculations are the values of μ/m and β; both of which enter quadratically. We limit our error in β to 15 percent and in its effect to 30 percent. Errors in the number of mesotrons and in our treatment of cascades should not be greater than 15 percent. Except for the mass uncertainty we would limit errors to ~ 40 percent. We believe $\mu/m=177$ is probably correct to within 15 percent and would thus assign an over-all limit of error of about 60 percent.

In view of these possible errors, we see that the experimentally observed burst frequency is consistent with either spin 0 or spin $\frac{1}{2}$ and moment $e\hbar/2\mu c$. Any appreciably different moment for spin $\frac{1}{2}$ would result in a burst frequency comparable to that from spin 1 moment $e\hbar/2\mu c$. Our calculations give slightly greater weight to the spin 0 hypothesis but do not justify an absolute distinction. On the other hand, spin 1 and moment $e\hbar/2\mu c$ (which gives the minimum electromagnetic effects for this spin) gives more bursts than are observed by at least a factor ten. Furthermore, this discrepancy becomes larger for large bursts where the data shows a more rapid decrease of burst frequency with burst size. Thus the experiments tend to exclude the possibility of spin 1 but are consistent with either spin 0 or spin $\frac{1}{2}$.

Further confirmation of this conclusion is given by Bhabha's calculation of the bursts produced in clay and his comparison with the experiments of Carmichael and Chou.[17] He found that the knock-on formula for spin 1 gave good agreement with observation. If he had then included the bremsstrahlung which must accompany a particle of spin 1, he would have found far too many large bursts. However it is apparent from our tables that the total burst frequency in clay for spin 0 or $\frac{1}{2}$ is roughly equal to that given by the spin 1 knock-on formula. With the conclusions from the data of Schein and Gill, this is significant, although the data of Carmichael and Chou do not involve sufficiently high energies alone to justify a sure conclusion.

[17] H. J. Bhabha, H. Carmichael and C. N. Chou, Proc. Ind. Acad. Sci. **10**, 221 (1939); H. Carmichael and C. N. Chou, Nature **144**, 325 (1939).

We have already indicated that in Nie's experiments the geometry was such as to preclude the possibility of exact calculation. The experiments have, however, contributed some doubt as to the Z dependence of bursts. Our calculations give Z^2 dependence for large bursts ($S>500$) where bremsstrahlung is dominant; this is reduced to Z dependence in the neighborhood of $S=100$ where knock on becomes important. For much smaller bursts Z independence may be expected since the mesotron spectrum flattens out at low energies.[18] Now Nie worked primarily with small bursts and appeared to find as many in iron as in lead although the transition curve for iron was not carried as far as the plateau. However, the effects of scattering, which was probably an important factor in decreasing the apparent burst size with his geometry, increase as Z^2—which would tend to decrease the difference between lead and iron. There is still a real need for a good experiment on burst production in a light element such as water.

The immediate conclusions to be drawn from the experiment of Schein and Gill are that the exponent in the mesotron spectrum at sea level must increase by several units in the range $10^{11}<E_0<10^{12}$ ev, and that the burst production cross section,

$$\int_0^{\epsilon_m} \epsilon^2 \sigma d\epsilon < 10^{-27}\ \text{cm}^2$$

at $E\approx 5\times 10^{10}$ ev. The burst frequencies calculated either with spin 0 or spin $\frac{1}{2}$ agree with experiment; those calculated with spin 1 do not. In critically examining the possibility of spin 1 we see that two points are perhaps questionable. In the first place, our calculations for bremsstrahlung of spin 1 which has been intended to give the minimum effect possible for this case, can, of course, be fully justified only by a more complete theory than we have at present. In the second place, if the mesotron mass were several times as great as here supposed, the experiments of Schein and Gill would no longer exclude this case.

In conclusion we wish to express our appreciation of the invaluable assistance of Professor J. R. Oppenheimer, with whom all parts of this work have been discussed.

[18] The essence of this discussion is to be found in Oppenheimer, Snyder and Serber, reference 8, and J. R. Oppenheimer, Rev. Mod. Phys. **11**, 264 (1939).