Note on the Radiation Properties of Heavy Nuclei

V. F. WEISSKOPF
University of Rochester, Rochester, New York

(Received December 23, 1940)

THERE is so far no way to calculate radiative transition probabilities between nuclear levels of heavy nuclei. We here propose a way of estimating the order of magnitude of these processes. In spite of its crude and over-simplified assumptions, it represents the experimental facts known so far.

We assume that the absorption probability of a nucleus in its ground state, averaged over an energy interval which contains a great number of excited levels, is proportional to the intensity at the nucleus of the spherical harmonic component of the light wave which is absorbed. This assumption is supposed to be valid for light wave-lengths for which $\lambda\!\!\!^{-} = \lambda(2\pi)$ is large compared to the nuclear radius.[1] It is equivalent to the statement that the absorption probability, if averaged over the level resonances, depends on the light intensity alone and not on the frequency, as long as the wave-length is large enough. (The corresponding assumption for the absorption of neutrons leads to the so-called "$1/v$-law" for slow neutrons.) In case of a 2^l-pole transition the light quantum absorbed is represented by the $(l-1)$st spherical harmonic. We therefore would obtain for the absorption cross section into an energy region of the nucleus, E Mev above the ground state:

$$\sigma^{(l)} \doteq C \cdot E^{2l-1}. \tag{1}$$

The constant C must be determined experimentally and is found to be roughly equal for all elements.* We further assume that most of the occurring nuclear transitions are quadrupole transitions. From the investigation of the γ-spectra of radioactive nuclei it is known that dipole and quadrupole transitions are about equally probable among lowest levels. It then follows from (1) that quadrupole radiation is dominant for higher frequencies.

The average 2^l-pole emission probability $\bar{\Gamma}^{(l)}$ from a state with an excitation energy E can be derived from the cross section $\sigma^{(l)}(E)$ of the reverse process: ($\bar{\Gamma}^{(l)}$ is measured in energy units after multiplication with $\hbar$)

$$\bar{\Gamma}^{(l)} = \frac{g}{2\pi^2} \frac{\sigma^{(l)}(E)}{\lambda\!\!\!^{-2}\omega(E)}. \tag{2}$$

Here g is the statistical weight of the ground state and $\omega(E)$ the level density of the combining levels with an excitation energy E (an n times degenerated level is considered as n levels) and $\lambda\!\!\!^{-}$ the wave-length of the emitted light (times $1/(2\pi)$).

There are three independent groups of experiments which all agree to the formulas derived. Bothe and Gentner[2] have made measurements of absorption cross sections of photo-effects on 9 identified heavy nuclei ($A > 60$) with γ-rays from proton bombarded lithium and boron. The former gives a γ-ray of 17 Mev, the latter mainly two rays of 16.6 and 11.8 Mev in the intensity ratio 1 : 7. According to (1) the ratio of the cross sections for the two radiations ought to be $\sigma_{\mathrm{Li}}^{(2)}/\sigma_{\mathrm{B}}^{(2)} = 2.12$. (For dipole radiation this ratio would be 1.18.) The actual ratios lie between 1.5 and 2.7 with one exception (Ga^{70}) which has the ratio 1.1. The average ratio is 2.3. The constant C is of the same order of magnitude for all elements investigated. The average value is $C = 0.65 \times 10^{-29}$ cm^2/(Mev)^3 and all values are found between 0.23 and 1.0×10^{-29}.

The second group are the photo-fission experiments by the Pittsburgh group.[3] Here U and Th were irradiated by a γ-radiation of 6.3 Mev produced by proton bombardment of fluorine. Fission was observed and the cross section for its production was found to be $3.6 \pm 1.0 \times 10^{-27}$ cm^2

[1] This assumption has been put forward by Weisskopf and Ewing, Phys. Rev. **57**, 472 (1940). In the present note it is worked out in more detail and applied to more recent experiments.

* Here and in the following considerations electric multipole radiation only is taken into account. Magnetic multipole radiation can be neglected except in special transitions where the electric transition is forbidden by selection rules (e.g., in isomeric transitions).

[2] W. Bothe and W. Gentner, Zeits. f. Physik **112**, 45 (1939).

[3] Haxby, Shoupp, Stephens and Wells, Phys. Rev. **57**, 1088 and **58**, 199 (1940); **59**, 57 (1941).

for U and $1.7 \pm 0.5 \times 10^{-27}$ cm^2 for Th.[4] Since the γ-ray energy is near to the binding energy of the neutron, (Wheeler[5] calculated the latter and found 6.1 Mev for U and 6.2 Mev for Th), the excitation of the nucleus would lead in most cases to fission rather than to neutron expulsion. The fission cross section should therefore be equal or slightly smaller than the absorption cross section. The above value of C would give $\sigma^{(2)}$(6.3 Mev) $= 1.6 \times 10^{-27}$ cm^2 which is as close to the observed values as one might expect. If dipole radiation were assumed, we would find a cross section of 1.2×10^{-26} cm^2 from Bothe and Gentner's result and formula (1).

Finally, actual life times of nuclear levels have been measured in radioactive nuclei as quoted by Bethe[6] and recently also in stable nuclei as for In^{115} and Pb by Waldman and Collins[7] and Guth.[8] They have been found to be all of the order of 10^{-12} sec. which corresponds to a transition probability to lower levels $\Gamma \sim 10^{-3}$ ev. The radiation emitted is of the order of 1 Mev in all cases observed. (The values of Γ in five elements investigated lie between 0.6 and 2 millivolts, the excitation energy between 0.61 and 1.8 Mev.) Formula (2) gives an average value of the transition probability from an excited state to the ground state and should be used only for higher excitations with close lying levels. If we still try to apply it for low levels we get $\bar{\Gamma}^{(2)} = 0.8 \times 10^{-3}$ ev by putting $\omega(E) = 1(\text{Mev})^{-1}$. The latter value for the level density is of the order of the expected average distance of lowest levels combining with the ground state.

The following consideration indicates that the experimental value of C in (1) is within the theoretical expectations. The average value of the quadrupole moment

$$Q = \sum_i \int r_i^2 \psi_g^* \psi_E d\tau$$

of the transition[9] from the ground state ψ_g to a state with the energy E can be calculated from the cross section $\sigma(E)$ by means of the formula:

$$|Q|^2 = \frac{15}{8\pi^3} \frac{C^3\hbar^3}{E^3} \frac{\sigma^{(2)}(E)}{e^2\omega(E)}.$$

Here the ground state is assumed to be nondegenerate. The following relation must hold:

$$\int_0^\infty |Q|^2 \omega(E) dE = \rho^4,$$

where ρ^4 is the average value of $(\sum_i r_i^2)^2$ in the ground state. We expect formula (1) to be valid until $\lambda\!\!\!^{-} \sim$ radius of the nucleus, that is for $E < 40$ Mev. If we assume $\sigma(E)$ falling off above that energy, we obtain

$$\rho^4 \sim \frac{15}{8\pi^3} \int_0^{40\text{ Mev}} \frac{c^3\hbar^3}{e^2} C dE \sim 10^{-48}\ \text{cm}^4,$$

which is of the expected order of magnitude.

[4] A rough estimate of the order of magnitude expected by the theory has been given by Bohr and Wheeler (N. Bohr and J. A. Wheeler, Phys. Rev. **56**, 426 (1939)) but without taking into account the frequency dependence of the nuclear photo-effect.

[5] J. A. Wheeler, personal communication.

[6] H. A. Bethe, Rev. Mod. Phys. **9**, 229 (1937).

[7] Waldman and Collins, Phys. Rev., in print.

[8] E. Guth, Phys. Rev., in print.

[9] The summation is taken over all protons.