near the average rather than the total excitation energy in the compound nucleus. The three types of states $^{\alpha}Ne^{20}$, $^{\pi}Ne^{20}$ and $^{\alpha\pi}Ne^{20}$ are differentiated on the basis of the relative intensities of the long and short range alpha-particles which result from their decay, but a complete explanation for this difference in behavior has not been proposed. Of course the irregular variation of the ratios of the yields of long range alphas to pair-alphas may be understood in part in terms of the fact that large changes in angular momentum would decrease the yield of the shorter range pair-alphas. But the consistently large ratios of gamma-alphas to pair-alphas can apparently not be understood on this basis, especially since $^{\gamma}Ne^{20}$ and $^{\pi}Ne^{20}$ can both be formed by *s* collisions and both $^{\gamma}O^{16}$ and $^{\pi}O^{16}$ by the ejection of an *s* alpha-particle. It is possible that a new selection rule is involved.[32]

The yield measurements must be supplemented by precise measurements of the total decay width Γ of each resonance and of the distribution in angle of the alpha-particles before a complete description of the properties of the levels of the intermediate nucleus can be given.

In conclusion we wish to express our appreciation to Professor J. R. Oppenheimer for numerous contributions to the theoretical aspects of this discussion.

[32] Streib, Fowler and Lauritsen, Phys. Rev. 58, 187(A) (1940).

FEBRUARY 1, 1941 PHYSICAL REVIEW VOLUME 59

Velocity-Range Relation for Fission Fragments

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Considerations indicated in an earlier note as regards the rate of velocity loss of fission fragments along the range are developed in greater detail and a comparison is given between the calculations and more recent experiments. Especially is a more precise estimate given for the charge effective in electronic encounters which are determining for the stopping effect over the first part of range, and for the screening distance in nuclear collisions which are responsible for the ultimate stopping. In the estimate of the effect of electronic interactions, use is made of a comparison with the stopping of α-particles of the same velocities. In this connection, however, a certain correction is necessary due to an intrinsic difference in the stopping formulae to be applied in the two cases. Moreover, fission fragment tracks show, in contrast to α-rays, a considerable range straggling originating in the end part of the range. It is shown that in this respect also the calculation agrees closely with the experimental data.

IN an earlier note[1] the peculiar velocity-range relation for fission fragments revealed by cloud-chamber studies of fragment tracks[2] has been briefly discussed. In particular, it was pointed out that in the different parts of the range we have to do with two essentially different stopping mechanisms. At the beginning of the range, where the total charge of the fragment is still large, the stopping is due practically only to energy transfer to the individual electrons in the atoms of the gas penetrated. With decreasing velocity, however, the fragment charge effective in electronic interactions will rapidly decrease and direct transfer of momentum from the fragment to the gas atoms through close nuclear collisions will gradually become of greater importance. In the last part of the range, such collisions will, in fact, be almost entirely responsible for the stopping effect. In the note it was shown that it is possible, from very simple considerations regarding the way in which the charge of the fragment varies with velocity, to account at least qualitatively for the characteristic features of the experimental velocity-range relation. The continuation of the work,

[1] N. Bohr, Phys. Rev. 58, 654 (1940).
[2] K. J. Brostrøm, J. K. Bøggild and T. Lauritsen, Phys. Rev. 58, 651 (1940).

however, has led to essential improvements of the various estimates entering into the calculations and it may, therefore, be of interest to give here a somewhat closer discussion of the question.

The problem of primary importance in the discussion is the estimate of the number of electrons carried with the fragment nucleus on its way through the gas. This number is determined by the balance between the continual capture and loss of electrons by the fragment in encounters with the gas atoms. Here we meet at once with a behavior essentially different from that of high speed particles with small nuclear charge, such as protons and α-rays. In fact, in the latter cases any captured electron will have an "orbital velocity" small compared with the velocity of the particle itself and the probability of electron capture will, therefore, be much smaller than the probability of subsequent electron loss, with the result that the particles will be practically stripped of electrons over nearly the whole range. In the case of fission fragments, however, a considerable number of the electrons in the neutral atom will have orbital velocities larger than the initial velocity of the fragment, and, as stressed in the earlier note, the capture and loss of such electrons will take place under conditions very different from those for electrons more loosely bound.

We shall, for brevity, refer in the following to the ensemble of the electrons in the neutral fragment with orbital velocities greater than the instantaneous fragment velocity V as the "electron core" of the fragment. In the first place, the probability of capture of electrons into states normally occupied by this core will be much larger than that for capture into states of looser binding. Indeed, in collisions with gas atoms sufficiently heavy to possess themselves a corresponding electron core, the probability of capture of the former kind will be quite considerable even in a single collision, if the cores penetrate each other. Moreover, while the electrons outside the core will be easily removed during encounters with the electrons and nuclei of the gas atoms, electrons belonging to the core can obviously not be removed during such encounters, at any rate if the charge of the gas nuclei is smaller than the nuclear charge of the fragment. With a high degree of approximation we may, therefore, assume that the fragment at any instant along its path carries with it a number of electrons just constituting the core.

In this connection it is interesting to note that even in the original fission process we may expect that both fragments escape with their electron cores practically intact. In spite of the violence of the rupture of the original heavy nucleus the initial fragment velocities will, in fact, be considerably smaller than the orbital velocities of the major part of the electrons in the original atom. As a consequence of the almost adiabatic influence on such electrons of the translatory motion of the nuclear fragments, a balance like that described will, therefore, be established, at any rate partially, before the fragments are separated by distances comparable with atomic dimensions.[3]

The calculation in the previous note of the rate of velocity loss of the fragment per unit path was based on the following formula (not stated explicitly there):

$$\frac{1}{N}\frac{dV}{dx}=\frac{4\pi e^4}{M_1 m V^3}(Z_1^{\text{eff}})^2\sum_s \ln\frac{mV^3}{2\pi\nu_s e^2 Z_1^{\text{eff}}} +\frac{4\pi e^4 Z_1^2 Z_2^2}{M_1 M_2 V^3}\ln\left(\frac{M_1 M_2}{M_1+M_2}\frac{V^2 a_{12}^{\text{scr}}}{Z_1 Z_2 e^2}\right), \quad (1)$$

where N is the number of gas atoms per unit volume, e and m are the electronic charge and mass, Z_1e, Z_2e and M_1, M_2 the charge and mass of the nuclei of the fragment and gas atoms,

[3] In an earlier attempt to estimate the influence of electron capture on the range-velocity relation of fission fragments, G. Beck and P. Havas [Comptes rendus **208**, 1643 (1939)] assumed that, immediately after the fission process, the fragment nucleus is almost stripped of electrons and that during its passage through the gas it gradually captures electrons at such a rate that the fragment charge decreases with time according to an exponential law. Assuming, further, that the rate of capture is so great that the fragment is practically neutralized before its velocity has fallen to half of its original value and neglecting the stopping and ionizing effects of direct nuclear collisions, they conclude that all ionizing effects of the fragment will disappear long before it is stopped. In particular, they see herein a possible explanation of the apparent discrepancy between range measurements based on the ionizing power of the fragment on the one hand and the transfer of its radioactivity on the other hand. From the excessive deflections of the paths of fragments near the end of the tracks in cloud-chamber pictures it is clear, however, that such discrepancies must rather be ascribed to ordinary thermal diffusion of the fragments in the gas within the period of radioactive decay which is extremely long compared with the time interval in which the fragment has lost all of its initial velocity.

respectively. Z_1^{eff} is the effective charge of the fragment nucleus in electronic encounters and a_{12}^{scr} is the distance between the nuclei where the electronic screening sets an effective limit to the action of their charges in close collisions. The summation in the first term is to be extended over the various electrons in the gas atoms or, rather, over the various virtual atomic oscillators of frequency ν_s, taken with their respective weights.

The first term, which accounts for the contribution to the velocity loss due to energy transfer to the individual atomic electrons, corresponds to the original formula[4] for the stopping of high speed particles, based on simple considerations of classical mechanics. It differs from the quantum-mechanical formula deduced by Bethe[5] by means of the Born approximation method by a factor in the logarithmic argument:

$$\kappa = hV/4\pi E_1E_2, \qquad (2)$$

where h is Planck's constant, and E_1 and E_2 are the charges of the particles considered; thus, here $E_1 = Z_1^{\text{eff}}e$ and $E_2 = e$. The reason that the classical formula and not the Bethe formula is to be applied in our case is that κ, as we shall see below, is small compared with unity over the whole part of the range where electronic interaction constitutes the essential stopping effect. In fact, only for $\kappa \gg 1$ can a simple wave-mechanical diffraction procedure be applied rigorously to a collision between two charged particles, while for $\kappa \ll 1$ classical orbital pictures can, at least with high approximation, be applied to such a collision.[6]

The second term in (1) accounts for the contribution to the velocity loss by direct transfer of momentum from the fragment to the gas atoms through close nuclear collisions. While a few of these collisions give rise to side branches to the fragment track, the main contribution to the stopping effect at the end part of the range is due to numerous collisions which individually are not violent enough to cause visible branching and only add to the ionization of the track. In this case, κ is very small (of the order 10^{-3}) and we have, consequently, to do with a problem in which classical pictures can be applied with an extremely high degree of approximation. In contrast to the case of electronic encounters, where the limit to the energy transfer is set by the dynamic properties of the atomic oscillators indicated by the dependence of the logarithmic argument on ν_s, the limit indicated by the parameter a_{12}^{scr} is here set by the screening of the nuclear charge of the colliding atoms by the static charge distribution of the bound electrons.

The formula for the rate of velocity loss of the fragment given in the previous note was obtained by introducing in (1) the rough estimates

$$Z_1^{\text{eff}} = V/V_0 \quad \text{and} \quad a_{12}^{\text{scr}} = a_0(1/Z_1 + 1/Z_2), \qquad (3)$$

where the conventional notations

$$V_0 = 2\pi e^2/h \quad \text{and} \quad a_0 = h^2/4\pi^2 me^2 \qquad (4)$$

refer to the velocity and radius of orbit of the electron in the hydrogen atom. A closer examination of the electron distribution in heavy atoms, however, based on the results obtained by the statistical method of Thomas and Fermi leads to the more accurate estimates

$$Z_1^{\text{eff}} = Z_1^{\frac{1}{3}} V/V_0 \quad \text{and} \quad a_{12}^{\text{scr}} = a_0(Z_1^{\frac{2}{3}} + Z_2^{\frac{2}{3}})^{-\frac{1}{2}}. \qquad (5)$$

The first of the expressions (5) represents the resultant charge of the fragment nucleus and the electron core for velocities not too close to V_0; the second expression represents the screening distance effective in nuclear collisions, which is practically independent of the fragment velocity in the whole interval considered.

Introducing the value (5) for Z_1^{eff} into (2) we get

$$\kappa = 1/2Z_1^{\frac{1}{3}}, \qquad (6)$$

which gives a value for κ quite small compared with unity, since for fission fragments $Z_1^{\frac{1}{3}}$ lies between 3 and 4; thus, the use of classical mechanics in deducing formula (1) is here amply justified. For the comparison with the stopping power of α-rays mentioned below, it is interesting to note that this value for κ is even considerably smaller than the values for κ^{-1} for protons and α-rays in the same velocity interval. As regards the justification of the application of the first term of formula (1) to the electronic stopping of fission fragments it may further be noted that the linear dimensions of the fragment core, for the

[4] N. Bohr, Phil. Mag. **25**, 10 (1913) and **30**, 581 (1915).
[5] H. A. Bethe, Ann. d. Physik **5**, 325 (1930).
[6] Compare F. Bloch, Ann. d. Physik **16**, 285 (1933); and E. J. Williams, Sci. Progress, **121** (1936). For a closer discussion, see reference 11.

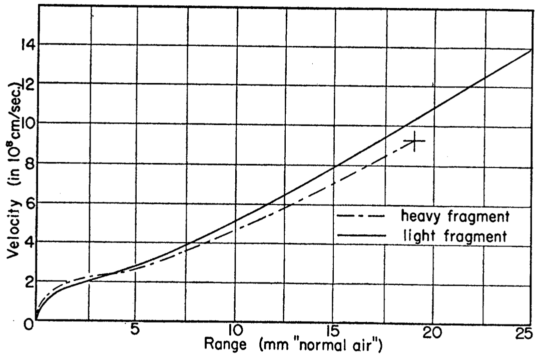

FIG. 1. Empirical velocity-range curves in argon.

radius of which we have approximately

$$r_c = a_0 Z_1^{\frac{1}{3}} V_0/V, \qquad (7)$$

are, of course, just of the same order of magnitude as the minimum distance of approach to a particle of charge Z_1^{eff} by an electron with velocity V, according to classical mechanics.

As pointed out in the previous note, formula (1) gives a value for the rate of velocity loss nearly independent of the velocity in the initial part of the range, corresponding to the almost constant slope of the experimental velocity-range curve in this region. This result follows from the linear dependence of Z_1^{eff} on V as well as from the fact that the sum of the logarithms in the first term of (1) is nearly proportional to V in the velocity interval considered. In estimating the absolute value of this sum by a comparison with the experimental data for stopping of α-rays with the same velocity, it must be taken into account that, owing to the occurrence in the logarithmic argument in (1) of the factor κ which does not appear in Bethe's formula to be applied to such light particles, a not inconsiderable correction has to be introduced. An estimate based on the statistical distribution of the oscillator frequencies ν_s in heavy atoms gives, in fact, that the value of the logarithmic sum will, as for α-rays, be nearly proportional to $Z_2^{\frac{1}{2}}$ but will have a numerical value only about $\frac{3}{5}$ of the value for α-rays with the same velocity.

Since the logarithmic sum is very insensitive to small changes in Z_1, the rate of velocity loss for fission fragments of different charge and mass should be proportional to $Z_1^{\frac{2}{3}}/M_1$ at the beginning of the range. In accordance with the empirical velocity-range curves in argon given in Fig. 1,[7,8] we shall, therefore, expect only a slightly greater slope for the lighter than for the heavier of the two main groups of fission fragments with mass and charge ratios of about 2 : 3. Moreover, as seen from the figure, we obtain a close estimate of the actual total range by extrapolating the initial linear slope of the curves to zero velocity. Using this fact in calculating the total range and applying for α-rays the well-known Geiger range formula we find by means of the first term in (1) for the ratio between the range R_F of a fission fragment and the range R_α of an α-particle with the same initial velocity V_i:

$$R_F/R_\alpha = 5(M_1/Z_1^{\frac{2}{3}})(V_0/V_i)^2, \qquad (8)$$

where in determining the numerical factor regard is taken of the difference mentioned above in the values of the logarithmic sum in the two cases considered. The relation (8) is actually found to be in close agreement with the experimental data.

This general agreement may be considered as a sensitive test for the estimate of the effective charge of the high speed fragment in electronic encounters. In this connection it must, however, be noted that it is not justified to assume on the basis of such arguments only that Z_1^{eff} is identical with the total fragment charge for the velocities considered. In fact, a closer consideration shows that, if the fragment carried a number of bound electrons in addition to the core, the reduction in the stopping and ionizing effect would be essentially less than would correspond to the reduction of the total charge. It is, therefore, very interesting that a direct measurement of the fragment charge by Perfilov[9] by means of the deflection in magnetic field of fission fragments expelled in vacuum from thin layers of U_3O_8 has yielded a value of about $20e$. This value agrees, in fact, closely with the expression (5) for Z_1^{eff} at the beginning of the range where V is about 5 V_0.

When we pass to the portion of the range where the velocity is nearing V_0 and where the empirical velocity-range curve exhibits an almost flat

[7] N. Bohr, K. J. Brostrøm, J. K. Bøggild and T. Lauritsen, Phys. Rev. 58, 839 (1940).

[8] J. K. Bøggild, K. J. Brostrøm and T. Lauritsen, Kgl. Danske Vid. Sels. Math.-fys. Medd. (Math.-phys. Comm., Acad. Sci. Copenhagen) 18, 4 (1940).

[9] N. A. Perfilov, Comptes rendus Acad. Sci. U.R.S.S. 28, 5 (1940).

plateau, several circumstances have to be taken into account in applying formula (1). In the first place, the estimate (5) for the fragment charge is, as already mentioned, only valid for a value of V considerably larger than V_0. For smaller velocities the charge will, in fact, decrease more rapidly and approach unity for velocities near V_0, since the very loosest bound electrons in heavy atoms are held almost as firmly as the electron in the hydrogen atom. Next, the basic assumptions in the calculation by which formula (1) is deduced, that the velocity of the moving particle is essentially higher than the orbital velocities of the atomic electrons and that the size of the particle is small compared with the orbital dimensions, are no longer fulfilled when V approaches V_0. Because of these circumstances, the rate of velocity loss will here be considerably smaller than the almost constant value for larger velocities, in agreement with the gradual diminishing of the slope of the velocity-range curve.

Just in the part of the range where the velocity is of the same order as V_0, the stopping effect of nuclear collisions which in the initial part of the range is very small compared with the effect of electronic interaction will, as explained in the previous note, gradually become preponderant, finally causing a steep descent of the velocity-range curve at the very end of the range. This character of the curve corresponds, in fact, to the very rapid increase with decreasing velocity of the factor before the logarithm in the second term of formula (1). Since the argument of the logarithm in this term is for $V = V_0$ still large compared with unity (about 15), the expression for the stopping effect of nuclear collisions will be valid for much smaller velocities than the first term in (1) and hold approximately down to velocities which are only a small fraction of V_0. While the logarithm is very insensitive to small changes in a_{12}^{scr} and has almost the same value for the heavier and lighter fragment group, the factor before the logarithm is essentially larger for the heavier fragment group, giving rise to a still steeper final descent of the velocity-range curve for this group in accordance with the experimental data.

Also in quantitative respects, the course of the velocity-range curves near the end of the range is in close agreement with the second term in formula (1). In fact, if we compare the range R_0 for fission fragments of velocity V_0, deduced from (1) by neglecting entirely the first term, with the total range R_F of the fragments with initial velocity V_i estimated from this term in the manner described above, we find

$$R_0/R_F = k(M_2/m)Z_1^{-4/3}Z_2^{-3/2}(V_0/V_i), \quad (9)$$

where k is a constant which depends on the logarithm in the two terms in (1) and the value of which is about 0.07. Putting $V_i = 5V_0$, we get from (9) for argon $R_0 = R_F/10$ which fits in very well with the run of the curves on Fig. 1.

As mentioned above, the total range of fission fragments compared with that of α-rays should be practically the same for light and heavy gases. However, we see from (9) that we shall expect that the ratio of the end part of the range (where the stopping depends only on nuclear collisions), to the whole range should (apart from the case of hydrogen, where the value of M_2/Z_2 is abnormally low), be inversely proportional to $Z_2^{\frac{1}{2}}$. This conclusion is also supported by recent measurements on the range of fission fragments in helium[10] which have given a range, relative to that of α-rays, about 20 percent longer than the corresponding range in argon. Such a difference would, in fact, just be explained if the ratio between R_0 and R_F is three times as large in helium as in argon, corresponding to the ratio of the inverse square roots of their nuclear charges.

Range measurements on fission fragments show a very considerable straggling which, as pointed out in the previous note, must be ascribed to the end part of the range. In fact, for the initial part of the range, where the stopping is due to electronic encounters, we shall, just as for α-rays, expect a very small degree of straggling, but at the end part, where the stopping is due to encounters with much heavier particles, the straggling effect will be far greater. Using the same calculations as originally applied for the estimate of α-ray straggling[4] we shall, for the straggling due to nuclear collisions, expect that the range will be statistically distributed according to the formula

$$W(R) = \frac{1}{(2\pi)^{\frac{1}{2}}\rho R_0} \exp\left[-\frac{(R-R_0)^2}{2\rho^2 R_0^2}\right], \quad (10)$$

[10] J. K. Bøggild, K. J. Brostrøm and T. Lauritsen, Phys. Rev. 59, 275 (1941) (following paper).

where $W(R)dR$ is the probability that the range has a value between R and $R+dR$; and R_0 is the mean value of the range, while ρ is a numerical constant approximately given by

$$\rho^2 = 3M_1M_2/4(M_1+M_2)^2. \quad (11)$$

For helium and argon (11) gives values of ρ equal to 0.16 and 0.37, respectively. Although the relative straggling is thus more than twice as large in argon as in helium, the absolute straggling of the range should be nearly the same, since the value for R_0 for the sensitive end part of the range should be about three times as large in helium as in argon. According to the above estimate of the fraction of the range where nuclear collisions constitute the preponderant stopping effect, we should expect $R_0\rho$ for both gases to be about 5 percent of the total range, in good agreement with the experiments which give for argon, as well as for helium, a straggling of this order of magnitude.[8, 10]

The various considerations here indicated are treated in greater detail in a paper shortly to appear in the Communications of the Copenhagen Academy of Science.[11] Especially is a closer discussion given there of the applicability of simple mechanical arguments for the treatment of the stopping and scattering of heavy highly charged atomic particles as well as of the ionization and electron capture by such particles.

Note added in proof.—After the present paper was sent from Copenhagen, we received here the issue of *The Physical Review* of October 15, 1940, which contains an article by W. E. Lamb on the passage of uranium fission fragments through matter. In main features the considerations of this article correspond to the arguments developed here and similar results are obtained. The treatment differs, however, at various points which will be commented upon in the fuller paper referred to above[11] where, also, the results of various experimental investigations not known in Copenhagen when the recent publications from this Institute were completed will be discussed.

[11] N. Bohr, Kgl. Danske Vid. Sels. Math.-fys. Medd. (Math.-phys. Comm., Acad. Sci. Copenhagen), **18**, 8 (1940).

FEBRUARY 1, 1941 PHYSICAL REVIEW VOLUME 59

Range and Straggling of Fission Fragments

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As reported in an earlier note to *The Physical Review*,[1] a study of the tracks of uranium fission fragments in a cloud chamber filled with argon gas has yielded evidence for two groups of tracks corresponding to the two types of fragments known from direct measurements of their kinetic energies and for chemical analysis of the radioactive products. Direct measurements of the ranges of fragments expelled in both directions simultaneously from thin uranium targets on thin foils in the cloud chamber gave some indication of two groups. Also, a statistical analysis of the number of side branches along the ranges of a large number of tracks from thick targets showed clearly the presence of two different kinds of tracks, of which the one had two to three times as many branches at the other over most of the range, while near the end, the numbers of branches were more nearly equal. On the basis of general considerations regarding the course of the range-velocity curves and the relative charges of the fragments, it was concluded that the heavier particle had the more branches and the shorter range, corresponding to its higher charge and lower initial velocity. Further experiments, both in argon gas[2] and in

[1] N. Bohr, J. K. Bøggild, K. J. Brostrøm and T. Lauritsen, Phys. Rev. **58**, 839 (1940).

[2] The work on argon is more fully reported in the Kgl. Danske Vid. Sels. Math.-fys. Medd. (Math.-phys. Comm., Acad. Sci. Copenhagen) **18**, 4 (1940).