The Alpha-Particle Model and the Properties of the Nucleus Be^8

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Reasons are given for renouncing any attempt to interpret the scattering of alpha-particles in helium in terms of a hypothetical law of force between alpha-particles. The analysis of scattering observations in the preceding paper is employed in conjunction with other information from theory and experiment to draw conclusions about the normal state and the first two excited levels of the compound nucleus Be^8:

ENERGY	WIDTH	LIFE	J
125 kev	1 to 100 ev	10^{-15} to 10^{-17} sec.	0
2.8 Mev	0.8 Mev	10^{-21} sec.	0
—	Very great	—	2

Difficulties remain in the interpretation of the reaction of B^{11} with protons to yield Be^8.

1. THE ALPHA-PARTICLE MODEL CONTRASTED WITH THE PICTURE OF ALPHA-PARTICLES AS PERMANENT UNITS IN NUCLEAR STRUCTURE

ALPHA-PARTICLES lost all claim to consideration as permanent units in nuclear structure when a general theory of nuclear constitution was developed.[1] Existence of such units, it was recognized, would be inconsistent with the large kinetic energy of zero-point motion of the elementary particles composing the nucleus and with the extremely close coupling between neutrons and protons. The nature of the elementary interactions, however, with their saturation character and apparent symmetry with respect to spin and charge, first clearly recognized by Breit, Feenberg and Wigner,[2] emphasized just as strongly the special stability of nuclei composed of even and equal numbers of neutrons and protons. The subsequent study of various simple laws of force having such properties, aided by instructive methods of group theory, led to a number of more detailed conclusions as to the energy and symmetry properties of low levels of light nuclei, including Be^8.

An alternative account of some of the symmetry properties of light nuclei is obtained, as the writer stressed,[3] by recognizing the existence within the nucleus of temporary alpha-particle groupings obeying the Bose-Einstein statistics. Indeed, for the symmetry of an alpha-particle structure to be able to manifest itself in the exclusion of certain states of oscillation and rotation otherwise to be expected, it is sufficient that the period of the motion in question be short in comparison with the time of rearrangement of the groupings. This condition was shown to be approximately satisfied in a number of cases. Thus low energy levels could be predicted in the even-even nuclei Be^8, C^{12}, O^{16},[3] and Ne^{20}.[4] This type of alpha-particle model was extended by Hafstad and Teller[5] to nuclei containing one extra elementary particle. For both kinds of nuclei the predictions of the simple model accorded to a striking extent with the results of the more abstract and apparently quite different calculations of Wigner, Feenberg, Phillips and Hund.[6,7] In combination with well-known selection rules these predictions have since proved of value in correlating a number of features of transformations observed in light nuclei.

The above-mentioned formulation of the alpha-particle model has to be sharply distinguished from the early conception of alpha-particles as permanent units in nuclear structure. That picture assumed not only that the interaction

[1] N. Bohr, Nature **137**, 344 (1936); N. Bohr and F. Kalckar, Kgl. Danske Vid. Sels. Math.-fys. Medd. **14**, No. 10 (1937).
[2] G. Breit and E. Feenberg, Phys. Rev. **50**, 850 (1936); E. Wigner, *ibid.* **51**, 106 (1937).
[3] J. Wheeler, Phys. Rev. **52**, 1083 (1937).
[4] E. Teller and J. A. Wheeler, Phys. Rev. **53**, 778 (1938).
[5] L. R. Hafstad and E. Teller, Phys. Rev. **54**, 681 (1938).
[6] E. Feenberg and E. Wigner, Phys. Rev. **51**, 95 (1937).
[7] E. Feenberg and M. Phillips, Phys. Rev. **51**, 597 (1937); F. Hund, Zeits. f. Physik **105**, 202 (1937).

between two alpha-particles could be described by a potential function, but also that forces were additive when several alpha-particles were present. For neither of these assumptions do present views of nuclear constitution give any well founded justification.[8] Any attempt to give a detailed account of the coupling between even two alpha-particles in terms of a potential function leads to unavoidable difficulties of principle, as is emphasized particularly in the accompanying paper of Margenau.[9] The alpha-particle model thus properly concerns itself, not with any special picture of the forces between the temporary alpha-particle groupings, but with the rotational and symmetry properties of the nucleus and a relatively small group of related questions.[10]

To attempt to determine a law of interaction of universal applicability from the scattering of alpha-particles in helium would be, from the above point of view, a mistaken endeavor. On the other hand, we shall be entirely justified in using the scattering analysis of the preceding paper[11] (a) to draw such conclusions as are there given about the range of the specific nuclear interaction, (b) to test the predictions of the alpha-particle model for the compound nucleus Be^8 and (c) in conjunction with this model to correlate the related experimental evidence and obtain as complete a picture as possible of the properties of this nucleus.

The nucleus Be^8, according to the alpha-particle model, may be compared to a symmetric diatomic molecule. In addition to a ground state of zero angular momentum ($^1S_0$), the properties of which are discussed in Section 2 below, it will on this view possess excited rotational states with even-valued angular momentum ($^1D_2$, $^1G_4$, $\cdots$). Energies of the order of 3 Mev and 9 Mev for the first two excited states were calculated[3] from an early estimate of the moment of inertia. A smaller inertial moment and a correspondingly increased energy of 5 or 6 Mev for the first stage of rotational excitation are suggested by Dennison[12] since he finds in the case of O^{16} that a similar alteration correlates the predictions of the model with observation. Besides the just mentioned rotational levels the beryllium nucleus will have other states some of the lowest of which according to the model, will be to a certain extent comparable with the first few vibrational levels of a diatomic molecule.

Dee and Gilbert[13] proposed that the nucleus Be^8 is formed, as an intermediate product in the reaction $B^{11}+H^1 \rightarrow 3He^4$, in a short-lived state with about 2.8 Mev of excitation. On this basis they found a satisfactory explanation for the distribution in energy of the alpha-particles observed by Oliphant, Kempton and Rutherford.[14] As a check Fink performed several interesting experiments[15] with counters in coincidence from the results of which it seemed necessary to doubt the interpretation in question. A closer study of the coincidence experiments, however, confirms the hypothesis of Dee and Gilbert (Section 3 below).

The level at 2.8 Mev had already, previous to Fink's experiments, been identified as[16] the $^1D_2$ state predicted both by the alpha-particle model[3] and by a more elaborate calculation[6] based on a Hartree model. Nevertheless, the observed level is actually one of zero angular momentum, according to the scattering analysis in the preceding paper.[11] Whether this particular state should now be correlated with the lowest excited vibrational state of the alpha-particle model is not certain. That the predicted $^1D_2$ level is much broader than the 2.8-Mev state is, however, clear from the analysis of the scattering of alpha-particles in helium. Evidence supporting this conclusion and indirectly confirming that the 2.8-Mev level has zero angular momentum is found by studying the distribution in energy of alpha-particles which result from the beta-ray decay of Li^8 (Section 4). The existence of a very broad $^1D_2$ level is one of the few checks on the alpha-particle model at present available in the case of Be^8.

[8] Arguments against additivity of forces are given by B. O. Grönblom and R. E. Marshak, Phys. Rev. **55**, 229 (1939).
[9] H. Margenau, Phys. Rev. **59**, 37 (1941).
[10] It has been applied for example to the treatment of nuclear spins and magnetic moments: H. A. Bethe, Phys. Rev. **53**, 842 (1938) and R. G. Sachs, *ibid.* **55**, 825 (1939).
[11] J. A. Wheeler, Phys. Rev. **59**, 16 (1941).
[12] D. M. Dennison, Phys. Rev. **57**, 454 (1940).
[13] P. I. Dee and C. W. Gilbert, Proc. Roy. Soc. **A154**, 291 (1936).
[14] M. L. E. Oliphant, A. E. Kempton and E. Rutherford, Proc. Roy. Soc. **A150**, 251 (1935).
[15] K. Fink, Ann. d. Physik **34**, 717 (1939).
[16] G. Breit and E. Wigner, Phys. Rev. **50**, 1191 (1936); **51**, 593 (1937).

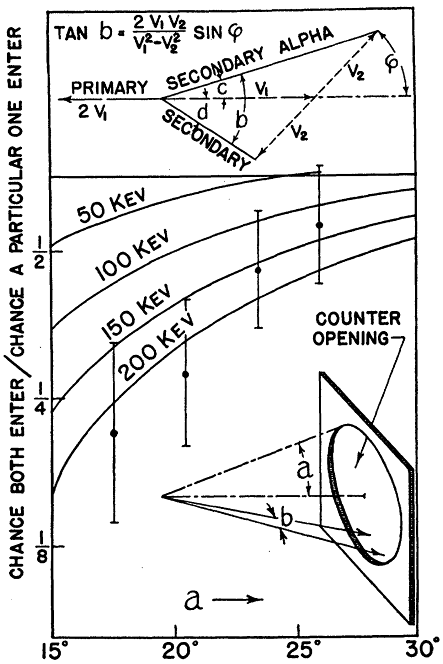

FIG. 1. The energy released in the disintegration of Be^8 determines the maximum angle of divergence between the secondary alpha-particles from the reaction of B^{11} with protons. The chance both secondaries shall enter a counter of aperture a is calculated for various assumed values of the energy of disintegration of normal Be^8, and compared with the observations of Laaff.

The application of standard selection rules gives a satisfactory account of most nuclear transformations which lead to the three states of Be^8 whose properties are investigated in Sections 2, 3 and 4. However, phenomena observed in the disintegration of B^{11} by protons lead to a number of unsolved questions (Section 5).

2. The Normal State of Be^8

The isotope Be^8 certainly is not present in natural beryllium to a relative abundance greater than 10^{-4}, according to Bleakney and his collaborators.[17] Nevertheless the question of the instability of this nucleus is left in doubt by even the most precise determination of its mass by the indirect method of energy balances; Allison, Skaggs and Smith[18] conclude Be^8 is stable by 65 ± 140 kilovolts.

That a normal Be^8 nucleus set in motion disintegrates into two alpha-particles before ionization losses have robbed it of appreciable energy was first shown by Kirchner, Laaff and Neuert[18a] and later confirmed by observations of Laaff[19] and of Fink.[15] From their ingenious experiments they could say that the energy of disintegration probably lies in the range 100 to 200 kev, but with no single value in this range could they obtain an account of their observations at all satisfactory. To propose an energy release indeterminate by a matter of almost 100 kilovolts or a probability of disintegration of Be^8 varying with direction would be equally unacceptable explanations of this difficulty. Theoretical estimates[20] give 10^{-16} to 10^{-19} second as the order of magnitude of the lifetime of a Be^8 nucleus unstable by 100 to 200 kev. This nucleus separates from the primary alpha-particle in the reaction

$$B^{11}+H^1\rightarrow C^{12}\rightarrow Be^8(\text{normal})+He^4\rightarrow 3He^4 \quad (1)$$

with a speed of 2.5×10^9 cm/sec. Its disintegration will therefore occur at a distance between 10^{-10} and 10^{-7} cm, too far away to be influenced in either direction or energy by the alpha-particle. Furthermore, the principle of uncertainty correlates with the lifetime an indeterminacy in energy or natural width of 0.01 kev to 10 kev, far too small to be significant in these experiments. However, a closer study reveals a much simpler source of the difficulties of interpretation encountered by Fink and Laaff.

The arrangement of Laaff consists of a counter which can distinguish between the entry of a single alpha-particle and a pair, and count the number of events of each kind. The proportion of pairs increases as the angular aperture a of the counter is enlarged (see Fig. 1). The chance a given alpha-particle shall enter the counter may be taken to be

$$S=\pi a^2/4\pi = a^2/4, \quad (2)$$

if we neglect the difference between an angle and its sine and assume random direction of emission. An accompanying alpha-particle which diverges from the first by an angle b may also enter, with a probability given by the fraction of a ring of angular radius b included within the

[17] W. Bleakney, J. P. Blewett, R. Sherr and R. Smoluchowski, Phys. Rev. **50**, 545 (1936).
[18] S. K. Allison, L. S. Skaggs and N. M. Smith, Jr., Phys. Rev. **57**, 550 (1940).
[18a] F. Kirchner, O. Laaff and H. Neuert, Naturwiss. **39**, 794 (1937).
[19] O. Laaff, Ann. d. Physik **32**, 760 (1938).
[20] H. A. Bethe, Rev. Mod. Phys. **9**, 167 (1937).

counter opening. Averaging this fraction over the possible points of entry of the first alpha-particle we obtain for the chance, C, of a double count the result

$$C=(a^2/2\pi)\{\arc\cos\,(b/2a)-(b/2a)(1-b^2/4a^2)^{\frac{1}{2}}\}. \quad (3)$$

For a given value of the energy of radioactive decay, the angle of divergence b will vary, as illustrated in the upper portion of Fig. 1, according to the angle φ between the velocity v_1 of the disintegrating nucleus and the velocity v_2 with which either secondary alpha-particle is ejected. With sufficient approximation we may write

$$b/2a=(1/2a)\arctan 2v_1v_2\sin\varphi/v_1^2-v_2^2 = v_1v_2\sin\varphi/a(v_1^2-v_2^2). \quad (4)$$

An isotropic distribution of disintegrations gives $\sin\varphi d\varphi$ as the probability that φ lie in any range $d\varphi$ between 0 and $\pi/2$ (Laaff assumed all values of φ are equally probable, thus underweighting the larger values of b). Averaging expression (3) over φ, we find as chance of a double count the result

$$D=(a^2/4)\{1+(4/3\pi)(c^{-1}-c)K(c)-(4/3\pi)(c^{-1}+c)E(c)\}, \quad (5)$$

when $\quad c=v_1v_2/a(v_1^2-v_2^2)$

is less than 1. When c is greater than 1 we have to replace the complete elliptic integrals $K(c)$ and $E(c)$ in (5) by $-cK(c^{-1})$ and $cE(c^{-1})$, respectively. Noting that an alpha-particle of velocity v_1 has an energy of 1.4 Mev[19] we can evaluate (5) for various assumed values of the energy of disintegration. The ratio, D/S, of the chance of a double count to the chance for a single chosen alpha-particle to enter the counter is plotted in Fig. 1 for comparison with Laaff's results. There is seen to be a range of values of the disintegration energy, any one of which now gives a satisfactory representation of the observations.

In interpreting the observations of Fink[15] it is necessary to take account of the isotropic distribution of the disintegration of normal Be^8 nuclei just as in the experiments of Laaff. Fink's arrangement allows a somewhat more accurate determination of the energy of disintegration. A counter (II), biased to respond only to long range primary alpha-particles, stands on one side of his thin boron target. On the other

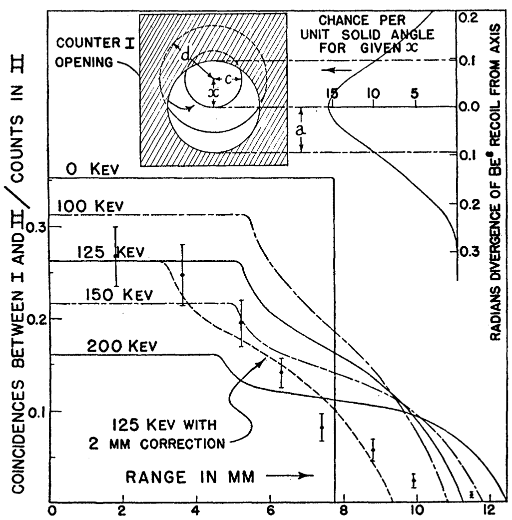

FIG. 2. Coincidences between two diametrically opposed counters, one of which responds only to fast primary alpha-particles, as increasingly thick foils are placed over the other counter (I). The observations of Fink come into satisfactory accord with curve calculated for 125 kev energy of disintegration if it be assumed that energy equivalent to 2 mm range is partly lost in the boron target, partly required to operate the counter.

side is a counter (I) into which should fly 35 percent of the recoiling Be^8 nuclei, if they were stable. The number of coincidences to be expected on this assumption between counts in I and counts in II is so small because counter I has an appreciable aperture, the target has a finite size, and the forward momentum of the compound nucleus C^{12} makes the direction of the recoiling Be^8 not quite opposite that of the primary alpha-particle. The distribution in direction of the recoils shown in the upper right-hand portion of Fig. 2 allows for all three effects and has, for simplicity, been averaged over azimuth. (The interpretation of the experiments would have been much simplified if counter I had had a considerably larger opening.) As stopping material is introduced before I there should come a moment, if normal Be^8 is stable, when all recoils are suddenly excluded, as shown by the curve marked 0 kev in Fig. 2. Fink's observations, also given in the figure, contradict this prediction. In fact, we know from Laaff's experiment that Be^8 disintegrates with an energy of the order of 150 kev. This value corresponds to a lifetime so short that already within the target the two secondary alpha-particles will have been produced and will be diverging from the original direction of the Be^8 by angles c and d, respectively, as indicated in Fig. 1. When a Be^8 nucleus starts off at an angle x with respect to the axis of counter I, a count will no longer be certain when x is smaller than the angular semiaperture a of the instrument, except in the case that $c+d$ is less than $2a$ and simultaneously x is less than $(a^2-cd)^{\frac{1}{2}}$. In every other case the probability of a pulse will be obtained by adding the fractions of the arcs of circles of radii c and d included within the opening. This probability then has to be averaged with respect to x in accordance with the curve in the upper right-hand portion of Fig. 2. We thus obtain the probability of a count in I simultaneous with one in II provided (a) that the line of disintegration of Be^8 is inclined at a definite angle x to the line of recoil and (b) that the normal velocities $v_1+v_2 \cos \varphi$ and $v_1-v_2 \cos \varphi$ of both secondary alpha-particles are sufficient to penetrate any foils over the counter. If only one secondary can enter to produce a pulse the chance of a count will be determined solely by the average fraction of arc of circle c included within the opening of the instrument. The chance of a count has finally to be averaged with respect to φ with the weight factor $\sin \varphi d\varphi$. The results of the rather long numerical calculations are plotted in Fig. 2 for comparison with Fink's results. No theoretical curve agrees entirely with his findings as they stand. If it be assumed, however, that energy corresponding to 2 mm of range is partly lost in the boron target and partly required to operate the counter, as does not seem unreasonable, then we obtain a satisfactory representation of the observations with an energy of disintegration of 125 kev, and a possible uncertainty in either direction of the order of 25 kev.

Combining all available evidence about the normal state of the nucleus Be^8, we conclude from the experiments of Laaff and Fink that the ground level is unstable by about 125 kev; from the alpha-particle model and also from the universal rule for even-even nuclei that it has zero angular momentum and even parity; and from the theory of radioactive decay[20] that it has a mean life of the order of 10^{-17} to 10^{-15} sec., corresponding to a natural width of the order of 1 to 100 ev.

3. The Hypothesis of Dee and Gilbert

Formation of Be^8 in the above-discussed normal state occurs in only a small percentage of the disintegrations of B^{11} by protons. Moreover, it leads to a quite characteristic distribution in energy of alpha-particles: primaries of 5.6 Mev, secondaries in the range $[\frac{1}{2}(5.6\text{ Mev})^{\frac{1}{2}} \pm\frac{1}{2}(0.125\text{ Mev})^{\frac{1}{2}}]^2$ or 1.02 to 1.86 Mev. Just below the primary group begins a very much more intense continuous spectrum of alpha-particles, extending down to the lowest energies at which observations have been made ($\sim$1.5 Mev) and effectively masking the just mentioned group of secondary alpha-particles. Dee and Gilbert[13] interpreted the strong spectrum as composed of two parts, one caused by primary alpha-particles recoiling from the Be^8 nuclei excited to a level at 2.8 Mev, the other caused by secondaries from the isotropic disintegration of such unstable nuclei. Working out this interpretation in more detail, Bethe obtained a satisfactory representation[20] of the observations of Oliphant, Kempton and Rutherford by assuming that the fraction of Be^8 nuclei, thus excited, with energy between E and $E+dE$, is given by

$$N(E)dE=\pi^{-1}\tfrac{1}{2}\Gamma dE/[(\tfrac{1}{2}\Gamma)^2+(E-E_0)^2], \quad (6)$$

with $E_0=2.8$ Mev, $\Gamma=0.8$ Mev.

To test Dee and Gilbert's hypothesis about an excited state of Be^8, Fink used the same arrangement of counters which he had employed in the experiments on normal Be^8, only increasing the thickness of covering foils to exclude the slow secondaries resulting from the decay of the latter nucleus. He varied the bias of the two counters with respect to energy and measured the ratio of coincidences to single counts. The ratios observed were less by an order of magnitude than those his calculations had led him to expect if an unstable Be^8 nucleus with about 2.8 Mev excitation were formed as an immediate step in the reaction of B^{11} with protons. Is there any escape from this difficulty?

Whether or not the hypothesis of a temporary Be^8 be true, the laws of conservation of momentum and energy can always be described by saying that a "primary" alpha-particle is ejected from an excited C^{12} nucleus; the remainder of the system recoils with equal and opposite total momentum, carrying within it any surplus energy not used as kinetic energy; this surplus goes into energy of separation of two "secondary" alpha-particles. If the second division occurs before the first alpha-particle has really left, the distinction between the two stages of the disintegration process is of course purely formal, the disintegration being really a three-body process. Nevertheless from measurements on the energy and momentum of the three alpha-particles from an individual transmutation, performed with whatever elaborate conceivable arrangement of counters in coincidence, it is in principle not possible to say whether or not the given reaction took place in one or two stages. Therefore one cannot, without further assumptions conclude that the peak in the energy distribution of Oliphant, Kempton and Rutherford indicates a two-step process. Nor even if one does assume that the process has this character can he deduce from their observations the probability distribution of energy of excitation of the intermediate Be^8 nuclei without information as to the correlation in direction between the two division processes. But any mechanism of three-body disintegration which could give the observed well-defined peak would be so specialized and improbable as to argue for the formation of a rather defined intermediate Be^8 nucleus; and in proportion as this nucleus is well separated from the primary alpha-particle, all directions for its dissociation into secondaries are to be expected as equally probable. Not only is the existence of such excited nuclei and their isotropic disintegration thus very probable, but also these assumptions lead, as already noted, to a consistent interpretation of the observations of the Cambridge group. In the light of Fink's verdict against the hypothesis of Dee and Gilbert we must, however, inquire whether they and Bethe have adopted for the distribution in energy of intermediate Be^8 nuclei the *only* law giving an acceptable representation of the observations.

Let $f(E)dE$ denote the fraction of Be^8 nuclei excited in the interval E to $E+dE$. Disintegration of such nuclei in every direction being assumed equally probable, and the total energy release in the boron reaction being $Q=8.8$ Mev, the secondaries from the given nuclei will be distributed uniformly in energy between the limits

$$Q/6+E/3\pm(E/3)^{\frac{1}{2}}(Q-E)^{\frac{1}{2}}. \tag{7}$$

Primaries and secondaries counted together, the total number of alpha-particles per disintegration in the energy interval A to $A+dA$ will be

$$g(A)dA=f(Q-3E/2)3dA/2 + \int_{E_{\min}}^{E_{\max}} [2dA/2(E/3)^{\frac{1}{2}}(Q-E)^{\frac{1}{2}}]f(E)dE, \tag{8}$$

where

$$E_{\max},\, E_{\min}=Q/4+3E/4 \pm E^{\frac{1}{2}}(9Q/8-27E/16)^{\frac{1}{2}}. \tag{9}$$

To obtain the excitation law $f(E)$ from a knowledge of the distribution function $g(A)$, introduce new variables θ, φ and t:

$$E=(Q/2)(1-\cos\theta)$$
$$A=(Q/2)(1-\cos\varphi).$$
$$g=3t/2.$$

Also extend the domain of definition of t and φ by writing

$$-t(-\varphi)=t(\varphi)=-t(2\pi-\varphi)$$
$$-f(-\varphi)=f(\varphi)=-f(2\pi-\varphi).$$

Equation (8) then reduces to

$$t(\varphi)=f(\varphi)+(2/3^{\frac{1}{2}})\int_{2\pi/3-\varphi}^{2\pi/3+\varphi} f(\varphi)d\varphi. \tag{10}$$

Analysis of this equation in terms of its characteristic functions $\sin n\varphi (n=1, 2, 3, \cdots)$ shows that it possesses a solution if and only if the distribution in energy of emergent alpha-particles satisfies the condition of energy conservation,

$$\int_0^{\pi} \cos\varphi t(\varphi)\sin\varphi d\varphi=0. \tag{11}$$

This solution is unique only up to an arbitrary additive multiple of $\sin 2\theta$:

$$f(\theta)=t(\theta)+\cos 2\theta\left\{\int_0^{\theta}(4/3) + \int_{2\pi/3-\theta}^{2\pi/3+\theta}(4/3)\right\}\sin 2\varphi t(\varphi)d\varphi - \sin 2\theta\left\{\int_0^{\theta}4+\int_{\theta}^{2\pi/3-\theta}(8/3) + \int_{2\pi/3-\theta}^{2\pi/3+\theta}(4/3)\right\}\cos 2\varphi t(\varphi)d\varphi + \text{const}\cdot\sin 2\theta. \tag{12}$$

An entirely general distribution function $t(\varphi)$ being given, the excitation function $f(\varphi)$ will generally be found to be negative somewhere in the interval $\varphi=0$ to $\varphi=\pi$, and therefore unacceptable from a physical point of view, *whatever* be the choice of the arbitrary constant. Consequently it is of interest that when the observed distribution[21] is put into the right side of Eq. (12) and $f(\theta)$ is calculated, one and *only* one value for the arbitrary constant leads to an acceptable excitation function. This unique law of distribution of excitation energy for immediate Be^8 nuclei must therefore be essentially identical with that of Bethe and, except that outside the resonance region proper $f(\theta)$ falls off somewhat more rapidly than would be expected from Eq. (6), the similarity is very close indeed.

Dee and Gilbert's hypothesis being thus satisfactory in every other respect, does it actually lead to discrepancy with the coincidence experiments of Fink? Denote the angular semiaperture of the two opposed counters by a_1 and a_2, respectively. They respond only to alpha-particles whose energy is greater than B_1 and B_2. In the course of a given observation let $Nf(E)dE$ disintegrations of B^{11} occur with ejection in equal probability in all directions of Be^8 nuclei excited between E and $E+dE$. Of the recoiling primary alpha-particles, on the average $(Na_1^2/4)f(E)dE$ or none at all will enter the counter I according as the primary energy $(\frac{2}{3})(Q-E)$ is greater than or less than the bias B_1. The secondaries will be distributed in energy uniformly between the limits of Eq. (7) and will therefore give a contribution $2(Na_1^2/4)f(E)dE$ if the bias is below the lower limit, 0 if it is above the upper limit, and

$$2(Na_1^2/4)f(E)dE\{Q/6+E/3+(E/3)^{\frac{1}{2}}(Q-E)^{\frac{1}{2}}-B_1\}/2(E/3)^{\frac{1}{2}}(Q-E)^{\frac{1}{2}},$$

if intermediate. The sum of the effects of primaries and secondaries, integrated over all values of the excitation energy E, will determine the number of counts Z_1 to be expected in counter I when it is set at a given bias, and similarly for counter II.

The intermediate nucleus Be^8 will have an excitation in the interval E to $E+dE$, the primary will enter counter I, and a secondary will enter counter II with a certain probability so that on the average such coincidences will occur to the number

$$(Na_1^2/4)2(a_2^2/4)(v_2+v_1/v_1)^2 f(E)dE, \quad (13)$$

provided that the energy of the primary exceeds the bias B_1 and that the bias B_2 is less than the energy, $Q/6+E/3+(E/3)^{\frac{1}{2}}(Q-E)^{\frac{1}{2}}$, of a secondary thrown off just opposite to the primary. The factor $2(a_2^2/4)(v_2+v_1/v_1)^2$, according to Fig. 1, represents the solid angle within which the line of disintegration of Be^8 must lie if the secondary is to deviate by a small angle a_2 or less from the prolongation of the course of the primary. To (13) has to be added a similar expression allowing for coincidences where the primary enters counter II. Coincidences caused by one secondary entering each counter are negligible in number because they also can only occur when the excited Be^8 disintegrates nearly parallel to the direction of the primary, and then both have simultaneously appreciable energy only if the Be^8 is highly excited, which happens only rarely.

The number C of coincidences, and its ratio to the number Z_2 of pulses in counter II, can be calculated analytically when the excitation function for Be^8 is given by expression (6). The fraction of a sphere covered by counter I and II, respectively, being $a_1^2/4=1/440$ and $a_2^2/4=1/740$, and each experiment of Fink having been continued until 2000 particles entered

TABLE I. *Comparison of coincidences in counters I and II. The observed values in the last two columns are the number of counts in counter I and number of coincidences between I and II observed by Fink when 2000 counts were registered in II.*

Bias in Mev B_1	B_2	Chance of Event $4Z_1/Na_1^2$	$16C/a_1^2a_2^2$	Calc. C	Z_1	Obs. Z_1	C
5.44	3.29	0.091	0.00	0	207	76	0
5.15	3.29	0.114	0.30	1	260	114	2
4.85	3.29	0.164	2.05	6	374	—	3
4.51	3.29	0.299	5.25	16	682	—	10
4.19	3.29	0.562	7.37	23	1280	—	16
3.84	3.29	1.060	10.21	31	2420	—	21
3.29	3.29	1.478	11.77	36	3370	3410	20
3.68	3.68	1.202	10.02	38	3370	—	19
4.04	4.04	0.785	5.82	34	3370	—	16
4.23	4.23	0.520	2.22	19	3370	—	26
4.40	4.40	0.366	0.00	0	3370	—	(1)
4.56	4.56	0.270	0.00	0	3370	—	0

[21] Relation (11) on the distribution function makes it possible to estimate fairly straightforwardly the distribution function below 1.5 Mev, where Oliphant, Kempton and Rutherford did not observe.

counter II, we can compute the numbers Z_1 and C to be expected and compare them with his results, as in Table I.

The hypothesis of Dee and Gilbert is seen to lead to results which by no means disagree by an order of magnitude with the observations of Fink. Details of the calculations which led to his contrary conclusions are not given in his very interesting paper, but the discrepancy presumably arose through identification of solid angle and circular angle as in the work by him and Laaff which was reanalyzed in Section 2. Even with proper allowance for geometry, predictions are not in complete accord with the observations, for at least one simple reason. In the first seven entries of Table I, counter II is used with constant bias and can be considered as monitoring counter I; so far as the measurement of Z_1 is concerned, Fink is to that extent repeating the observations of Oliphant, Kempton and Rutherford on the over-all distribution in energy of emergent alpha-particles. The fact that the first two entries of Z_1 do not agree thus only reemphasizes our earlier conclusion that expression (6) for the law of excitation of Be^8 does not fall off sufficiently rapidly outside the resonance region proper; the incomplete agreement is no argument against the disintegration theory proper of Dee and Gilbert. The *real* consequence of their hypothesis, the calculated number of coincidences, checks in order of magnitude with the observations. The discrepancies are outside the statistical errors only when the smaller values of bias are used. These discrepancies are certainly due in part to the oversimplified form assumed for the excitation function. Also allowance should be made in the calculation of the coincidences for possible asymmetry of the distribution in direction of the primary alpha-particles from the compound nucleus C^{12}. A recalculation is prevented by lack of information about the distribution for the primaries in which we are interested. It is known well only for the primaries from the reaction giving normal Be^8. That reaction, however, is responsible for no coincidences, and for a negligible number of single counts, in the above experiments.

We conclude that the hypothesis of Dee and Gilbert is consistent with all available observations when the limits of our information are taken into account. Knowledge that Be^8 has a well-defined resonance level at an excitation of about 3 Mev allows a unique analysis[11] of the scattering of alpha-particles in helium, with the result that the level in question possesses zero angular momentum.

4. Excited State with Two Units of Angular Momentum

Beta-ray decay of normal Li^8 leads to a broad unstable excited level of Be^8, according to Breit and Wigner,[22] who thus accounted qualitatively for the continuous distribution in energy of alpha-particles observed to accompany the electron emission. This explanation became more definite when Dee and Gilbert gave evidence for an excited level at about 2.8 Mev with a breadth of the order of 1 Mev. On the other hand, analysis in the preceding paper of observation on the scattering of 6-Mev to 7-Mev alpha-particles in helium indicates that Be^8 may possess a very broad resonance level with two units of angular momentum at an excitation of 4 or 5 Mev. Does either state furnish a satisfactory account of the observed distribution in energy of alpha-particles?

The electron and neutrino taking away a total energy between B and $B+dB$, the Be^8 nucleus will be left with an excitation in the interval dB at an energy given by $E=Q-B$. Here $Q=16$ Mev corresponds to the difference in mass between Li^8 and two free alpha-particles at rest. The probability per second that such a process shall occur will be nearly proportional to B^5, according to Fermi's theory of beta-decay, multiplied by the sum of the squares of the matrix elements between the initial state and all states in the interval E to $E-dB$. The behavior of the latter factor near a well-defined resonance level will follow the standard dispersion formula. Thus, the number of alpha-particles in a unit energy interval at $E/2$ will be expected to vary with E approximately as

$$B^5/[(E-E_0)^2+(\tfrac{1}{2}\Gamma)^2]. \quad (14)$$

This formula, with $E_0=2.8$ Mev, $\Gamma=0.8$ Mev,

[22] G. Breit and E. Wigner, Phys. Rev. **50**, 1191 (1936); **51**, 593 (1937).

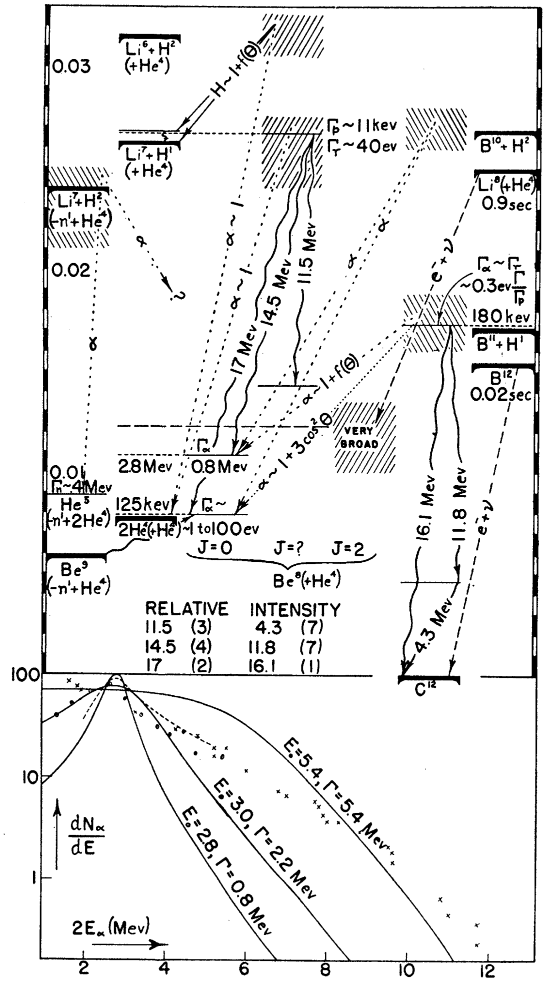

FIG. 3. The upper diagram summarizes certain transformations which lead to the nucleus Be^8. In the lower chart the circles and crosses represent observations of Rumbaugh, Roberts and Hafstad, reference 23, on the distribution in energy of alpha-particles which follow the decay of Li^8, and the dashed curve is based on the results of Fowler and Lauritsen. Disagreement between the measurements and the lower smooth curve shows that the end state of the decay process cannot be the 2.8-Mev level of Be^8.

gives a variation with energy in definite disagreement with the observations[23] (Fig. 3). Therefore we conclude that the beta-ray decay of Li^8 leads to a state of Be^8 which possesses a different angular momentum, and has much greater breadth, than the 2.8-Mev level. Assignment to the final state of an angular momentum of two units is most reasonable in view of analysis of the scattering of alpha-particles in helium. It is found impossible to assign to the end level a single energy E_0 and width Γ such that expression (14) will fit the observations. This situation, while at first sight unsatisfactory, is in the last analysis reasonable; the dispersion formula will not apply, and no satisfactory definition of level width and energy will in general be possible, when the "level" has a breadth comparable with its own excitation. To go further in such a case one would in principle have to solve a many-body problem and then carry through a calculation of matrix elements like that given by Kittel.[24]

On the above view the failure of Li^8 to decay with observable probability into either the 2.8-Mev level or the 0.12-Mev state of Be^8 is to be understood as caused by the action in each case of the same selection rule. Except for changing the role of the 2.8-Mev level we therefore come back to the interpretation of Breit and Wigner: The ground level of Li^8 is the component of highest angular momentum in the triplet $^3P_0$, $^3P_1$, $^3P_2$ predicted by Feenberg and Wigner[16] as lowest states of Li^8; a beta-ray transition in which the angular momentum changes by two units will be of negligible importance in comparison with a transition of the type 2→2. Absence of transitions to the 2.8-Mev level support the conclusion drawn from the alpha-particle scattering that this state has zero angular momentum.

Although the transition 2→2 is observed, the absolute value of the decay probability is, according to Kittel,[24] of the order of a hundredth what would off-hand have been expected from the lifetimes of nuclei of nearly that same mass. As explanation he suggests the change in spin (triplet→singlet) involved in the transformation, a change which is associated with the completely different symmetry character predicted for the two levels in question by Feenberg and Wigner.

5. SUMMARY: THE PROBLEM OF THE BORON REACTION

Analysis of observations on the scattering of alpha-particles in helium and of certain coincidence experiments has led to information about the first three levels of the nucleus Be^8. Figure 3

[23] W. A. Fowler and C. C. Lauritsen, Phys. Rev. **51**, 1103 (1937); L. H. Rumbaugh, R. B. Roberts and L. R. Hafstad, Phys. Rev. **51**, 1106 (1937); **54**, 672 (1938).

[24] C. Kittel, Phys. Rev. **55**, 515 (1939).

summarizes these results, together with some features of transformations in which Be^8 is involved. Satisfactory interpretations already given in the literature[25] for yields and angular distributions in most of these reactions need be changed only little, and in a fairly obvious manner, to take account of the new conclusions. However, to understand the selection rules for the reaction $B^{11}+H^1$ is at least as difficult now as when Oppenheimer and Serber treated the question[26] on the assumption that the 2.8-Mev level was of the type $^1D_2$. The evidence from the alpha-particle scattering has forced us to assign to this state the same angular momentum and parity as the $^1S_0$ ground level of Be^8, and the distribution of alpha-particles following the decay of Li^8 has indirectly helped to confirm this reassignment. Yet the disintegration of B^{11} leads to the ground state with a probability which shows a sharp resonance for 180-kev protons, while there is no apparent resonance in the much greater yield of Be^8 nuclei excited to the 2.8-Mev level. The intensity of both groups has been studied by various observers, particularly by Williams, Wells, Tate and Hill.[27] Still no available observations make it clear whether the apparent selection rule is absolute: (1) Does that state of the compound nucleus C^{12} which emits long (short) range alpha-particles also emit a few of short (long) range? If further observation shall reveal that the selection rule is not absolute, then we will have to ask, (2) why the probabilities are so different for C^{12}, in a given level, to break up into two states of Be^8 of the same angular momentum and parity; and (3) what must be the properties of the resonance level of C^{12} that it gives rise to a distribution in angle of long range alpha-particles approximately proportional to $1+3\cos^2\theta$.[28] If on the other hand, the selection rule shall be shown to be absolute, then an extremely difficult problem will arise, and it will be necessary to ask: (4) What is the possibility that the observations on the scattering of alpha-particles in helium, analyzed in the previous article, are in error by an amount much more than the limits stated in the original experimental papers? Further progress appears to depend on answering some of the above-mentioned questions. *Note added in proof.*—In contrast to the results of Neuert,[28] Jacobs and Whitson[29] now find that the angular distribution of both long and short range alpha-particles is spherically symmetric. They have kindly reported to the author that the curve for yield as a function of proton energy for *each* group of alpha-particles shows a resonance superposed on an exponentially increasing background. This result speaks against the operation of any absolute selection rule in the boron reaction and is consistent with the conclusion that the 2.8-Mev level of Be^8 has zero angular momentum. Moreover, the resonance and nonresonance processes, respectively, can now be interpreted in terms of singlet and triplet states of C^{12} formed by protons incident in *s* states. As principal problem there appears to remain only the explanation for the intensity difference between long and short range groups.

[25] Summarized in H. A. Bethe, Rev. Mod. Phys. **9**, 69 (1937); M. S. Livingston and H. A. Bethe, Rev. Mod. Phys. **9**, 245 (1937).

[26] J. R. Oppenheimer and R. Serber, Phys. Rev. **53**, 634 (1938).

[27] J. H. Williams, W. H. Wells, J. T. Tate and E. L. Hill, Phys. Rev. **51**, 434 (1937).

[28] H. Neuert, Ann. d. Physik **36**, 447 (1939).

[29] J. A. Jacobs and W. L. Whitson, Phys. Rev. **59**, 108A (1941).