Correlation Between Cosmic-Ray Intensity at Cheltenham and the Air Temperatures and Pressures for 1939

NIEL F. BEARDSLEY
University of Chicago, Chicago, Illinois
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A calculation has been made of correlations between the cosmic-ray data from Cheltenham and radiosonde data from the Anacostia Naval Air Station. The variance of the cosmic-ray intensity is found to depend 15 percent on total air pressure, 40 percent on the distribution of the air mass as correlated with the surface temperature, 10 percent on world-wide changes with 30 percent still unaccounted for. Increased air mass in the upper air reduces the surface cosmic-ray intensity more than a corresponding increase in air mass at lower levels. The calculations use 220 days in 1939.

THERE have recently been published several papers having to do with the effect of the temperature distribution and air mass distribution of the atmosphere above a cosmic-ray meter on the readings of that meter.[1–3]

We have made a number of correlation computations between the 1939 data from a cosmic-ray meter at Cheltenham and radiosonde balloon data from the nearby Naval Air Station at Anacostia. There were 220 days in 1939 on which the balloons reached a height of 12 km and at the same time data were being taken by the cosmic-ray meters at both Cheltenham and also at Huancayo, Peru. We have used the two-hour mean from a type C ionization meter shielded with 12 cm of lead located at Cheltenham, taken at the hour of the radiosonde flight and, as described later, the daily mean of data from a similar meter at Huancayo, Peru. Most of the flights were made at 5 to 6 o'clock in the morning E.S.T.

For these computations, the mass of the air was divided into four layers. The annual mean pressures were

$\bar{P}_{\text{surface}} = 1015.7$ millibars $\quad \bar{P}_{8000} = 362.2$
$\bar{P}_{4000\text{ meters}} = 620.1 \quad \bar{P}_{12,000} = 198.7.$

The variations from the year's means of each layer are indicated by the notation:

P_2 = variation from the year's mean of $P_{\text{surface}} - P_{4000\text{ meters}}$
P_3 = variation from the year's mean of $P_{4000} - P_{8000\text{ meters}}$
P_4 = variation from the year's mean of $P_{8000} - P_{12,000\text{ meters}}$
P_5 = variation from the year's mean of $P_{12,000\text{ meters}}$
P_0 = variation from the year's mean of total barometric pressure
T = variation from the year's mean of surface temperature.

Pressures are measured in millibars and temperatures in degrees centigrade. To avoid double subscripts, 0, 2, 3, 4, 5 as subscripts refer to P_0, P_2, P_3, P_4 and P_5, respectively.

The first computations were purely meteorological in nature and are represented in Fig. 1. Correlation coefficients (r_{ij}) and "path coefficients" (β)[4] were computed between the total pressure and the pressure differences in the different layers. Also similar calculations were made using the surface temperature *vs.* these pressure differences. The regression coefficients were expressed as "b's" computed from equations of the form

$$T = +b_{T2}P_2 + b_{T3}P_3 + b_{T4}P_4 + b_{T5}P_5,$$

giving

$$T = -0.5696P_2 - 0.1121P_3 - 0.2681P_4 + 0.3069P_5.$$

The similar equation for P_0 would, of course, have all b's equal to unity. The regression coefficients were also expressed by the standard partial regression coefficients, the nondimensional "β's" computed from equations of the form

$$P_0/\sigma_0 = \beta_{02}P_2/\sigma_2 + \beta_{03}P_3/\sigma_3 + \beta_{04}P_4/\sigma_4 + \beta_{05}P_5/\sigma_5.$$

There is then between b and β the relation that

[1] N. Beardsley, Phys. Rev. **57**, 336 (1940).
[2] Y. Nishina, Y. Sekido, H. Simamura and H. Arakawa, Phys. Rev. **57**, 663 and 1050 (1940).
[3] D. Loughridge and P. Gast, Phys. Rev. **57**, 938 (1940).
[4] Sewall Wright, Ann. Math. Statistics **5**, 161 (1934).

$b_{ij}=\beta_{ij}\sigma_i/\sigma_j$ where σ_i is the standard deviation, $(P_i^2/N)^{\frac{1}{2}}$. This use of standard units $[(x-\bar{x})/\sigma]$ reduces the variance (square of the standard deviation) of each to unity. Then the square of the total correlation between P_0 or T with P_2, P_3, P_4, P_5 is the total contribution of P_2, P_3, P_4 and P_5 to the variance of P_0 or T expressed as a fraction. This total correlation (R) is given by[5,6]

$$\sum_{i=2}^{i=5} r_{0i}\beta_{0i}=R^2_{0.2345} \quad \text{and} \quad \sum_{i=2}^{i=5} r_{Ti}\beta_{Ti}=R^2_{T.2345}$$

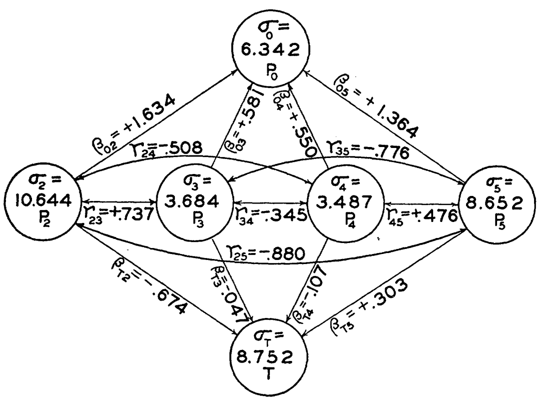

FIG. 1. Correlation diagram connecting P_2, P_3, P_4, P_5 with P_0 and also with T.

in which $r_{0i}\beta_{0i}$ is the contribution of P_i to the variance of P_0. The variance of one quantity because of a second quantity is a better measure of the influence of that second quantity on the first than is either the regression coefficient or the correlation coefficient alone since it depends on both. Correlation coefficients alone measure the closeness of fit of the changes of the two quantities but do not indicate the magnitude and hence the importance of those changes. Regression coefficients give the relative magnitude of the changes of the two quantities but do not indicate the closeness with which these changes follow each other.

Since $P_2+P_3+P_4+P_5=P_0$, $R^2_{0.2345}=1$ and no other factors are involved. However, $R^2_{T.2345}=0.8824$. Since the variance (in standard units) of T is unity, this shows that $1-0.8824=0.1176$ or about 12 percent of the variance of T is not associated with a change of the pressures used. The influence of this unknown agent "U," which is assumed to be independent of the P_i's may be written as

$$r_{TU}\beta_{TU}=1-R^2_{T.2345}.$$

This grouping of unknown effects into a single number causes that number to include (1) all experimental errors, (2) any changes in mass distribution within one (or more) of the chosen layers and (3) any lack of validity in our assumed linearity of regression.

Correlation of Cosmic-Ray Intensity with Air Pressure and Temperature

The bi-hourly means of the departures from balance of the meter at Cheltenham, with bursts deducted but not corrected for barometer or temperature, expressed as tenths percent of the total mean intensity (82 ions cm^{-3} sec.^{-8}) were taken at the hour of the radiosonde flight.

I = Variation of these means from their average value.

I was correlated with the pressure intervals and separately with P_0 and T. For dimensional equations we obtained

$$I=-0.8567P_2-0.9120P_3-1.4070P_3-1.9937P_5$$

and

$$I=-1.5517P_0-1.4507T.$$

In all of these computations we have minimized only the variations of the independent variables of the regression lines. This is equivalent to assuming that the errors of the cosmic-ray data, P_0 and T are negligible compared with the errors of P_2, P_3, P_4, P_5. This was done both because of the simplification of such computations and because it seemed reasonable that data taken at the surface would be much more accurate than balloon data. Figure 2 shows the results in terms of correlation coefficients and path coefficients.

$r_{I2}=+0.4298$	$r_{I2}\beta_{I2}=-0.2598$
$r_{I3}=+0.3520$	$r_{I3}\beta_{I3}=-0.0805$
$r_{I4}=-0.5074$	$r_{I4}\beta_{I4}=+0.1695$
$r_{I5}=-0.6243$	$r_{I5}\beta_{I5}=+0.7331$
	$R^2_{I.2345}=+0.5623\sim 56\%$

$$r_{IU}\beta_{IU}=1-0.5623=0.4377\sim 44 \text{ percent.}$$

[5] H. L. Rietz, *Handbook of Mathematical Statistics*, page 141.
[6] Henry Schultz, *Statistical Laws of Demand and Supply*, page 174.

It appears that about 56 percent of the variance of I is the result of changes of the four pressure intervals leaving about 44 percent unaccounted for. This 44 percent includes all the experimental error, both of radiosonde instruments and of the cosmic-ray meters. Because of the method of computation used, any other quantities not taken into the computations specifically but which are correlated with the pressure intervals and which directly affect the cosmic-ray intensity would be included in the pressure effect. Any other factors which are not correlated with the pressures would appear in this 44 percent. For the P_0T correlation,

$$r_{I0}=-0.2238 \qquad r_{I0}\beta_{I0}=+0.1500\sim 15\%$$
$$r_{IT}=-0.5186 \qquad r_{IT}\beta_{IT}=+0.4483\sim 45\%$$
$$R^2_{I.0T}=+0.5983\sim 60\%.$$

Then $r_{IU}\beta_{IU}=1-0.5983=0.4017\sim 40$ percent, indicating that 15 percent of the variance of I is caused by total pressure and 45 percent is caused by changes of distribution of air mass as measured by the surface temperature leaving only 40 percent unaccounted for. The difference between 40 percent and 44 percent implies that P_0 and T are better sources of correlation than P_2, P_3, P_4, P_5 though the difference is too small to be significant.

It may be that there are other effects (e.g. magnetic) which by being correlated with the surface temperature cause the temperature to be more important than its effect on mass distribution would indicate. As a test of this point, the Cheltenham data were changed by subtracting 1.11 times the variation of the daily means of intensity at Huancayo from the 1939 annual mean. This 1.11 factor is taken from Forbush[7] and is probably not the best factor for 1939 but at least its use reduces the effect of world-wide magnetic changes. Figure 3 shows the results of this new computation. Here the correction for T and P_0 leaves 42 percent of the variance of $I'=I-1.11H$ unaccounted for while P_2, P_3, P_4, P_5 corrections show only 37 percent unaccounted for.

[7] S. E. Forbush, Phys. Rev. **54**, 983 (1938).

$$r_{I'2}=+0.4108 \qquad r_{I'2}\beta_{I'2}=-0.2892$$
$$r_{I'3}=+0.3190 \qquad r_{I'3}\beta_{I'3}=-0.1007$$
$$r_{I'4}=-0.5074 \qquad r_{I'4}\beta_{I'4}=+0.1711$$
$$r_{I'5}=-0.6334 \qquad r_{I'5}\beta_{I'5}=+0.8470$$
$$R^2_{I'.2345}=+0.6281\sim 63\%.$$
$$r_{I'0}=-0.2865 \qquad r_{I'0}\beta_{I'0}=+0.2034$$
$$r_{I'T}=-0.4541 \qquad r_{I'T}\beta_{I'T}=+0.3727$$
$$R^2_{I'.0T}=+0.5761\sim 58\%.$$

The increase in variance caused by P_2, P_3, P_4, P_5 from 56 to 63 percent and the decrease in variance caused by P_0, T from 60 to 58 percent which occur when the cosmic-ray data are corrected for world-wide changes indicate that these changes are correlated with surface temperature. (Such correlation in no way implies that one is the cause of the other.)

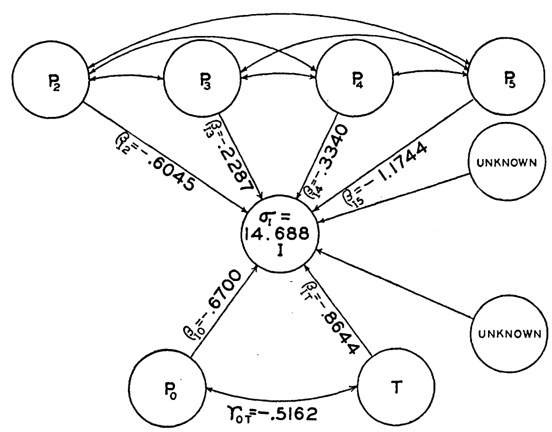

FIG. 2. Correlation diagram connecting the uncorrected cosmic-ray intensity at Cheltenham with P_2, P_3, P_4, P_5 and also with P_0, T.

This temperature-magnetic effect may be tested directly as in Fig. 4 where I is correlated with P_0 and T on the one hand and with P_0, T and H (Huancayo) on the other.

$$r_{I0}\beta_{I0}=+0.1552 \qquad r_{IH}=+0.3260$$
$$r_{IH}\beta_{IH}=+0.1050 \qquad r_{I0}=-0.2238$$
$$r_{IT}\beta_{IT}=+0.4407 \qquad r_{IT}=-0.5186$$
$$R^2_{I.0HT}=+0.7009.$$

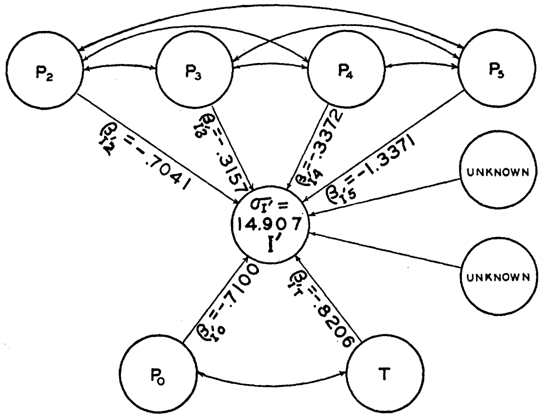

FIG. 3. Correlation diagram connecting the cosmic-ray intensity at Cheltenham which has been corrected for the intensity at Huancayo with P_2, P_3, P_4, P_5 and also with P_0, T.

$r_{TH} = -0.0830$ and $r_{0H} = +0.0962$ are below the level of significance.[8]

Figure 4 attributes about 15 percent of the variance of I to the surface pressure, 45 percent to the effect of temperature and 10 percent to changes in world-wide magnetism as such changes are measured by variations in the intensity at Huancayo where there is little seasonal or even daily change in air conditions.

In a further calculation an equation of the form

$$C_0 = b_{C2}P_2 + b_{C3}P_3 + b_{C4}P_4 + b_{C5}P_5$$

was used where C_0 represents the Cheltenham data which have been corrected for both barometer and Huancayo. This equation is not quite a proper assumption since one P can change with the others held constant only if the total pressure changes, yet the data had been corrected for variations in total pressure! Only the relations between these regression coefficients and not their absolute value can have a meaning. Evaluation of this equation gives

$$C_0 = +0.2113P_2 - 0.1128P_3 - 0.1962P_4 - 1.0923P_5. \quad \text{(A)}$$

The smaller of these coefficients are of doubtful significance. A similar equation with data not corrected for barometer but corrected for Huancayo gave

$$C = -1.0126P_2 - 1.2776P_3 - 1.4423P_4 - 2.3038P_5. \quad \text{(B)}$$

The almost constant differences between these regression coefficients; 1.2239, 1.1648, 1.2452, 1.2114 are nearly the same as the single regression coefficient of

$$C_0 = C + 1.2187P_0. \quad \text{(C)}$$

It is not possible to separate rigorously the total mass effects from the effects produced by changing the distribution of the air mass since the total mass of air is correlated with the distribution. However, the increasing negative magnitude of the coefficients of Eqs. (A) and (B) with height and the fact that they differ essentially only by the total mass coefficient (with its correlated distribution effect) is significant. Evidently for a constant total amount of air, the concentrating of that air in the upper layers results in a greater reduction in cosmic-ray intensity than concentrating it in the lower layers. This is, of course, in agreement with a negative temperature coefficient as is ordinarily found and is just what is to be expected since mesotrons are supposed to suffer both mass absorption and also a loss in intensity caused by their disintegration.

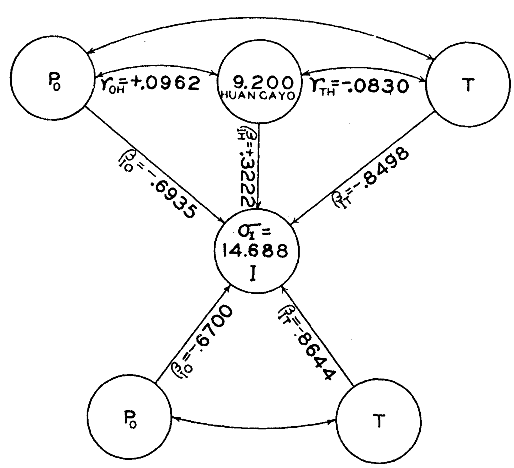

FIG. 4. Correlation diagram connecting the uncorrected cosmic-ray intensity at Cheltenham with P_0, H, T and also with P_0, T.

[8] "Student's" t test tables from R. A. Fisher, *Statistical Methods for Research Workers* (Oliver and Boyd, 1932).

Conclusion

The division of the variance of the cosmic-ray intensity into 15 percent caused by barometric pressure, 45 percent caused by distribution of air mass as correlated with surface temperature and 10 percent caused by world-wide changes leaving 30 percent unaccounted for is, we believe, new, and if it can be corroborated by the 1940 data when they are available, certainly important.

We wish to thank Professor A. H. Compton for discussing this with us, Dr. John A. Fleming of the Department of Terrestrial Magnetism for making available the Cheltenham and Huancayo data and Mr. D. M. Little of the Aerological Division of the U. S. Weather Bureau for supplying us with the necessary air data.

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A Precise Determination of the Energy of the Neutrons from the Deuteron-Deuterium Reaction*

T. W. Bonner†
Rice Institute, Houston, Texas
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A precise determination of the energy of the neutrons from the deuteron-deuterium reaction has been made. The energy of the neutrons emitted in the forward direction to the 0.52-Mev deuterons which produced the disintegrations was found to be 3.58 ± 0.03 Mev. The disintegration Q value of the reaction is 3.31 ± 0.03 Mev. The mass of He^3 calculated from this Q value is 3.01698 ± 0.00006 mass units.

THE neutrons produced when deuterium is bombarded by deuterons are known to come from the reaction

$$H^2+H^2 \rightarrow He^3+n^1+Q_1 \qquad (1)$$

and those neutrons are now known to be monoenergetic.[1] Such a source of monoenergetic neutrons is most important for neutron scattering experiments as well as for disintegration experiments by neutrons. Furthermore an accurate knowledge of the energy of the neutrons involved in such experiments is essential. For this reason it is important to find their energy accurately at one bombarding energy and angle of observation so that their energies under different bombarding conditions may be accurately calculated.

An accurate knowledge of the energy of the neutrons from reaction (1) also gives one of the best methods of determining the mass of He^3. The value of Q_1 together with the energy liberated in the other deuteron-deuterium reaction

$$H^2+H^2 \rightarrow H^3+H^1+Q_2 \qquad (2)$$

gives a direct comparison of the binding energy of He^3 and H^3 as well as a means of finding the stability of He^3 and H^3. Several determinations[2] have already been made on the value of Q_1 and it was the purpose of the present experiment to improve on the accuracy of these determinations.

The method of determining neutron energies in this experiment is the same as that previously used.[3] The energy of the neutrons is obtained from the range of recoil-protons in a methane-filled cloud chamber. Because of the near equality of the mass of proton and neutron, a proton recoiling in the forward direction gets

* A preliminary report of these results was given in Nature **143**, 681 (1939).

† The experimental work was carried out at the Cavendish Laboratory, Cambridge, England, while the writer held a Guggenheim Fellowship.

[1] E. Hudspeth and H. Dunlap, Phys. Rev. **57**, 971 (1940); R. D. Park and J. C. Mouzon, *ibid.* **58**, 43 (1940); H. T. Richards and E. Hudspeth, *ibid.* **58**, 382 (1940); H. H. Barschall and M. H. Kanner, *ibid.* **58**, 590 (1940). All the neutrons produced do not have exactly the same energy. They are monoenergetic only when they are produced in a thin target by deuterons of one energy and are projected at the same angle to the deuteron beam.

[2] P. I. Dee, Proc. Roy. Soc. **148**, 623 (1935); T. W. Bonner and W. M. Brubaker, Phys. Rev. **49**, 19 (1936); E. Baldinger, P. Huber and H. Staub, Helv. Phys. Acta **11**, 245 (1938); T. W. Bonner, Phys. Rev. **53**, 711 (1938).

[3] T. W. Bonner and W. M. Brubaker, Phys. Rev. **47**, 910 (1935); **48**, 742 (1935); **49**, 19 (1936); **50**, 308 (1936).