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Variation of the Rate of Decay of Mesotrons with Momentum

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In order to determine the dependence of the probability of decay on momentum, mesotrons with range between 196 and 311 g/cm² of lead and mesotrons with range larger than 311 g/cm² of lead were investigated separately. The softer group of mesotrons was found to disintegrate at a rate about three times faster than the more penetrating group, in agreement with the theoretical predictions based on the relativity change in rate of a moving clock. A new value of the proper lifetime of mesotrons of $(2.4\pm0.3)\times10^{-6}$ sec. is determined, based upon measurements with particles with momentum of approximately 5×10^{8} ev/c.

INTRODUCTION

RECENT experiments on the variation of cosmic-ray intensity with altitude have shown that the rate of decrease of the mesotron component with increasing atmospheric depth cannot be accounted for completely by ordinary ionization losses. It has been established, namely, that the number of mesotrons is much more strongly reduced by a layer of air than by a layer of condensed material which is equivalent to the air layer with regard to ionization losses.[1-5]

The anomalous absorption in air is interpreted on the hypothesis that mesotrons disintegrate spontaneously with a proper lifetime of the order of a few microseconds. According to this assumption, a considerable fraction of the mesotron beam will disappear by disintegration while traveling in the atmosphere. No appreciable number of mesotrons, however, will disintegrate within a condensed absorber, even equivalent in mass to the whole thickness of the atmosphere, because the time required for the traversal of such an absorber is very short compared with the lifetime of mesotrons.

A simple relativistic consideration shows that if the absorption anomaly of mesotrons is due to spontaneous decay it must be more pronounced for mesotrons of low energy than for mesotrons of high energy. In fact, let τ_0 be the "proper lifetime" of mesotrons; i.e., the lifetime measured in a frame of reference in which the mesotron is at rest, and τ the lifetime measured in a frame of reference in which the mesotron is moving with a velocity β.[6] Then

$$\tau=\tau_0/(1-\beta^2)^{\frac{1}{2}} \qquad (1)$$

and the "average range before decay" L; i.e., the average distance traveled by the mesotrons before disintegrating, becomes

$$L=\beta\tau=p\tau_0/\mu, \qquad (2)$$

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[1] B. Rossi, N. Hilberry and J. B. Hoag, (a) Phys. Rev. **56**, 837 (1939); and (b) Phys. Rev. **57**, 461 (1940).
[2] W. M. Nielsen, C. M. Ryerson, L. W. Nordheim and K. Z. Morgan, Phys. Rev. **57**, 158 (1940).
[3] M. Ageno, G. Bernardini, N. B. Cacciapuoti, B. Ferretti and G. C. Wick, Phys. Rev. **57**, 945 (1940).
[4] H. V. Neher and H. G. Stever, Phys. Rev. **58**, 766 (1940).
[5] A. Ehmert, Zeits. f. Physik **115**, 333 (1940).
[6] We shall use throughout the paper the system of units described by B. Rossi, Phys. Rev. **57**, 660 (1940).

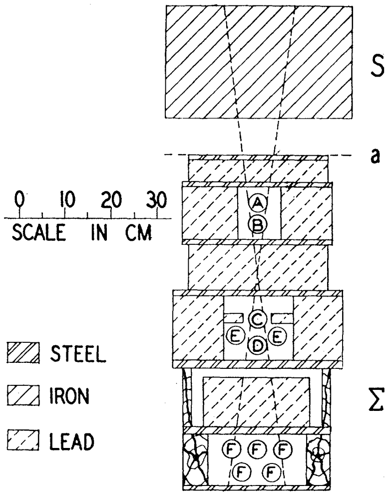

FIG. 1. Experimental arrangement.

where μ is the mass and $p=\mu\beta/(1-\beta^2)^{\frac{1}{2}}$ is the momentum of the mesotrons. The probability of decay per centimeter path is obviously $1/L$. It is seen that the average range is directly proportional, and the probability of decay inversely proportional, to the momentum.

The experiments described in the present paper were primarily designed to test the dependence of disintegration probability on momentum expressed by Eq. (2). The purpose was to provide an additional check of the disintegration hypothesis and simultaneously to verify the relativistic transformation formula for time intervals. Further experimental evidence on the subject of the decay was particularly desirable in view of Fermi's recent theory showing that the energy losses of fast particles in condensed materials are appreciably reduced by the dielectric polarization of the medium.[7] According to this theory even *stable* mesotrons are absorbed by gases more strongly than by solid or liquid materials of the same mass per cm^2. The difference in absorption due to polarization increases with increasing mesotron momentum; i.e., varies oppositely from the difference due to decay. The polarization effect, as calculated by Fermi, was quantitatively inadequate to account for the experimental results already obtained. Yet it was interesting to investigate whether the observed absorption anomaly was a decreasing function of the mesotron momentum, as the anomaly attributed to the decay, or an increasing function, as the anomaly attributed to the polarization.

An attempt to determine the rates of decay of mesotrons of different momenta has been reported by Nielsen, Ryerson, Nordheim and Morgan.[2] Mesotron groups of different average momentum were selected by taking the difference between the intensities of the mesotron beam after filtration through various thicknesses of lead. By this method, however, it is hardly possible to reach a sufficient accuracy, since the difference is small compared with the quantities directly measured. In our experiments the statistical precision was greatly improved by measuring the difference directly; i.e., by recording only mesotrons which can traverse a given thickness of lead but are stopped by a certain additional absorber.

Experimental Method

The experimental arrangement is schematically represented in Fig. 1. The Geiger-Müller counter tubes were of the self-quenching type. Their internal diameter was 4 cm and their effective lengths were as follows: counters A, B, C and E, 27 cm; counter D, 20 cm; counters F, 60 cm. The five counters F and the two counters E were all connected in parallel. The counter battery F covered completely the solid angle subtended by counters A, B, C and D. In order to cut off the soft component, 5 cm of lead was permanently placed above counter A and 10 cm of lead between counters B and C. Including the material of the frame, the permanent absorber above or between counters A, B, C and D was equivalent in absorption power to 186 g/cm^2 of lead, while that between D and F was equivalent to 10 g/cm^2 of lead. Counters A, B, C and D were protected on the side by lead walls 11.5 cm thick. An additional lead absorber Σ of 115 g/cm^2 could be introduced between D and F and an absorber S made of iron plates could be arranged above the apparatus so as to cover the whole solid angle subtended by counters A, B, C and D. The apparatus was set up in a moving van which could be taken to different altitudes on mountain roads. The whole system, except for the absorber S, was enclosed in a thermostatic box.

By means of an appropriate vacuum-tube

[7] E. Fermi, Phys. Rev. 57, 485 (1940).

circuit, the following events were simultaneously recorded: (1) Fivefold coincidences between counters A, B, C, D and one of counters F or E (coincidences $[ABCD(E+F)]$); (2) coincidences between counters A, B, C and D not accompanied by a pulse either of counters F or of counters E (anticoincidences $[ABCD-(E+F)]$). The coincidences $[ABCD(E+F)]$ were mainly caused by mesotrons going through counters A, B, C, D and F. After entering the apparatus; i.e., after crossing the surface indicated by a in Fig. 1, these mesotrons had to traverse 196 g/cm^2 of lead when there was no absorber in Σ, or 311 g/cm^2 of lead when 115 g/cm^2 of lead were placed in Σ. Chance coincidences were negligible and coincidences due to air showers were certainly rare on account of the heavy lead shield at the side of the counters. Coincidences caused by ionization showers generated by mesotrons in the various absorbers could not introduce any error because they were a small and *constant* fraction of the coincidences caused directly by mesotron traversals.[8] Thus, one is justified in taking the counting rate $[ABCD(E+F)]$ as a measure of the number N of mesotrons entering the apparatus with a residual range larger than the total amount of matter present above or between the counters.

Anticoincidences $[ABCD-(E+F)]$ could be accounted for by one of the following events. (a) A mesotron has traversed A, B, C and D and has been stopped between D and F. (b) A mesotron has gone through A, B, C, D and F, but the counter battery F has failed to detect it for lack of efficiency. (c) A chance coincidence between pulses of counters A, B, C and D has occurred. (d) A mesotron has traversed counters A, B, C and D, but has been scattered through a wide angle so as to miss the counter battery F.

The stopping of mesotrons between D and F (case (a)) is certainly the main origin of the anticoincidences recorded with lead in Σ, which are, as we shall see, several times more frequent than those recorded without lead. The events described under (b) and (c) are about equally frequent with and without the absorber Σ. The scattering (case (d)) may contribute a small number of anticoincidences, which is not necessarily the same with and without the absorber. Since, however, only slow mesotrons are appreciably scattered as well as absorbed, the difference in the number of anticoincidences due to scattering is a small and constant fraction of the difference in the number of anticoincidences due to absorption. Thus, the difference between the number of anticoincidences recorded with and without lead in Σ is proportional, if not accurately equal, to the number of mesotrons which traverse 10 g/cm^2 of lead and are absorbed by 125 g/cm^2 of lead between D and F. These mesotrons are those which enter the apparatus with a residual range between $R_a=196$ and $R_b=311$ g/cm^2 of lead.

[8] This was not always the case for the experimental arrangements previously used. See the discussion on p. 464, reference 1(b).

The Measurements

Measurements were taken alternately at Denver, Colorado, and at Echo Lake, approximately 30 miles west of Denver. The geomagnetic latitude is practically the same (49° N) for both locations. The difference in altitude is 1624 m. The difference in atmospheric pressure, as measured during the experiments, was 108 mm Hg, equivalent to 147 g/cm^2.

The measurements at Echo Lake were performed partly with an iron absorber of 200 g/cm^2 in S and partly without this absorber. No iron absorber was used at Denver. Three complete sets of measurements were carried out at Denver and two at Echo Lake. The deviations of the single readings from the averages were within the statistical fluctuations. The final results are summarized in Table I. The errors given are the standard statistical deviations.

TABLE I. *Summary of the measurements at Denver and Echo Lake.* $[ABCD(E+F)]$ *and* $[ABCD-(E+F)]$ *are the numbers of coincidences and anticoincidences per minute.* Δ *is the difference between the numbers of anticoincidences per minute recorded with and without 115 g/cm*^{2} *of lead in* Σ. *The errors are the standard statistical deviations.*

Location	Absorber g/cm^2 S(Fe)	Σ(Pb)	Time Min.	$[ABCD \times(E+F)]$	$[ABCD -(E+F)]$	Δ
Denver $z=1616$ m $h=856$ g/cm^2	0	0	3384	5.16±0.048	0.091±0.0052	0.276 ±0.009
	0	115	6783	4.79±0.027	0.367±0.0074	
Echo Lake $z=3240$ m $h=709$ g/cm^2	0	0	308	6.87±0.15	0.15 ±0.02	0.53 ±0.03
	0	115	1469	6.49±0.066	0.68 ±0.021	
	200	0	2846	5.73±0.045	0.119±0.0064	0.394 ±0.012
	200	115	5362	5.43±0.032	0.513±0.0098	

According to the discussion in the foregoing section, the counting rates $[ABCD(E+F)]$ with 115 g/cm^2 of lead in Σ can be taken as a measure of the number N of mesotrons entering the apparatus with a residual range larger than $R_b=311$ g/cm^2 of lead, while the figures listed under Δ can be taken as a measure of the number n of mesotrons entering the apparatus with a residual range between $R_a=196$ and $R_b=311$ g/cm^2 of lead.

Let N_1, N_1' and N_2 be the values of N at Echo Lake under 200 g/cm^2 of iron, at Echo Lake without the iron absorber and at Denver without the iron absorber, respectively. Let n_1, n_1' and n_2 be the corresponding values of n. Considering first the measurements taken without the iron absorber, we have

$$n_1'/N_1'=0.082\pm0.005, \quad n_2/N_2=0.058\pm0.002.$$

It appears that the fractional number of slow mesotrons increases rapidly with altitude, in agreement with the results of the absorption measurements in carbon by Rossi, Hilberry and Hoag.[1] Because of a possible effect of scattering on the determination of n, the above figures cannot be trusted to represent accurately the absolute values of the ratios n_1'/N_1' and n_2/N_2. However, the ratios between values of $[ABCD(E+F)]$ or Δ at different depths should not be appreciably affected by scattering or by other disturbing effects. Thus we have

$$N_2/N_1=0.883\pm0.007 \quad n_2/n_1=0.698\pm0.031$$
$$N_2/N_1'=0.738\pm0.009 \quad n_2/n_1'=0.520\pm0.035$$

where the actual errors should not exceed the statistical errors indicated.

Discussion

In order to discuss our experimental results, we need a relation between ranges and momenta for mesotrons. The *momentum* loss due to collision is given by the Bethe-Bloch formula

$$-\frac{dp}{dx}=2\pi r_0{}^2NZ\mu_e\frac{1}{\beta^3}\left[\log\frac{W_m\mu_e\beta^2}{I^2Z^2(1-\beta^2)}+1-\beta^2\right], \quad (3)$$

where r_0 is the classical radius of the electron, N the number of atoms per cm^3, Z the atomic number, μ_e the rest energy of the electron, β the velocity of the mesotron, W_m the maximum transferable energy, and $I=13.5$ ev (this expression differs by a factor $1/\beta$ from the expression for the *energy* loss). A correction has to be applied to account for the polarization effect pointed out by Fermi. The correction, however, is very small for the mesotron momenta in which we are interested. According to some recent calculations of Halpern and Hall, it is of the order of 2 percent for iron and of 3 percent for lead.[9] Numerical integration of the equation for the momentum loss yields the range as a function of the momentum. The ranges $R_a=196$ g/cm^2 of lead and $R_b=311$ g/cm^2 of lead, which define the mesotron groups considered in the present experiments, are thus found to correspond to the momenta $p_a=3.1\times10^8$ and $p_b=4.5\times10^8$ ev/c, respectively. Mesotrons reaching Denver with momenta equal to p_a and p_b had momenta equal to 5.9×10^8 and 7.3×10^8, respectively, at the altitude of Echo Lake. For mesotrons with momentum between 3.1×10^8 and 7.3×10^8 ev/c the ratio between momentum losses per g/cm^2 of air and of iron is very nearly a constant and equal to 1.23. Thus, as far as collision losses are concerned, 200 g/cm^2 of iron is approximately equivalent to 147 g/cm^2 of air.[10] Consequently, if the mesotrons were stable, one should observe the same mesotron intensity at Echo Lake under 709 g/cm^2 of air plus 200 g/cm^2 of iron as at Denver under 856 g/cm^2 of air alone. This applies to the mesotron band between 3.1×10^8 and 4.5×10^8 ev/c as well as to the whole mesotron spectrum above 4.5×10^8 ev/c.

Our experimental results show that both N and n are larger at Echo Lake under the iron absorber than at Denver without this absorber. The difference is accounted for by the decay of mesotrons on their way down from 3240 m to 1616 m. Let us define the *probability of survival* w_{12} between two elevations z_1 and z_2 as the

[9] See O. Halpern and H. Hall, Phys. Rev. **57**, 459 (1940). We are greatly indebted to the authors for kindly communicating to us the numerical results of their calculations, which are not yet published.

[10] The thickness of the iron absorber is actually slightly larger than it should be. When the experiments were performed, the results of Halpern and Hall on the polarization effect were not yet available and previous calculations had given a larger correction for this effect (see reference (7)). However, both the coincidences $[ABCD(E+F)]$ and the anticoincidences $[ABCD-(E+F)]$ change so slowly with the thickness of the absorber S, that it is hardly necessary to apply any correction to the experimental results.

TABLE II. *Comparison between various determinations of the average range before decay L from measurements on the absorption anomaly for vertical mesotrons; z_1 is the elevation of the higher station, p_2 the momentum of the recorded mesotrons, p the effective momentum. The data of Rossi and Hall for $p_2>3.0\times 10^8$ ev/c have been obtained from Table I, adding the counting rates [ABCD(E+F)] and [ABCD −(E+F)]. This sum represents the number of mesotrons with range >186 g/cm² of lead.*

Authors	z_1 Meters	p_2 10^8 ev/c	p 10^8 ev/c	L 10^5 cm	Compensating Absorber
Rossi *et al.*[1]	4300	>2.5	>3.3	9.4±0.9	carbon above the counters
	3240	>2.5	>3.3	9.9±1.2	
	1616	>2.5	>3.3	9.5±1.7	
	180	>2.5	>2.5	9.4±1.6	
Nielsen *et al.*[2]	2040	>1.8	>3.5	6.2±0.5	carbon above the counters
	2040	>3.5	>5.3	8.9±1.2	
	2040	>5.9	>7.8	15.1±3.5	
	2040	1.8→3.5	3.5→5.3	2.45	
	2040	3.5→5.9	5.3→7.8	2.26	
Ageno *et al.*[3]	3460	>2.2	>4.5	31±5	lead between the counters
Rossi and Hall	3240	>3.0	>4.3	12.3±0.6	iron above the counters
	3240	>4.5	>5.8	13.3±0.9	
	3240	3.1→4.5	4.4→5.8	4.5±0.6	

probability that a mesotron present at the higher level z_1 does not disintegrate before reaching the lower level z_2. Then $w_{12}=n_2/n_1=0.698$ is the experimental value for the average probability of survival between $z_1=3240$ m and $z_2=1616$ m of the mesotrons which reach z_2 with momenta between p_a and p_b, and $W_{12}=N_2/N_1=0.883$ is the corresponding value for the mesotrons which reach z_2 with momenta larger than p_b. One sees that w_{12} is much smaller than W_{12}, which shows that slow mesotrons disintegrate at a much faster rate than the more energetic ones. This result is in agreement with the predictions based upon the disintegration hypothesis (see Eq. (2)) and affords strong support to the hypothesis itself. For a mono-energetic group of mesotrons, the probability of decay has a very simple theoretical expression, provided the momentum loss in the air layer between z_1 and z_2 can be neglected. In this case, Eq. (2) gives

$$\log w_{12} = -(z_1 - z_2)/L. \tag{4}$$

It is convenient to use Eq. (4) as a definition of L also when the momentum loss cannot be neglected. It can easily be proved that L is still related to the lifetime τ_0 by an expression of the type of Eq. (2)

$$L = p\tau_0/\mu, \tag{2}$$

where, however, p has now the following meaning:

$$p = (p_2 + ah_2) \times \left[1 + \log \frac{p_2 + a(h_2 - h_1)}{p_2} \Big/ \log \frac{h_2}{h_1}\right]^{-1}. \tag{5}$$

Here h_1 and h_2 are the atmospheric depths at the elevations z_1 and z_2, p_2 is the momentum of mesotrons at z_2 and a is the momentum loss per g/cm² of air.[11] The momentum p is intermediate between the initial momentum $p_1 = p_2 + a(h_2 - h_1)$ and the final momentum p_2. We shall refer to it as the *effective momentum.*

Let us first consider the experimental results concerning mesotrons which have residual momenta between 3.1×10^8 and 4.5×10^8 ev/c at the lower elevation. The corresponding effective momenta are 4.4×10^8 and 5.8×10^8 ev/c and we may take 5.0×10^8 ev/c as an average. For this mesotron group the experimental value of the probability of survival between 3240 and 1616 m is $w_{12}=0.698\pm 0.031$, and therefore $\mathbf{L=(4.5\pm 0.6)\times 10^5}$ **cm.** It then follows from (2): $\tau_0/\mu=(9.07\pm 1.3)\times 10^{-4}$ cm c/ev and accordingly, taking $\mu=8\times 10^7$ ev/c², $\tau_0=(7.2\pm 0.9)\times 10^4$ cm/c, or $\boldsymbol{\tau_0=(2.4\pm 0.3)\times 10^{-6}}$ **sec.** We shall next consider the continuous mesotron spectrum which reaches 1616 m with a residual momentum larger than 4.5×10^8 ev/c. The probability of survival for this mesotron group is $W_{12}=0.883$, and if we take Eq. (4) as an experimental definition of L we get $\mathbf{L=(13.3\pm 0.9)\times 10^5}$ **cm.** Assuming $\tau_0=2.4\times 10^{-6}$ sec., we then calculate formally, from Eq. (2), $p=1.5\times 10^9$ ev/c. This momentum should represent a sort of average effective momentum for the mesotron group considered. The value $p=1.5\times 10^9$ ev/c is quite compatible with our present knowledge of the momentum spectrum of mesotrons.[12] Thus, while a quantitative proof of Eq. (2) is still wanting, its approximative validity may be considered as established.

In evaluating the experimental results we have only considered mesotrons coming in the vertical direction. As a matter of fact, our experimental arrangement was strongly selective for vertical

[11] This follows immediately, for instance, from Eq. (14) of the paper "The disintegration of mesotrons" by B. Rossi, Rev. Mod. Phys. **11**, 296 (1939).

[12] Cf., e.g., P. M. S. Blackett, Proc. Roy. Soc. **A159**, 1 (1937); D. J. Hughes, Phys. Rev. **57**, 592 (1940).

mesotrons, but detected also mesotrons coming in directions inclined up to an angle of almost 45°. The inclined mesotrons travel a longer distance and have on that account a smaller probability of survival than the vertical ones. The increase in the path length, however, is partially compensated by an increase in the effective momentum. Thus the correction is not large and can be disregarded at the present state of the experimental accuracy.

Comparison with Previous Results

Table II summarizes the data on the mesotron decay which can be deduced from the measurements so far reported on the absorption anomaly for vertical mesotrons. L is calculated, according to Eq. (4), from the experimental values of the probability of survival. The results of Nielsen, Ryerson, Nordheim and Morgan on the mesotron groups with p_2 from 1.8×10^8 to 3.5×10^8 ev/c and from 3.5×10^8 to 5.9×10^8 ev/c are not accurate enough for a quantitative comparison with our data on the mesotron group with p_2 from 3.1 ± 10^8 to 4.5×10^8 ev/c. No other measurements on selected groups of mesotrons are available. All the remaining data in Table II refer to mesotrons for which only the lower limit of the momentum is defined. A comparison between these data is not straightforward because L depends not only on the minimum effective momentum $p_{\min}$ of the mesotrons recorded, but also on the shape of the momentum spectrum, which is probably different at different altitudes. One may expect, however, an approximate correlation to exist between the values of $p_{\min}$ and L in the various experiments. Table II shows that this is actually the case if we exclude the measurements of Ageno, Bernardini, Cacciapuoti, Ferretti and Wick, who found a value of L much larger than that determined by other authors for nearly the same value of $p_{\min}$. The reason for the disagreement is not completely clear. It may be noted that Ageno and collaborators used a lead absorber placed *between* the counters to compensate for the difference in atmospheric depth between the higher and the lower station. With this arrangement an appreciable number of mesotrons may have been removed from the beam by scattering, and this may have reduced the magnitude of the absorption anomaly due to decay. We may add that the results recently obtained by Neher and Stever[4] with an ionization chamber, concerning mesotrons coming in all directions, agree better with our present results and with those of Rossi, Hilberry and Hoag and of Nielsen, Ryerson, Nordheim and Morgan than with the results of Ageno, Bernardini, Cacciapuoti, Ferretti and Wick.

Conclusion

The experiments described have shown, in agreement with previous results, that the number of cosmic-ray mesotrons is more strongly reduced by a layer of air than by a dense absorber equivalent to the air layer with regard to ionization losses. The indication from earlier experiments[13] that the difference in stopping power between air and condensed materials increases when the mesotron momentum is decreased has been definitely established. This result verifies a theoretical prediction based upon the disintegration hypothesis, thus confirming the view that the absorption anomaly is caused by spontaneous decay of mesotrons in the atmosphere. A value of the proper lifetime $\tau_0=2.4\times10^{-6}$ sec. is deduced from measurements on a fairly monokinetic group of mesotrons.

We are greatly indebted to Professor A. H. Compton for discussions of the problem and for the encouragement given to this work. We also express our appreciation to Professor J. C. Stearns of Denver University for his friendly collaboration, and to Professor N. Hilberry and Mrs. Jane E. Hamilton for their valuable assistance throughout the experiments. To the Willet Company for making available for two months a suitable truck, to the Denver City Parks for their helpful cooperation and to the Carnegie Institution of Washington for financial support, we wish to express our sincere gratitude. One of us (B. R.) acknowledges with thanks the financial support granted to him by the Committee in aid of Displaced Foreign Scholars.

[13] See reference 2. Also M. A. Pomerantz, Phys. Rev. 57, 3 (1940).