Mesons in a Coulomb Field

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THE theory of charged mesons of spin 1 leads in its present form to divergent or otherwise unreasonable results not only in the majority of nuclear problems, but also in the treatment of the interaction of mesons with the electromagnetic field. The simplest instance is the scattering of mesons by a point charge. Massey and Corben[1] and Oppenheimer, Snyder and Serber[2] have shown that in the Born approximation the cross section for this scattering does not decrease with the energy E of the meson, if $E \gg \mu c^2$, but tends to a finite limit. Still one could hope that the exact solution of the problem would lead to a reasonable result. However, this is not the case.

The meson wave function has 6 independent components which correspond to the components of the vector potential and of the electric intensity of the electromagnetic field. Let us investigate the stationary solutions ($\Psi \sim e^{i\omega t}$) of the Proca equations in a central field of force for $\hbar|\omega| > \mu c^2$ (unbound states). The complete set of these solutions consists of three independent eigenfunctions for any given set of values of ω, j and m, such that $|m| \leq j$ and $j \geq 1$ (for $j=0$ there is only one eigenfunction). Here $j\hbar$ denotes the total angular momentum of the meson and $m\hbar$ its projection on the Z axis. The three eigenfunctions correspond to three possible orientations of the spin s of the meson with respect to its orbital momentum l. However s and l are not constants of the motion: One of the eigenfunctions is of the form

$$\Psi_\lambda = f_\lambda{}^0(r) \cdot Y_j{}^{m_\lambda}(\theta, \varphi) \cdot e^{i\omega t}, \quad \lambda = 1, 2 \cdots 6, \quad (1)$$

where $m_\lambda = m$ or $m_\lambda = m \pm 1$, while two others are of the form

$$\psi_\lambda = \{f_\lambda{}^+(r) \cdot Y_{j+1}{}^{m_\lambda}(\theta, \varphi) + f_\lambda{}^-(r)\, Y_{j-1}{}^{m_\lambda}(\theta, \varphi)\} e^{i\omega t}, \quad (2)$$

where $f_\lambda{}^+$ and $f_\lambda{}^-$ are not independent of one another.

If the potential energy $V(r)$ of the meson has no singularity at $r=0$ and if $j \geqslant 1$, then for fixed ω, j and m there is one regular eigenfunction of the type (1) and two of the type (2). However, in the field of a point charge $e'V = ee'/r$ there is again one regular eigenfunction of the type (1), but the solutions of the equations for $f_\lambda{}^+$ and $f_\lambda{}^-$ are of the form:

$$f_\lambda{}^\pm \sim r^{-7/4} \cdot e^{2\eta\sqrt{s}},$$

where

$$s = \frac{[j(j+1)]^{\frac{1}{2}}}{\mu c^2} \cdot \frac{ee'}{r} \quad \text{and} \quad \eta = 1^{\frac{1}{4}},$$

i.e., $\eta = \pm 1$ or $\eta = \pm i$. Not two, but only one of these four solutions is finite at $r=0$; the three others lead to an infinite value of the energy $E = \int \psi^x H \psi \cdot d\tau$ of the meson.[3]

Thus the regular solutions of the Proca equations in a field of a point charge do not form a complete set of functions and the problem of the Coulomb scattering of mesons has no solution.

The reason for this breakdown of the theory may be sought in the neglect of the finite size of elementary particles. If that is the case, the Coulomb scattering of mesons with energies $E \geqslant \hbar c/r_0$ must substantially depend on the size r_0 of elementary particles. This will also be true for the bremstrahlung and probably also for the production by mesons of fast β-electrons. Thus the estimates of the cross sections for these processes based on the results of the Born approximation cannot be valid for $E \geqslant \hbar c/r_0$.

We hope to discuss in a further note some further consequences of the peculiar behavior of mesons in a Coulomb field (in particular the bound states of the mesons).

The present note is a summary of a part of a paper which will appear in the *Journal of Physics*, edited by the Academy of Sciences of the USSR.

[1] H. F. W. Massey and H. C. Corben, Proc. Camb. Phil. Soc. **35**, 463 (1939).

[2] J. R. Oppenheimer, H. Snyder and R. Serber, Phys. Rev. **57**, 75 (1940).

[3] If one calculates the charge density ρ of mesons with the help of the least objectionable nonregular solutions ($\eta = \pm i$) one finds that in the neighborhood of the point charge $e'\rho \sim r^{-5/2}$ and that the sign of ρ is uniquely determined by the sign of e' and is opposite to it. This means, that if e.g. $e' > 0$ an infalling negative meson may be caught by e' in a spiral orbit, whereas an infalling positive meson produces pairs, the negative components of which are caught by e'.