

FIG. 1. Tangent point  $X_0$  as function of cube root of atomic number Z.

initial point  $\psi_0$  as well as the initial slope  $\psi_0'$  can be determined accurately as desired. Of course, this integration curve does not satisfy the condition  $\psi(0) = 1$ . Dimensional consideration shows that the equation admits the following substitution with any constant parameter a:

$$\psi \rightarrow \frac{\psi}{a}, \quad x \rightarrow a^{\frac{1}{3}}x, \quad \psi' \rightarrow \frac{\psi'}{a^{4/3}}, \quad \beta \rightarrow \frac{\beta}{a^{\frac{3}{3}}}, \quad Z \rightarrow aZ.$$

Putting  $a = \psi_0$  we get an exact solution satisfying the condition  $\psi(0) = 1$  for the element of the atomic number  $Z = \psi_0 Z'$  which has the tangent point at the distance  $X_0 = \psi_0 \frac{1}{4} X_0'$  and the initial slope of the exact value  $-B_0 = \psi_0'/\psi_0^{4/3}$ . The tangent point  $X_0$  is plotted in Fig. 1 as the function of  $Z^{\frac{1}{2}}$ . The curve is approximately a straight line. Fortunately, the initial slope  $-B_0$  satisfies the practically linear dependency to the parameter  $\beta$ , as stated by Slater and Krutter.<sup>2</sup> But, both curves deviate slightly from a straight line for small values of Z.

We will publish shortly in *Journal of the Faculty of Science* of the Hokkaido Imperial University the detailed numerical table of the solution for every existing element and the values of various physical quantities, computed by the use of the obtained accurate solutions.

In conclusion the author wishes to express his sincere thanks to Professor Slater and Professor Jensen for the kindness to send him the detailed numerical tables of their results. He is also grateful to Dr. Nishina in Tokyo and to Professor Matsukuma in Sendai for their continued interest and much valuable advice. Further, he is much indebted to Mr. T. Sato in Fukuoka and to Mr. S. Nagai in Muroran for their kind help in the calculation. The financial support of The Japan Society for Promotion of Scientific Research and of The Department of Education is gratefully acknowledged.

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<sup>1</sup> Li, Na, Cu: J. C. Slater and H. M. Krutter, Phys. Rev. 47, 559 (1935). Ar: H. Jensen, Zeits. f. Physik 110, 277 (1938); K. Umeda, unpublished. Kr, Xe: H. Jensen, Zeits. f. Physik 110, 277 (1938). <sup>2</sup> J. C. Slater and H. M. Krutter, reference 1.

## Photographic Reversal

The interesting letter from J. E. Nafe and G. E. M. Jauncey1 "The Cause and Removal of Photographic Reversal" cites N. F. Mott<sup>2</sup> as propounding the hypothesis that "photographic reversal (or solarization) produced by light is due to the reaction of bromine atoms released from the crystal lattice during the process of latent image formation with the silver of the concentration specks which makes a coating over the grain." The view that bromine released from relatively interior parts of the grain causes reversal by recombination with external latent image silver is known in photographic theory as the "regression theory of reversal" and was first definitely formulated by Lüppo-Cramer.<sup>3</sup> This "regression" theory long antedates the Gurney-Mott theory of latent image formation, and does not appear either to support this latter-whether reversal be effected by blue light or by x-rays-nor to contradict it, although the x-ray reversal curve shown by Baltzer and Nafe<sup>4</sup> does not exhibit the retardation which Mott<sup>2</sup> anticipated.

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June 14, 1940.
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<sup>1</sup> J. E. Nafe and G. E. M. Jauncey, Phys. Rev. **57**, 1048–49 (1940). <sup>2</sup> N. F. Mott, Phot. J. **78**, 286 (1938). <sup>3</sup> Lüppo-Cramer, Phot. Korr. (1905), p. 259; *Photogr. Probleme* (Halle, 1907), p. 338, et seq.; cf. also J. M. Eder, Ausführl. Handbuch d. Phot. **2**, Part 1, 586 (1927).

<sup>4</sup> O. J. Baltzer and J. E. Nafe, Phys. Rev. 57, 1048 (1940).

## Spectra of Highly Ionized Argon

Previous attempts to excite the spectra of high stages of ionization in the rare gases, either by electrodeless or condensed discharges, have failed to produce results beyond the fourth or fifth stage of ionization. Boyce<sup>1</sup> has located two multiplets in A V near 710A and 830A, but states that lines from A VI were definitely absent. One multiplet in Kr IV near 840A has been identified.<sup>2</sup> Paul<sup>3</sup> has reported the excitation of Ne III and IV (previously observed by Boyce<sup>4</sup>) and has tentatively identified two multiplets as radiations from Ne V.

A very simple and effective method for the excitation of the higher stages of ionization of the rare gases has recently been developed in this laboratory. It is particularly convenient in that it requires only slight modifications of the source chamber regularly used for solid or cored electrodes. A small hole is drilled centrally into the aluminum or carbon lower electrode. A fine bronze capillary connected to a gas reservoir feeds into this hole through the base of the electrode. In this way the gas can be fed directly into the spark gap. The gas in the reservoir is kept at a pressure of from 20 to 50 mm of Hg, and the flow is adjusted to a convenient rate (in the case of argon this was roughly one cc per minute) by means of a needle valve. The pumping speed is adjusted so as to remove the gas rapidly enough to prevent the formation of an arc discharge.

The spectra were photographed in the 21-foot, grazingincidence vacuum spectrograph, which covers the range

INT.	λ(vac.)	ν(CM <sup>-1</sup> )	Stage of Ioniz.	CLASSIFICATION
3	596.69	167,590	VI	$3s^23p \ ^2P_{3/2} - 3s3p^2 \ ^2S_{1/2}$
2	588.93	169,800	VI	${}^{2}P_{1/2}$ ${}^{2}S_{1/2}$
2	555.65	179,970	Ϋ́Ι	${}^{2}P_{3/2}$ ${}^{2}P_{1/2}$
10	551.36	181,370	VI	${}^{2}P_{3/2}$ ${}^{2}P_{3/2}$
6	548.91	182,180	· VI	${}^{2}P_{1/2}$ ${}^{2}P_{1/2}$
3	544.72	183,580	VI	${}^{2}P_{1/2}$ ${}^{2}P_{3/2}$
4	527.70	189,500	V	3523 2 3P2 -353 23 3S1
0	526.87	189,800	VIII	$3p  {}^{2}P_{3/2} - 3d  {}^{2}D_{3/2}$
4	526.45	189,950	VIII	$^{2}P_{3/2}$ $D_{5/2}$
2	524.19	190,770	V	3523 p2 3P1 -353 p3 3S1
1	522.08	191,540	v	${}^{3}P_{0}$ ${}^{3}S_{1}$
3	519.43	192,520	VIII	$3p \ ^2P_{1/2} - \ 3d \ ^2D_{3/2}$
10	479.39	208,600	VII	$3s3p ^{3}P_{2} - 3s3d ^{3}D_{3}$
6	475.65	210,240	VII	$^{3}P_{1}$ $^{3}D_{2}$
3	473.93	211,000	VII	${}^{3}P_{0}$ ${}^{3}D_{1}$
4	260.33	384,130	VIII	$3d \ ^2D_{5/2} - 3f \ ^2F_{7/2}$
2	260.25	384,250	VIII	${}^{2}D_{3/2}$ ${}^{2}F_{5/2}$
4	250.95	398,480	VII	3s3p 3P2 -3s4s 3S1
2	249.89	400,170	VÍI	<sup>3</sup> P <sub>1</sub> <sup>3</sup> S <sub>1</sub>
1	249.38	401,000	VII	<sup>3</sup> P <sub>0</sub> <sup>3</sup> S <sub>0</sub>
4	230.86	433,160	VIII	$3p \ ^2P_{3/2} - 4s \ ^2S_{1/2}$
2	229.43	435,860	VIII	${}^{2}P_{1/2}$ ${}^{2}S_{1/2}$
5	192.63	519,120	VII	$3s3p ^{3}P_{2} - 3s4d ^{3}D_{3}$
3	192.04	520,720	VII	$^{3}P_{1}$ $^{3}D_{2}$
2	191.76	521,490	VII	${}^{3}P_{0}$ ${}^{3}D_{1}$

TABLE I. Classified argon lines.

from 30A to 680A. Twenty thousand sparks (at the rate of about 100 sparks per minute) gave a very satisfactory spectrogram.

In the argon spectra lines arising from argon IV, V, VI, VII, and VIII have been identified. Preliminary results for some of the stronger lines in the spectra are listed in Table I. In all cases the frequencies agree very closely with those predicted from the laws governing isoelectronic sequences.

A more complete report on the argon spectra and one on the neon and krypton spectra will be ready soon.

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Department of Physics, University of Illinois, Urbana, Illinois, May 31, 1940.

J. C. Boyce, Phys. Rev. 48, 396 (1935).
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## Concentration of Isotopes by Thermal Diffusion: Rate of Approach to Equilibrium

The purpose of this note is to make some additions and corrections to an article<sup>1</sup> of the same title which appeared in a recent issue of this journal. The equation numbers given below refer to this article.

(1). In Eq. (10) we used the expression for the transport,  $\tau_1$ , which was derived by Furry, Jones and Onsager<sup>2</sup> for the stationary condition. The detailed justification for its use in the nonequilibrium case follows. We want a solution of Eq. (4), which may be written in the form

$$\frac{\partial(\rho c_1)}{\partial t} = -\rho v \frac{\partial c_1}{\partial z} + \rho D \frac{\partial^2 c_1}{\partial z^2} + \frac{\partial}{\partial x} \left[ \rho D \left( \frac{\partial c_1}{\partial x} - \frac{\alpha c_1 c_2}{T} \frac{\partial T}{\partial x} \right) \right], \quad (4')$$

subject to the boundary conditions (5) and (6):

$$c_1 v_{1x} = -D \left[ \frac{\partial c_1}{\partial x} - \frac{\alpha c_1 c_2}{T} \frac{\partial T}{\partial x} \right] = 0$$
 (5')

at x=0 and x=d, and

$$\int_0^d \rho v(x) dx = 0. \tag{6}$$

Let us integrate Eq. (4) with respect to x from x=0 to x=d. Making use of (5), we find:

$$\int_{0}^{d} \rho \frac{\partial c_{1}}{\partial t} dx = \int_{0}^{d} \rho D \frac{\partial^{2} c_{1}}{\partial z^{2}} dx - \int_{0}^{d} \rho v \frac{\partial c_{1}}{\partial z} dx.$$
 (a)

In the integral on the left-hand side and in the first integral on the right-hand side we may replace  $c_1$  by its mean value across the column without appreciable error. The fractional variation of  $c_1$  with respect to x is of the order  $\alpha \Delta T/T$ , so that the error involved is certainly less than this amount. The last integral would vanish on account of (6) if  $c_1$  were independent of x, so that it is not possible to use the mean value here. It is, of course, just the variation of the concentration across the column which gives rise to the transport.

0.

The integral may be evaluated as follows. Let  $^{3}$ 

$$G(x) = \int_0^x \rho v dx; \qquad (b)$$

G(o) = G(d) = 0. (c)

An integration by parts gives:

$$-\int_{0}^{d} \rho v \frac{\partial c_{1}}{\partial z} dx = \int_{0}^{d} G(x) \frac{\partial^{2} c_{1}}{\partial x \partial z} dx.$$
 (d)

The value of  $\partial^2 c_1 / \partial x \partial z$  may be obtained from Eq. (4) which, after integration with respect to x, may be written

$$\rho D \frac{\partial c_1}{\partial x} = \rho D \frac{\alpha c_1 c_2}{T} \frac{\partial T}{\partial x} + \int_0^x \left\{ \rho' v' \frac{\partial c_1'}{\partial z} + \rho' \frac{\partial c_1'}{\partial t} - \rho' D' \frac{\partial^2 c_1'}{\partial z^2} \right\} dx'. \quad (e)$$

The prime indicates that the various quantities in the integrand are functions of x'. Differentiating (e) with respect to z, we find

$$\frac{\partial^2 c_1}{\partial x \partial z} = \frac{\alpha}{T} \frac{\partial T}{\partial x} \frac{\partial}{\partial z} (c_1 c_2) + \frac{1}{\alpha D} \int_0^x \left\{ \rho' v' \frac{\partial^2 c_1'}{\partial z^2} + \rho' \frac{\partial^2 c_1'}{\partial z \partial t} - \rho' D' \frac{\partial^3 c_1'}{\partial z^3} \right\} dx'.$$
(f)

The transport equation is obtained by substituting (f) into (d) and then (d) into (a). After multiplying through by the mean circumference, B, the resulting equation may be expressed in the following form:

$$B \int_{0}^{d} \rho \left( \frac{\partial}{\partial t} - D \frac{\partial^{2}}{\partial z^{2}} \right) \left( 1 - I(x) \frac{\partial}{\partial z} \right) c_{1} dx$$
$$= -\frac{\partial}{\partial z} \left( H c_{1} c_{2} - K \frac{\partial c_{1}}{\partial z} \right), \quad (g)$$

where I(x) is introduced by an integration by parts;

$$I(x) = \int_0^d \frac{dG(x')}{\rho' D'} dx', \qquad (h)$$

then