Electron Path Lengths in Multiple Scattering

An approximate solution of the problem of multiple scattering of electrons has been given by Goudsmit and Saunderson.[1] For an initially collimated beam incident normally on a foil of thickness t the angular distribution, per unit solid angle, was given by

$$f(\vartheta, t)=\frac{1}{4\pi}\sum(2l+1)e^{-\nu Q_l}P_l(\cos\vartheta), \tag{1}$$

where Q_l, which is given explicitly in reference 1, is $[1-P_l(\cos\vartheta)]_{\text{Av}}$, averaged over the angular distribution for single (elastic) scattering. ν is the average number of collisions which the electrons undergo in traversing the foil and was given by

$$\nu=\sigma Nt, \tag{1a}$$

where σ is the total scattering cross section for single scattering and N is the number of scattering atoms per unit volume.

A more exact value for the average number of collisions would be

$$\nu=\sigma N s_{\text{Av}}, \tag{2}$$

where s_{Av} is the average length of path for the electrons traversing the foil. The approximation involved in (1) is therefore equivalent to the assumption of linearity of the electron paths and the Goudsmit-Saunderson result is thus valid only for thin foils or for very fast electrons for which the scattering of the beam is small. Stated otherwise, the assumption of linear paths is equivalent to replacing $\cos\vartheta$ by unity in the convection term of the Boltzmann equation which describes the variation in the angular distribution with penetration into the foil.[2]

A calculation of the average path length of electrons which have penetrated a given distance into the scattering foil is of importance not only for the angular distribution but also for the evaluation of experiments on energy loss.[3] The following is an attempt to make such a calculation. The restriction will be made to the case of foils thin enough, or electrons fast enough, so that back scattering is negligible. Thus, while the region of validity of (1) is to be extended, the extension is not to be made beyond the region of prediffusion. The average path is approximately given by

$$s_{\text{Av}}=\int_0^t dx\int f(\vartheta, x)\,|\sec\vartheta|\,d\Omega. \tag{3}$$

In order to avoid divergence difficulties in the angular integration $\sec\vartheta$ is expanded in a series containing a *finite* number of Legendre polynomials. This is of course equivalent to limiting the series

$$\sec\vartheta=\sum\xi^n,\quad \xi\equiv 1-\cos\vartheta<1 \tag{4}$$

to a like number of terms. By comparing the series

$$\sec\vartheta\cong\sum_0^n\xi^l=\sum_0^n a_l P_l(\cos\vartheta) \tag{5}$$

the coefficients are readily evaluated as shown in Table I. The relative error in $\sec\vartheta$ is ξ^{n+1} so that a series of six terms represents $\sec\vartheta$ with an error of only 2 percent for angles as large as 60°.

TABLE I.

n	a_0	a_1	a_2	a_3	a_4	a_5
0	1					
1	2	−1				
2	10/3	−3	2/3			
3	16/3	−33/5	8/3	−2/5		
4	128/15	−13	152/21	−2	8/35	
5	208/15	−171/7	352/21	−58/9	48/35	−8/63

From (3) and (5) the path length-thickness ratio is

$$s_{\text{Av}}/t=1+\sum_1^n a_l(n)\left(\frac{1-e^{-\nu Q_l}}{\nu Q_l}-1\right) \tag{6}$$

with ν given by (1a). As an indication of the consistency of the above procedure s_{Av}/t may be evaluated for various special cases for different values of n. For example, for medium fast electrons ($\sim$1 Mev) traversing Al foils of thicknesses about 0.05 cm the path length-thickness ratio for various values of n (2 to 5) shows an extreme variation of only 2 percent.

The above result for the average path length may be compared with the "first approximation" result of Goudsmit and Saunderson.[1] According to these authors one may take

$$s_{\text{Av}}/t=1/\langle\cos\vartheta\rangle_{\text{Av}}\cong 1+\xi_{\text{Av}}=e^{\nu Q_1}.$$

This is equivalent to limiting the series (5) to two terms and in addition the average cosine is evaluated only at the end of the path whereas a summation over the entire trajectory should be taken (cf. 3). For a given n this latter approximation overestimates the mean path length. Thus for very thin foils, for which the Goudsmit-Saunderson result is most nearly valid, (6) gives ($n=1$)

$$s_{\text{Av}}/t=1+\tfrac{1}{2}\nu Q_1$$

instead of $1+\nu Q_1$.

Numerical applications of the above will appear in a subsequent issue of the *Physical Review*.

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[1] S. A. Goudsmit and J. L. Saunderson, Phys. Rev. **57**, 24 (1940); Phys. Rev. **57**, 552 (1940).
[2] W. Bothe, Zeits. f. Physik **54**, 161 (1929). H. A. Bethe, M. E. Rose and L. P. Smith, Proc. Am. Phil. Soc. **78**, 573 (1938).
[3] E.g. see A. J. Ruhlig and H. R. Crane, Phys. Rev. **53**, 618 (1938); M. M. Slawsky and H. R. Crane, Phys. Rev. **56**, 1203 (1939).

Effect of the Nuclear Coulomb Field on the Capture of Slow Mesons

In their note on the absorption of slow mesons in matter Yukawa and Okayama[1] have shown, that (a) the majority of slow mesons are captured by atomic nuclei only after having been stopped by losing their energy through ionization, and (b), if the material is dense, the capture takes place nearly always before they disintegrate spontaneously. In deriving these conclusions they assumed mesons to be free. But for slow mesons, especially for those which have been stopped by ionization, the Coulomb force of the atomic nuclei plays an important part in the problem.