Electron Path Lengths in Multiple Scattering

An approximate solution of the problem of multiple scattering of electrons has been given by Goudsmit and Saunderson.[1] For an initially collimated beam incident normally on a foil of thickness t the angular distribution, per unit solid angle, was given by

$$f(\vartheta, t) = \frac{1}{4\pi} \sum (2l+1) e^{-\nu Q_l} P_l(\cos \vartheta), \tag{1}$$

where Q_l, which is given explicitly in reference 1, is $[1 - P_l(\cos \vartheta)]_{Av}$, averaged over the angular distribution for single (elastic) scattering. ν is the average number of collisions which the electrons undergo in traversing the foil and was given by

$$\nu = \sigma N t, \tag{1a}$$

where σ is the total scattering cross section for single scattering and N is the number of scattering atoms per unit volume.

A more exact value for the average number of collisions would be

$$\nu = \sigma N s_{Av}, \tag{2}$$

where s_{Av} is the average length of path for the electrons traversing the foil. The approximation involved in (1) is therefore equivalent to the assumption of linearity of the electron paths and the Goudsmit-Saunderson result is thus valid only for thin foils or for very fast electrons for which the scattering of the beam is small. Stated otherwise, the assumption of linear paths is equivalent to replacing $\cos \vartheta$ by unity in the convection term of the Boltzmann equation which describes the variation in the angular distribution with penetration into the foil.[2]

A calculation of the average path length of electrons which have penetrated a given distance into the scattering foil is of importance not only for the angular distribution but also for the evaluation of experiments on energy loss.[3] The following is an attempt to make such a calculation. The restriction will be made to the case of foils thin enough, or electrons fast enough, so that back scattering is negligible. Thus, while the region of validity of (1) is to be extended, the extension is not to be made beyond the region of prediffusion. The average path is approximately given by

$$s_{Av} = \int_0^t dx \int f(\vartheta, x) |\sec \vartheta| d\Omega. \tag{3}$$

In order to avoid divergence difficulties in the angular integration $\sec \vartheta$ is expanded in a series containing a *finite* number of Legendre polynomials. This is of course equivalent to limiting the series

$$\sec \vartheta = \sum \xi^n, \quad \xi \equiv 1 - \cos \vartheta < 1 \tag{4}$$

to a like number of terms. By comparing the series

$$\sec \vartheta \leqq \sum_0^n \xi^l = \sum_0^n a_l P_l(\cos \vartheta) \tag{5}$$

the coefficients are readily evaluated as shown in Table I. The relative error in $\sec \vartheta$ is ξ^{n+1} so that a series of six terms represents $\sec \vartheta$ with an error of only 2 percent for angles as large as $60°$.

TABLE I.

n	a_0	a_1	a_2	a_3	a_4	a_5
0	1					
1	2	−1				
2	10/3	−3	2/3			
3	16/3	−33/5	8/3	−2/5		
4	128/15	−13	152/21	−2	8/35	
5	208/15	−171/7	352/21	−58/9	48/35	−8/63

From (3) and (5) the path length-thickness ratio is

$$s_{Av}/t = 1 + \sum_1^n a_l(n) \left(\frac{1 - e^{-\nu Q_l}}{\nu Q_l} - 1 \right) \tag{6}$$

with ν given by (1a). As an indication of the consistency of the above procedure s_{Av}/t may be evaluated for various special cases for different values of n. For example, for medium fast electrons ($\sim$1 Mev) traversing Al foils of thicknesses about 0.05 cm the path length-thickness ratio for various values of n (2 to 5) shows an extreme variation of only 2 percent.

The above result for the average path length may be compared with the "first approximation" result of Goudsmit and Saunderson.[1] According to these authors one may take

$$s_{Av}/t = 1/\langle \cos \vartheta \rangle_{Av} \gtrsim 1 + \xi_{Av} = e^{\nu Q_1}.$$

This is equivalent to limiting the series (5) to two terms and in addition the average cosine is evaluated only at the end of the path whereas a summation over the entire trajectory should be taken (cf. 3). For a given n this latter approximation overestimates the mean path length. Thus for very thin foils, for which the Goudsmit-Saunderson result is most nearly valid, (6) gives ($n=1$)

$$s_{Av}/t = 1 + \tfrac{1}{2}\nu Q_1$$

instead of $1 + \nu Q_1$.

Numerical applications of the above will appear in a subsequent issue of the *Physical Review*.

<div align="right">M. E. Rose*</div>

Sloane Physics Laboratory,
  Yale University,
    New Haven, Connecticut,
      May 20, 1940.

* Sterling Fellow.
[1] S. A. Goudsmit and J. L. Saunderson, Phys. Rev. 57, 24 (1940); Phys. Rev. 57, 552 (1940).
[2] W. Bothe, Zeits. f. Physik 54, 161 (1929). H. A. Bethe, M. E. Rose and L. P. Smith, Proc. Am. Phil. Soc. 78, 573 (1938).
[3] E.g. see A. J. Ruhlig and H. R. Crane, Phys. Rev. 53, 618 (1938); M. M. Slawsky and H. R. Crane, Phys. Rev. 56, 1203 (1939).

Effect of the Nuclear Coulomb Field on the Capture of Slow Mesons

In their note on the absorption of slow mesons in matter Yukawa and Okayama[1] have shown, that (a) the majority of slow mesons are captured by atomic nuclei only after having been stopped by losing their energy through ionization, and (b), if the material is dense, the capture takes place nearly always before they disintegrate spontaneously. In deriving these conclusions they assumed mesons to be free. But for slow mesons, especially for those which have been stopped by ionization, the Coulomb force of the atomic nuclei plays an important part in the problem.

In consequence of the Coulomb attraction the capture probability will increase for negative mesons, while for positives it will be greatly reduced by the potential barrier. The competition between nuclear capture and spontaneous disintegration must in this way be different for mesons of different signs.

The effect of the Coulomb force on the capture of mesons in motion can roughly be taken into account by multiplying the capture probability, which was derived by various authors on the assumption of free mesons, by the factor

$$\frac{2\pi\alpha Zc/v}{1-e^{-2\pi\alpha Zc/v}} \quad \text{or} \quad \frac{2\pi\alpha Zc/v}{e^{2\pi\alpha Zc/v}-1} \quad (1)$$

for negative or positive mesons respectively, where Z is the atomic number of the material, and v is the velocity of the incident meson.

We have thus calculated the probability for a meson of an incident energy E, being captured along its path before it is brought to rest. The results for various values of E and Z are given in Table I.

TABLE I. Capture probabilities along the path.

	$E=10^8$	10^7	10^6	10^5 (VOLTS)	SIGN OF THE MESON
Pb	0.001	2×10^{-6}	5×10^{-15}	7×10^{-39}	+
	0.1	0.01	10^{-3}	10^{-4}	−
Al	0.017	6×10^{-4}	3×10^{-6}	4×10^{-11}	+
	0.032	2×10^{-3}	2×10^{-4}	2×10^{-5}	−
Air	0.013	5×10^{-4}	6×10^{-6}	6×10^{-9}	+
	0.019	10^{-3}	6×10^{-5}	6×10^{-6}	−

One sees from the table that this probability is, notwithstanding the Coulomb attraction, very small for slow negative mesons, and still less for positives. Consequently the capture in almost all cases does not take place before mesons come to rest. We have therefore calculated the probability per unit time for mesons being absorbed by nuclei after having come to rest. In this calculation one must realize that the factor (1) does not apply to such slow mesons as have their wave-length larger than the atomic radius. The capture cross section for mesons, the energy of which is smaller than about 1 volt, will show a very complicated dependence on v, because the screening of the nuclear Coulomb field begins now to come in (Ramsauer effect). The general feature, however, would roughly be given by assuming the cross section to vary in this energy range according to the $1/v$ law. The results obtained with this simplifying assumption are tabulated in Table II.

Since the probability for negative mesons being captured is seen always to be larger than the probability of disintegration, which is of the order of 10^6 sec.$^{-1}$, the negative mesons will be much more likely captured by nuclei than

TABLE II. Capture probabilities per sec. for negative and positive mesons.

	NEGATIVE MESONS	POSITIVE MESONS
Pb	2.5×10^{12}	—
Al	1.2×10^{11}	—
Air	3×10^7	10^{-950}

disintegrate spontaneously, not only in dense materials but also in gases. On the other hand, practically all positive mesons will disintegrate spontaneously because of the extremely small capture probability due to the existence of the potential barrier. Practically all positive mesons, which come to rest, should therefore be necessarily accompanied by a disintegration electron at the end of their range.

Experimental materials are now rather scanty, but it does not seem to us merely accidental that all the Wilson tracks, which could so far be definitely identified as disintegration electrons, are positives,[2] and none of the photographs, in which a negative meson track terminates within the cloud chamber, shows such a disintegration electron.[3]

If our theory is right, the experiments of Montgomery and others,[4] who could not find disintegration electrons, seem hardly to be understood, unless we assume that slow mesons they observed are not identical with the ordinary cosmic-ray mesons and have much smaller lifetime.

The detailed calculation and discussions will shortly appear elsewhere.

S. TOMONAGA
G. ARAKI

Institute of Physical and Chemical Research,
Tokyo, Japan,
May 24, 1940.

[1] H. Yukawa and T. Okayama, Sci. Papers Inst. Phys. and Chem. Research 36, 385 (1939).
[2] P. Kunze, Zeits. f. Physik 83, 1 (1933); S. H. Neddermeyer and C. D. Anderson, Phys. Rev. 54, 88 (1938); P. Ehrenfest, Comptes rendus 206, 428 (1938); E. J. Williams and F. R. S. Roberts, Nature 145, 102 (1940).
[3] Y. Nishina, M. Takeuchi and T. Ichimiya, Phys. Rev. 52, 1198 (1937); H. Maier-Leibnitz, Zeits. f. Physik 112, 569 (1939).
[4] C. G. Montgomery, W. E. Ramsey, D. H. Cowie and D. D. Montgomery, Phys. Rev. 56, 635 (1939).

Radio-Isotopes of Ba and Cs

With the 37-inch Berkeley cyclotron as a neutron source for irradiating Ba, a chemically identified Ba isotope of half-life 30 ± 1 hour was found.[1] The emitted radiations consisted of a "monochromatic" group of electrons at 250 KV, x-rays of approximately the characteristic energy for Ba K x-rays, and strong gamma-rays of about 250 KV, in addition to a soft complex spectrum of gamma-rays. This soft spectrum made it impossible to identify the x-rays definitely by using critical absorbers. Paraffin shielding decreased the yield of this activity, and Li was found to be a less effective source of neutrons than Be. This would seem to indicate that it is a neutron loss reaction, but that extremely high energy neutrons are not required. Deuteron bombardment of Ba metal did not give this period at all.

The 2.5-minute Ba period[2] was prepared by irradiating Ba with Li+H[2] neutrons and was proved to be chemically Ba.

The 87 ± 1-minute Ba period[3] was strongly activated by deuteron bombardment of Ba and gave a β-ray upper limit of about 1 MV and a gamma-ray of about 0.6 MV according to absorption coefficients in lead and copper. There are no very strong soft "monochromatic" electrons with this period.

Cs bombarded with deuterons or neutrons consistently gave a 3-hour±10-minute period rather than the previously reported period[4,5,6] of 1.5 hours. The normal