by N listed in Table I for the pressures above 1 mm were calculated from the slopes of these straight lines. The values for these higher pressures do not therefore represent true widths at half-maximum. They are merely convenient parameters for indicating the form of one wing of each component in terms of formula (1).

The fact that the shorter wave-length wing of the $^2P_{1/2}$ component and the longer wave-length wing of the $^2P_{3/2}$ component deviate strongly from the dispersion formula (1) is not only due to the asymmetry but is also due to the appearance at these higher pressures of a faint structureless band near each of the component lines. Because the bands are diffuse and faint and are located near to the doublet lines the position of the bands could not be determined with great accuracy. The first of these bands extends for approximately 6A to the red of a fairly sharp edge at 7964.5A while the second extends about 10A to the violet of an edge at 7817.6A. While these bands have apparently not been observed before, similar bands have appeared in the absorption spectra of other alkalis.

In conclusion the author wishes to express his gratitude to Professors I. S. Bowen and W. V. Houston for their supervision and encouragement. He is also indebted to Dr. P. E. Lloyd for his generosity in discussing his experiences during research on K resonance lines.

The Energy Losses Attending Field Current and Thermionic Emission of Electrons from Metals*

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The energy losses accompanying field emission and thermionic emission have been measured in the same experimental tube. This was accomplished by observing the thermal electromotive force in a junction from which the emission currents were drawn. No measurable temperature change was observed for field emission up to thermionic temperatures. For thermionic emission, temperature changes were observed which, when correlated with the power losses from the filament as a function of temperature, yield a calorimetric value for the work function of tungsten of 4.46±0.09 volts. A theoretical expression is given for the average net energy loss per electron emitted in thermionic and field current emission. The experiment gives strong additional evidence that in field emission the electrons escape by penetrating rather by surmounting the surface potential barrier as in thermionic emission.

INTRODUCTION

IT has frequently been observed that during the emission of large thermionic currents of electrons there is a very noticeable decrease in the temperature of the emitting body. This phenomenon provides direct evidence that the electrons furnishing the emission current partake of the thermal energy of the solid. Evidently the electrons leaving a metal at the high temperatures occurring in thermionic emission have more temperature energy than those which are supplied to replace them. The difference in the average energy of the electrons emitted by the cathode and of those supplied to the cathode results in a loss of power and in a corresponding decrease of temperature. The power loss producing the temperature change, when translated in terms of the average net energy loss per electron emitted, yields a value of the work function of the emitting metal in very satisfactory agreement with that obtained by independent methods.[1]

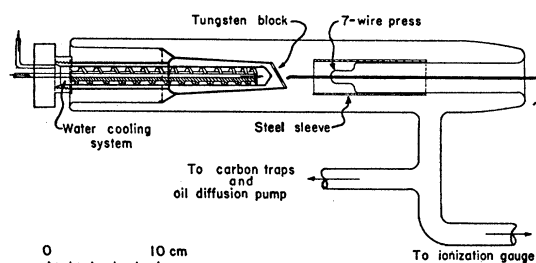
[1] Saul Dushman, Rev. Mod. Phys. **2**, 394 (1930).

FIG. 1. Section through the experimental tube.

Since the knowledge of the energy losses by a metal emitting electrons provides information concerning the mechanism of emission it is important to make measurements in the case of field current emission where electrons are supposed to escape from the metal by penetrating rather than by surmounting the barrier as in thermionic emission. The only at all satisfactory theoretical explanations[2] of the current-voltage relationships for field emission, as well as the form of the energy distribution curves[3] for field emission support this mechanism. The almost complete independence of these energy distributions as a function of temperature[4] as well as the remarkably small temperature dependence of field emission characteristics as obtained by Ahearn[5] and others[6] makes the region extending to thermionic emission of particular interest. The present experiment deals with the problem by measuring the cooling of a field current emitter in this region. The amount of cooling depends, as in thermionic emission, on the amount of temperature energy possessed by the emitted electrons.

METHOD

The essential feature of the method is to incorporate a thermocouple junction in the emitting cathode and to use the changes in the electromotive force of this junction to measure changes in temperature of the cathode during emission. This information coupled with a knowledge of the physical properties of the cathode is

sufficient to determine the rate at which energy is carried away by the emission current. The cathode can be designed to permit measurements at temperatures extending from room temperature into the thermionic range.

APPARATUS

Since large power losses must be dissipated at the target in this experiment, it was necessary to provide a water-cooled anode. Consequently, the thermocouple cathode assembly was installed in place of the regular cathode of a General Electric XP–DF x-ray tube the anode of which is water cooled. Figure 1 shows a section through the experimental tube. A seven-wire press was installed to permit the testing of various cathode-thermocouple arrangements. Of the four different arrangements that were tried, only the one which yielded the best experimental results will be described here. Since very good vacuum conditions are necessary for stable field current emission, the tube was evacuated with a two-stage oil diffusion pump used with charcoal.[7] Consistent space vacuums of the order of 10^{-8} mm of Hg were maintained throughout the experiments.

The potentials necessary for field emission were provided by a filtered, valve rectified x-ray supply, of approximately 30 kv. To insure steady currents, a radiator type x-ray tube was used in series with the experimental tube. In the portion of the experiment dealing with thermionic emission the potential was provided by batteries.

Of the various cathode-thermocouple combinations tried, the one illustrated in Fig. 2 proved most satisfactory. The thermocouple was formed by wires of 0.004-inch tungsten and 0.005-inch
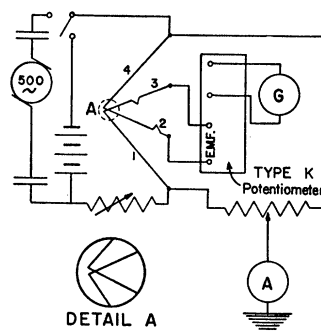

FIG. 2. Schematic diagram of the point assembly and measuring circuit.

[2] R. H. Fowler and L. Nordheim, Proc. Roy. Soc. A119, 173 (1928).
[3] J. E. Henderson and R. K. Dahlstrom, Phys. Rev. 55, 473 (1939).
[4] J. E. Henderson and R. K. Dahlstrom, Phys. Rev. 45, 764 (1934).
[5] A. J. Ahearn, Phys. Rev. 44, 277 (1933); 50, 238 (1936).
[6] R. A. Millikan and C. F. Eyring, Phys. Rev. 27, 51 (1926).

[7] J. E. Henderson, Rev. Sci. Inst. 6, 66 (1935).

tantalum spot-welded together near A. These were bent and shaped to the form of points close to the weld and placed opposite the anode to serve as the field emitter. Legs 1 and 3 are one continuous wire of tungsten and legs 2 and 4 are continuous tantalum. The thermal electromotive force of the junction formed by legs 2 and 3 was measured by a type K potentiometer capable of reading to 10^{-6} volt. Either leg, 1 or 4, furnished the path from ground for the emitted electrons.

For temperatures above room temperature, the cathode was heated either by a current from a battery or from a 500-cycle generator. The 500-cycle current was found convenient, since it permitted isolation of the measuring circuits across various portions of the cathode by means of a choke. Temperatures were determined by resistance measurements up to where a Leeds and Northrup optical pyrometer could be used. Since the temperatures of the various parts of the cathode were not the same, the reference temperature is taken as that of the region of maximum temperature, t_m, along the heated legs. The temperature of the junction may be as much as 200°C from this for the highest temperatures quoted. Corrections were made for absorption of light by the glass walls by measurements upon the temperature of a similar filament observed through both walls of the tube. The experimental plots in Figs. 3 and 4 show, respectively, the values
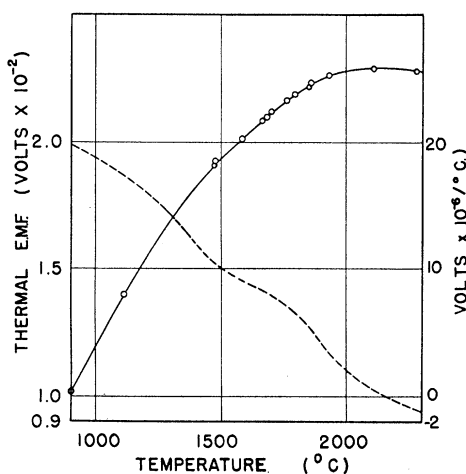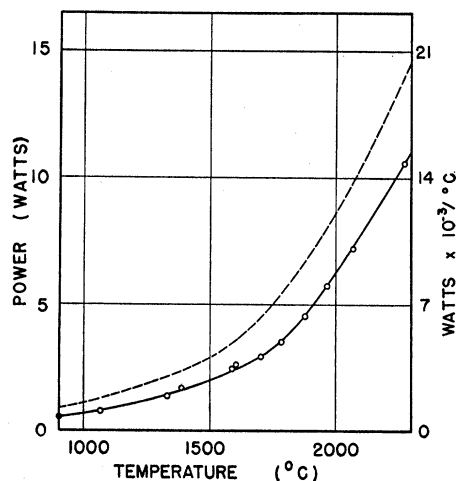

FIG. 4. The power, P, supplied by the heating current to the point assembly as a function of the maximum temperature, t_m. The dotted curve shows dP/dt_m as a function of t_m.

of the thermal electromotive force, K, as a function of t_m, and the power, P, supplied by the heating current as a function of t_m. In the same figures are shown the derivative curves giving the rate at which the thermal electromotive force and power supplied by the heating current change with the maximum temperature t_m. These data were taken when the heating was produced by the current supply. In the experiment itself the changes in power corresponding to changes in t_m as well as to changes in the e.m.f. of the junction will depend upon the distribution of this supply to the cathode. For example, during emission of electrons the power associated with this current is drained from the region of the cathode from which emission occurs, thus changing the distribution of the net power supplied to the cathode, and the temperature, t_m, may not be that corresponding to the same total power supplied solely by the heating current. That is, $(dP/dt_m)(W)$, where the power change is produced through the heating current, is in general different from $(dP/dt_m)(i)$, where the change in power occurs with the change of emission current. The relation between these two quantities depends upon the region from which emission current is drawn.

Use was made of the values of the ir drops in the cathode in estimating the distribution of the areas active in field current emission. It was

FIG. 3. The thermal electromotive force, K, of the tungsten-tantalum junction of the cathode shown in Fig. 2 as a function of the maximum temperature, t_m, along the heated wires. The dotted curve shows dK/dt_m or the thermal electromotive power as a function of t_m.
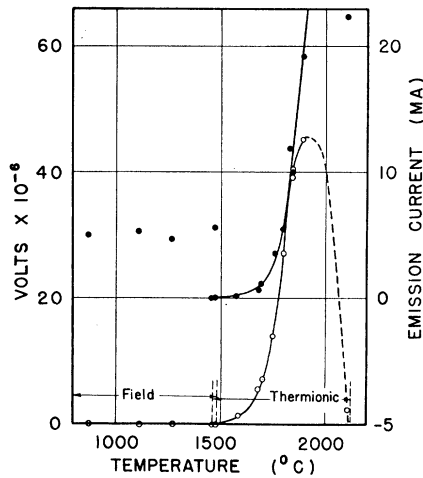
FIG. 5. The change in electromotive force produced by emission. The full circles represent the emission current and the open circles the corresponding changes in electromotive force.

found that at room temperatures emission occurred from the tips of the cathode, very close to the junction and nearest the anode; thus, in this case, the effectiveness of power changes due to emission to produce a change in temperature of the junction was twice that of power changes due to the heating current. The effect of higher temperatures of the cathode was, in general, the redistribution of the emission, frequently to regions of the surface at which the field strength was less than the maximum, and the predominant emission was not correlated with the position of highest temperature. This shifting with temperature was perhaps due to the migration of an impurity patch of lower work function or to a change of the crystal structure along the surface. The estimation of the actual distribution of current could be roughly made, but the effectiveness of power losses in producing a change in temperature of the junction was uncertain because of the ambiguity of the position along the cathode at which the emission occurred. Correspondingly the temperature of the portion of the metal from which the emission took place was uncertain within limits. At the highest temperatures for which measurements were made with field currents the effect of power losses was uncertain by a factor of about two, and the temperature of the emitting region by about 200°C. Because of the nature of the results

obtained, this uncertainty is unimportant except for the highest value of the temperature given for field emission.

In the case of thermionic emission both the position of emission and the temperature can be specified because the emission current occurs predominantly at the region of highest temperature, thus allowing a calculation of the effect of the power losses due to emission. Approximate expressions for $(dP/dt_m)(W)$ and $(dP/dt_m)(i)$ can be calculated from an equation obtained giving the temperature variations along the wire. The relation between them is:

$$\left(\frac{dP}{dt_m}\right)(i) = \frac{1}{1.16}\left(\frac{dP}{dt_m}\right)(W).$$

The highest temperatures along legs 1 and 4 occurred at positions about one-third of the length of the leg away from the junction, and for thermionic emission the maximum along leg 1, of tungsten, was higher than that along the tantalum leg by a small amount varying from 30° to 50°C. The value of the ir drops in the cathode during emission showed that only a small part of the total emission current was furnished by the tantalum leg, this fraction varying from practically zero at lower temperatures to one-fifth at the highest temperature used. The way in which the temperature of the junction, or the equivalent electromotive force, K, varies with t_m for the two types of power changes likewise must be known. With sufficient accuracy, calculations showed that

$$(dK/dt_m)(i) = (dK/dt_m)(W).$$

RESULTS

Data were taken by measuring the difference in thermal e.m.f. of the junction with the emission current alternately on and off. Figure 5 shows the results for both field current and thermionic emission. The remainder of the curve for field currents, omitted in the figure, extends for the range from 800°C down to room temperature along the axis. The temperatures used in plotting are again those of the position of maximum temperature along the cathode. The emission currents are represented by full circles and the corresponding changes in e.m.f. by open circles. The field currents were produced by the

high voltage supply and were usually adjusted to be about 5 milliamperes. The thermionic currents, except for the one at the highest temperature, were produced with 140 volts accelerating potential. The last value, for which about 90 volts was used, shows lack of saturation by departure from the curve rising exponentially at lower temperatures. The changes in electromotive force, K, for emission due to the high electric fields were less than the experimental error at all temperatures used. In the thermionic range the effects were easily measurable. Figure 6 shows the trend of the points, giving the values of ΔK per unit emission current in microvolts per milliampere. The decrease in the ordinate for the thermionic range with increasing temperature is associated with two characteristics of the cathode. First, as the temperature of the cathode increases, the amount of power change necessary to produce a given temperature change increases (see Fig. 4), and, second, the thermoelectric power of the junction decreases in this range (see Fig. 3).

The power loss per unit current for field current emission is less than that measurable by the apparatus. Therefore, only an upper limit can be given for the average net energy loss per electron emitted. At room temperature the sensitivity of the apparatus sets this limit at 0.002 ev/electron. At higher temperatures the sensitivity of the apparatus is less, and for the highest temperature at which field emission can be said to have been produced in this case, 1250°C, the energy loss per electron emitted was not greater than 0.08 ev/electron.
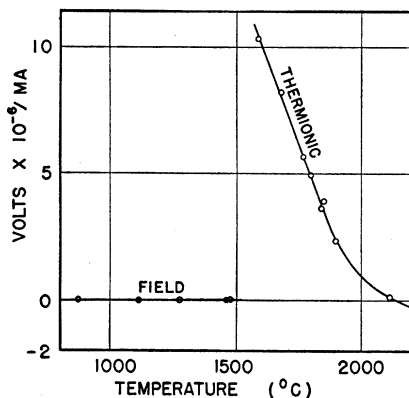
Fig. 6. The change in electromotive force per unit emission current as a function of temperature for field current and thermionic emission.
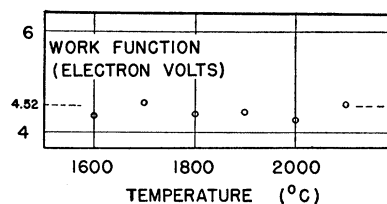
Fig. 7. Values of the work function of tungsten calculated from the experimentally measured energy losses found at different temperatures.

The values of $\Delta K/i$ found for thermionic emission are used to calculate the associated power loss per unit emission current or the equivalent energy loss per electron emitted. The power change per unit current, $\Delta P/i$, corresponding to $\Delta K/i$ is

$$\frac{\Delta P}{i} = \left(\frac{dP}{dt_m}\right)(i)\frac{\Delta K/i}{(dK/dt_m)(i)}$$

$$= \frac{1}{1.16}\left(\frac{dP}{dt_m}\right)(W)\frac{\Delta K/i}{(dK/dt_m)(W)},$$

where the quantities on the right can be obtained at each temperature from the curves in Figs. 3, 4, and 6. The result, expressed in watts per milliampere, can be alternatively given in terms of electron volts per electron by the relation:

$$w = 10^3 \cdot \Delta P/i,$$

where w is the loss of energy per electron.

The values of w as a function of temperature can be compared with those given by theory in the relation:

$$w = \phi + 3.2kT,$$

ϕ being the work function of the emitting metal, and T, the absolute temperature. Conversely, by use of this relation a value of ϕ may be obtained which can be compared with the known value for the emitting metal. The values for ϕ obtained in this way are shown in Fig. 7. The accepted value for the work function of tungsten, 4.52 ev,[8] is indicated. The average of the experimental results displayed is 4.46 ± 0.09 ev.

THEORETICAL DISCUSSION

It is interesting to compare the values obtained by theory with the experimental values of the net

[8] See A. L. Hughes and L. A. DuBridge, *Photoelectric Phenomena* (McGraw-Hill, 1932), p. 76.

energy loss per electron emitted. The energy losses occurring during electron emission from metals can be computed for a simple triangular type of potential barrier such as assumed by Fowler and Nordheim.[2] The calculations are made for electrons within the metal having Fermi-Dirac statistics.

The number of electrons per unit volume having velocity v in the range dv is

$$f(v)dv = \frac{2m^3}{h^3}\frac{dv_x dv_y dv_z}{e^{-(\mu-E)/kT}+1} = \frac{8\pi m^3}{h^3}\frac{v^2 dv}{e^{-(\mu-E)/kT}+1}$$

given by the Fermi-Dirac distribution function. The electron parameter μ is determinable from the number of electrons per unit volume. Assume that the emitting metal has a temperature T and is connected to a metal at 0°K from which the supply of electrons to the cathode is taken. Electrons passing between the regions of different temperature undergo a redistribution in energy to correspond to the temperature of the region occupied. The group of electrons passing from a region of lower to one of higher temperature gains heat energy from the metal, while the group passing in the opposite direction gives up energy to the cooler metal. When no emission occurs the net current within the metal is zero and the energy lost by the hot portion is gained by the cool portion of the metal. During emission the net current in the metal is not zero but is equal to the emission current. Those of the emitted electrons which possess temperature energy, that is, for which $E>\mu$, are replaced by electrons originally from the cool supply which have acquired heat energy on arriving at the emitting region at temperature T. A net loss of energy occurs as a result of this replacement equal to the difference between the energy of the emitted and the added electrons.

In order to calculate the heat energy which must be supplied to the electrons emitted it is necessary to find the average energy of the levels in the current supply out of which these electrons are raised. The number of electrons per unit time of each energy which gains temperature energy is proportional to the rate at which electrons of this energy enter the warmer metal and to the fraction of the number of levels of this energy which are vacant at temperature T.

The current of electrons of energy E coming from the 0°K supply is computed as follows. The number of electrons per unit volume having velocity v in dv which also have velocity v_x (in the direction of increasing temperature) in dv_x is

$$f(v)dv \cdot dv_x/2v = (4\pi m^3/h^3)v dv dv_x$$

and the current per unit area in this direction is

$$(4\pi m^3/h^3)v dv dv_x \cdot v_x = (4\pi m/h^3)dEdW$$

with $W=\frac{1}{2}mv_x^2$. The total current density of electrons of energy $E=\frac{1}{2}mv^2$ in dE is

$$\int_{W=0}^{W=E} \frac{4\pi m}{h^3}dEdW = \frac{4\pi m}{h^3}EdE.$$

The fraction of the total number of levels which should be vacant at T is

$$e^{-(\mu-E)/kT}/(e^{-(\mu-E)/kT}+1)$$

and, therefore, the rate at which electrons of energy E increase their energy on entering the region of temperature T is

$$\frac{4\pi m}{h^3}E\frac{e^{-(\mu-E)/kT}}{e^{-(\mu-E)/kT}+1}dE.$$

The average energy of the levels from which the electrons in the current are removed is

$$\int_0^\mu \frac{Ee^{-(\mu-E)/kT}}{e^{-(\mu-E)/kT}+1}dE \cdot E \Big/ \int_0^\mu \frac{Ee^{-(\mu-E)/kT}}{e^{-(\mu-E)/kT}+1}dE = \mu - \frac{0.81}{\ln 2}kT = \mu - 1.2kT.$$

In the emitted current the energy distribution is modified by the transmission coefficient of the barrier. The current density of emitted electrons at E is given by

$$\frac{4\pi m}{h^3}\frac{dE}{e^{-(\mu-E)/kT}+1}\int_0^E D(W)dW,$$

where $D(W)$ is the transmission coefficient of the barrier for electrons of normal energy W. The net loss of energy associated with this current is, then,

$$\frac{4\pi m}{h^3}\int_\mu^\infty \frac{(E-\mu+1.2kT)dE}{e^{-(\mu-E)/kT}+1}\int_0^E D(W)dW$$

and the average energy loss per electron emitted in the total current is

$$w=\int_\mu^\infty\frac{(E-\mu+1.2kT)}{(e^{-(\mu-E)/kT}+1)}dE\int_0^E D(W)dW\Big/\int_0^\infty\frac{dE}{e^{-(\mu-E)/kT}+1}\int_0^E D(W)dW.$$

The above expression is to be evaluated for two cases: for field current emission at temperatures up to the thermionic range, and for thermionic emission. For field emission Fowler and Nordheim[2] give the transmission coefficient as

$$D(W)=(4/C)W^{\frac12}(C-W)^{\frac12}\exp\left[-4\kappa(C-W)^{\frac32}/3F\right],$$

where

$$\kappa=2\pi(2m)^{\frac12}/h.$$

The temperature of the metal from which field currents were produced was always below the range for which appreciable thermionic emission occurred, and therefore the upper limit of the integral on E can be put at C, the height of the barrier. Substituting for $D(W)$, setting $\mu-W=x$ and dropping terms of higher power than the first in x in the expansion about $x=0$, we find

$$w=\frac{\displaystyle\int_\mu^C\frac{(E-\mu+1.2kT)}{e^{-(\mu-E)/kT}+1}dE\int_{\mu-E}^\mu\left[1+x\left(\frac{1}{2\phi}-\frac{1}{2\mu}\right)\right]e^{-2\kappa\phi^{\frac12}x/F}dx}{\displaystyle\int_0^C\frac{dE}{e^{-(\mu-E)/kT}+1}\int_{\mu-E}^\mu\left[1+x\left(\frac{1}{2\phi}-\frac{1}{2\mu}\right)\right]e^{-2\kappa\phi^{\frac12}x/F}dx}.$$

These approximations are valid for the values of the field used[9] where the greatest contribution to the emission current comes from the range around $W=\mu$. The net energy loss per electron emitted is upon integration:

$$w=\frac{kT\left[1.2\left(\dfrac{1}{1-\alpha kT}-\dfrac{1}{2-\alpha kT}+\cdots\right)+\left(\dfrac{1}{(1-\alpha kT)^2}-\dfrac{1}{(2-\alpha kT)^2}+\cdots\right)\right]}{\left(\dfrac{1}{\alpha kT}-\dfrac{1}{1+\alpha kT}+\cdots\right)+\left(\dfrac{1}{1-\alpha kT}-\dfrac{1}{2-\alpha kT}+\cdots\right)}$$

with

$$\alpha=2\kappa\phi^{\frac12}/F=1.03\times10^8\phi^{\frac12}/F.$$

For $T=300°$K and $F=10^8$ volts/cm, $\alpha kT\ll1$, and

$$w=1.6\alpha(kT)^2=0.0025\text{ ev/electron}.$$

[9] J. E. Henderson and R. K. Dahlstrom quote values of the field as high as 10^8 volts/cm; other estimates range from this value down to 10^7 volts/cm.

At 1550°K with $F = 10^8$ v/cm, $\alpha kT = 0.29$, and

$$w = 0.07 \text{ ev/electron.}$$

The magnitude of the computed energy loss per electron at 300°K is much too small to be detected by this experiment, but that calculated for 1550°K is very close to the limit detectable.

In the thermionic range the accelerating potentials used were relatively small, and the transmission coefficient is chosen as:

$$D(W) = 0, \qquad\qquad W < C,$$
$$D(W) = \bar{D}, \text{ a constant, } \quad W \geqslant C.$$

The energy loss per electron is then readily seen to be

$$w = \phi - 3.2kT; \quad \phi = C - \mu.$$

The use of this relation between w and ϕ in obtaining a satisfactory value of the thermionic work function of tungsten from the experimental data establishes that the agreement between theory and experiment is good.

DISCUSSION OF RESULTS

This experiment demonstrates clearly that there is no net measurable loss in energy from the emitter in field emission over the temperature range from room temperature to that temperature at which thermionic emission begins to furnish more than a negligible fraction of the total emission. Corresponding to this temperature and above there is a definite measurable energy loss. These conclusions show that there is a fundamental difference involved in the mechanism responsible for the two types of emission.

The calculations show that the Fowler-Nordheim theory of metals provides a picture essentially in agreement with these results. The theory describes field current emission as taking place by penetration of the surface barrier and, consistent with experiment, predicts a cooling effect at room temperatures much smaller than that measurable and only slightly within the limits of error of the method at 1250°C. In the thermionic range, where emission occurs without the application of high electric fields, only the electrons of normal energy greater than that corresponding to the top of the barrier are emitted and thus carry away heat energy. On this basis from the net energy loss per electron emitted, the thermionic work function of the emitting metal has been computed, agreeing well with the accepted value.

The relation between the temperature of the emitter and field emission has been attacked by two other types of experiments. In the first of these the striking independence of temperature of the total current in field emission was established first by Millikan and Eyring[6] and later for pure metals by Ahearn.[5] The other type of experiment deals with the energy distribution of the emitted electrons. Henderson and Dahlstrom[4] show that the energy distribution for the electrons emitted under the action of intense electrical fields changes only slightly with temperature. Their work shows a small but detectable increase in the high energy groups with temperature. These changes in the high energy groups doubtless represent temperature energy carried away from the metal but the small number of electrons involved make the total energy they represent undetectable in this experiment.

Both of these experiments differ from this investigation in their fundamental approach in that they deal with the properties of the emitted electrons whereas this experiment deals with the effect of the emission upon the metal itself. Evidently the majority of the electrons involved in field emission are practically independent of the temperature of the emitter, at least up to that temperature at which thermionic emission becomes the predominant form of emission, and carry away an inappreciable amount of temperature energy above that possessed by the electrons that replace them.