# Note on the Neher-Stever Experiment

ELDRED NELSON
*Department of Physics, University of California, Berkeley, California*
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Three problems important to the quantitative interpretation of experiments on the decay of the penetrating component of cosmic rays are discussed: (a) The correct treatment of the altitude effect in the energy distribution is found to be important in the determination of the decay constant of the mesotron; (b) the fraction of mesotrons decaying is shown to be independent of the zenith angle; (c) the absorption in water of cascades produced by decay electrons is calculated from the shower theory of Serber and is in agreement with the experimental results of Neher and Stever. This last point shows that one but not both of the decay products is shower producing.

THE experiment of Neher and Stever[1] on the relative stopping power of air and water for the penetrating cosmic rays indicates the presence of a component that decays. They immersed an ionization chamber in two neighboring lakes, differing in elevation by 3616 meters, to such depths that the total mass of absorber was approximately the same in both cases. The difference in the readings obtained should be a measure of the number of mesotrons decaying between the two lakes. In order to make a quantitative determination of the lifetime of the mesotron from their measurements, certain considerations must be met: (a) the variation of the energy distribution of the mesotrons with altitude; (b) the variation of the intensity of the mesotrons with the zenith angle; (c) the absorption of the soft component in the water. The soft component is composed chiefly of cascades from degraded primaries, knock-on secondaries, and decay electrons. Since the first of these will be approximately the same at corresponding depths in the two lakes[2] and the second will follow closely the absorption of the hard component, the main correction to the readings of the ionization chamber will come from the decayed mesotrons. Euler and Heisenberg[3] have calculated the energy distribution and the effect of decay of the mesotron; Bruins[4] has prepared extensive tables and graphs of this energy distribution for various altitudes and for several values of the decay constant; and Snyder[5] and Serber[6] have worked out the theory of multiplicative showers. With this material at hand, it is possible to give an adequate treatment to these problems.

Since the mesotrons are produced with an energy distribution $\sim E^{-2.93}$, lose energy in proportion to the amount of matter passed through, and decay spontaneously in a co-moving coordinate system, the probable number of mesotrons having an energy between $E$ and $E+dE$ and decaying between $l$ and $l+dl$ is[3,4]

$$N_d dEdl=\frac{rN_0}{E(E+x-x_0)^{2.93}}\times\left(\frac{x_0}{x}\frac{E}{E+x-x_0}\right)^{rH/(E+x)}dEdl. \quad (1)$$

Here $N_0$ is the normalizing constant, $x-x_0$ is the energy loss of a mesotron produced at a height $h$ above sea level,

$$x=\beta\sec\theta He^{-(h-l)/H},\quad x_0=\beta\sec\theta He^{-h/H}.$$

$H$ is the height of the homogeneous atmosphere, $\beta$ is the energy loss per cm of the homogeneous atmosphere, $\theta$ is the zenith angle,

$$r=mc\sec\theta/\tau,$$

[1] H. V. Neher and H. G. Stever, Phys. Rev., this issue. The author wishes to express his appreciation to Dr. Neher and Mr. Stever for the use of their data.

[2] The intensity of the cascades from degraded primaries in addition to being the same in both lakes will be small compared to the intensity of the mesotrons and may be neglected.

[3] H. Euler and W. Heisenberg, Ergeb. d. exakt. Naturwiss. **17**, 1 (1938).

[4] E. M. Bruins, Proc. Kon. Ned. Akad. v. Wetensch. **42**, 54 (1939); **42**, 740 (1939); **43**, 75 (1940).

[5] H. Snyder, Phys. Rev. **53**, 960 (1938).

[6] R. Serber, Phys. Rev. **54**, 317 (1938).

$m$ is the rest mass of the mesotron, $\tau$ its lifetime in the rest coordinate system, and $c$ the velocity of light. The intensity of the penetrating cosmic rays (number of mesotrons) at a point $l$ in the atmosphere is then

$$\int_{E_0}^{\infty} dE\int_{l}^{\infty} N_d dl = N_0\int_{E_0}^{\infty}\frac{dE}{(E+x-x_0)^{2.93}} \times\left\{\frac{x_0}{x}\frac{E}{E+x-x_0}\right\}^{rH/(E+x)} \quad (2)$$

$E_0$ is the minimum energy detectable by the measuring apparatus. In this integrand,

$$(E+x-x_0)^{-2.93}$$

represents the initial energy distribution and

$$\left(\frac{x_0}{x}\frac{E}{E+x-x_0}\right)^{rH/(E+x)}$$

represents the effect of decay. It is important to use the value of $x$ corresponding to the altitude at which the experiment is performed in both parts of the integrand, since the resultant distribution of energies and the intensity is sensitive to it. The use of sea level energy losses in the initial energy distribution or of sea level average energies in interpreting data obtained at higher altitudes will give a larger value of the decay constant $m/\tau$ than is consistent with Eqs. (1) and (2).

In regard to point (b) the fraction of mesotrons decaying is independent of the zenith angle if the absorbers in the apparatus have a plane surface (minimum energy $=E_0\sec\theta$). This is readily seen by replacing $E$ by $E'\sec\theta$ in Eqs. (1) and (2), the remaining dependence on $\theta$ being only the $\cos^{1.93}\theta$ of the initial energy distribution.[4]

$$\begin{aligned}\int_{E_0\sec\theta}^{\infty} dE\int_{l}^{\infty} N_d dl &= \cos^{1.93}\theta N_0\int_{E_0}^{\infty}\frac{dE'}{[E'+\beta h e^{-h/H}(e^{l/H}-1)]^{2.93}} \times\left[\frac{E'e^{-l/H}}{E'+\beta h e^{-h/H}(e^{l/H}-1)}\right]^{\gamma} \\ &= \cos^{1.93}\theta\left\{\int_{E_0\sec\theta}^{\infty} dE\int_{l}^{\infty} N_d dl\right\}_{\theta=0},\end{aligned}$$

where $$\gamma=\frac{mcH}{\tau(E'+\beta H e^{(l-h)/H})}.$$

The absorption of the soft component, however, depends on $\theta$. This will be treated quantitatively in the discussion of point (c).

In the experiment of Neher and Stever the depths of immersion of their ionization chamber in the upper lake (4.88, 5.88, and 6.88 m) were so great that the contribution of the soft radiation is obviously negligible; in the lower lake the corresponding depths (1.31, 2.31, and 3.31 m) were small enough so that the showers of electrons were not entirely absorbed out. As pointed out in the first paragraph, a good estimate of this contribution may be obtained by treating only those cascades of charged particles produced by the electrons into which the mesotrons have decayed.[7]

Assuming that the mesotron decays into an electron and a neutrino, the probability that the decay electron has an energy between $\mathcal{E}$ and $\mathcal{E}+d\mathcal{E}$ is given by[7]

$$d\mathcal{E}/E \quad (3) \qquad \mathcal{E}\leqslant E.$$

According to Serber[6] the number of charged particles at a point $l_2$ in a shower produced by an electron of energy $\mathcal{E}$ at $l$ is given by

$$\int_C dy e^{-k(l_2-l)\sec\theta}\mathcal{E}^y G(y). \quad (4)$$

$G(y)$ is a complicated function of the parameter $y$ on which $k$ also depends. The integration is carried out along the contour $C$ parallel to the imaginary axis and to the right of the origin. Then from (1), (3), and (4) the number of charged particles at $l_2$ arising from a mesotron decaying between $l$ and $l+dl$ is

$$Ndl = rN_0 dl\int_0^{\infty}\frac{dE}{(E+x-x_0)^{2.93}} \times\left[\frac{x_0E}{x(E+x-x_0)}\right]^{rH/(E+x)}\int_0^{E}\frac{d\mathcal{E}}{E} \times\int_C dy\mathcal{E}^y e^{-k(l_2-l)\sec\theta}G(y). \quad (5)$$

[7] A similar calculation was made by B. Feretti, Nuovo Cimento **15**, 421 (1938) using the shower theory of Bhabha and Heitler to estimate the energy spectrum of the soft component at sea level.

TABLE I. *Results of calculation.*

| Depth in Lower Lake | Ionization Due to Mesotrons | Ionization Due to Showers from Decay Electrons | Total Ioniza-tion | Observed Ioniza-tion |
|---|---|---|---|---|
| 3.31 m | 1.738 | 0.004 | 1.742 | 1.742 |
| 2.31 m | 1.917 | 0.020 | 1.937 | 1.936 |
| 1.31 m | 2.180 | 0.052 | 2.232 | 2.226 |

The integral over $\mathcal{E}$ is readily done, but the integral over $E$ in this form can apparently only be carried out numerically. However, since only high energy electrons contribute to the final result, (1) may be approximated by

$$\frac{N_0 r e^{-rl/E}}{E(E+x-x_0)^3} dEdl$$

and the integral over the energy

$$f=\int_0^\infty dE \frac{e^{-rl/E}E^{y-1}}{(E+x-x_0)^3}$$

may be carried out analytically. It can be expressed in terms of the Whittaker function.[8]

$$f=(rl)^{\frac{1}{2}(y-1)}(x-x_0)^{\frac{1}{2}(y-5)}\Gamma(3-y)e^{rl/2(x-x_0)} \times W_{\frac{1}{2}(y-5),\,\frac{1}{2}(-y)}[rl/(x-x_0)].$$

The $y$ integral is then evaluated by the saddle point method. To obtain the total number of charged particles arising from decay electrons, (5) is integrated down to the surface $l_1$ of the lower lake, neglecting the mesotrons decaying in the lower lake as they are too few to give any appreciable contribution.

$$N_e=\int_0^{l_1} Ndl. \qquad (6)$$

This integral was done graphically.

As the absorption of the shower depends on the zenith angle, (6) must be averaged over all directions of the incident particles. This correction reduces (6) by approximately 40 percent.

[8] E. T. Whittaker and G. N. Watson, *Modern Analysis* (Cambridge University Press, fourth edition, 1935), p. 340.

In doing the numerical work the following values of the constants were used: $H=8\times10^5$ cm of air at standard conditions, $\beta=2.5\times10^3$ ev per cm of air at standard conditions, $h=18.4\times10^5$ cm (this corresponds to assuming that the mesotrons are produced at 0.1 atmosphere), and $mc^2/\tau=2.86\times10^{13}$ ev per sec. This decay constant corresponds to $\tau=2.8\times10^{-6}$ sec. and $m\mp160$ electron masses and is that obtained from the experimental results of Neher and Stever. Another choice of the lifetime, such as $2.4\times10^{-6}$ sec., for the same mesotron mass would change the absorption in water only within the limits of experimental error but would lead to a significant discrepancy between the observed and calculated absorption in air. The results of this calculation are shown in Table I.

The intensity of the mesotrons was determined from Eq. (2) and the contribution of the decay electrons from Eq. (5) in both of which the normalization factor $N_0$ was chosen to fit the observed intensity at the 3.31 m depth. The correction due to the decay electrons at this depth is less than the probable error of the measurements and hence may be neglected. This also confirms the assumption that the ionization due to the decay electrons did not affect the readings taken in the upper lake. The ionization due to the showers from decay electrons is 15 percent at 1.3 m, 6 percent at 2.3 m, and 1.5 percent at 3.3 m of the ionization lost by mesotron decay. These figures would be little altered by any reasonable choice of the decay constant.

The agreement of the calculated and observed absorption in water of the cascades from decay electrons indicates that one but not both of the decay products, that is on the average only half of the mesotron energy, produces showers. This would contradict the hypothesis that the mesotron decays into a photon and an electron and tend to confirm Yukawa's original suggestion that it is $\beta$-radioactive, giving an electron and neutrino on decay.

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