Internal Pair Production in Radium C′

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Ra C′ shows a sharp line of 1.4-Mev conversion electrons with no corresponding quanta. The nuclear transition therefore is probably between states of zero angular momentum and like parity. We compute here the ratio of pair production to internal conversion of both K electrons for this process. Various approximations to the complicated exact formula are given for different Z values. For the case of Ra C′ this ratio is about 0.6 percent. This should be observable in a trochoid focusing spectrometer.

RADIUM C′ shows a very sharp line of conversion electrons corresponding to a nuclear energy change of 1.42 Mev. Investigation has shown that there are no gamma-quanta of this energy. This situation would not occur if the nuclear transition were between two states differing greatly in angular momentum, since for this, energy, the internal conversion coefficient is always small.[1] Gamow[2] has suggested that this may be due to a $0_{\text{exc}}\rightarrow 0_{\text{norm}}$ transition, in which case one quantum emission would be forbidden. These levels must have the same parity since otherwise the matrix elements of charge and current density vanish, and internal conversion would not take place if only electromagnetic coupling were involved.[3] In this Ra C′ transition, therefore, electron-positron pairs would be expected. We calculate here the ratio of pairs to conversion electrons.

The probability of conversion of both K electrons is readily obtained from Dirac wave-functions by summing over initial and final spin states:

$$w_K=8M^2\frac{[(1-\gamma)+3^{\frac{1}{2}}(1+\gamma)]^2}{27[\Gamma(2\gamma+1)]^3}Z^3\alpha^5[4Z\alpha PR^2]^{2\gamma-2}P(W+\gamma)\exp\left\{\pi\alpha Z\frac{W}{P}\right\}\left|\Gamma\left(\gamma+i\alpha Z\frac{W}{P}\right)\right|^2,$$

where we have chosen units such that m, c, h are equal to unity. Here $\gamma=[1-(z\alpha)^2]^{\frac{1}{2}}$ and α is fine structure constant. $W=E+\gamma$ is the energy of the outgoing electron, $P=[w^2-1]^{\frac{1}{2}}$ is the momentum of the electron, and R is the nuclear radius. In the nuclear matrix element $M=(\psi_{\text{exc}}, \sum_i \mathbf{r}_i{}^2\psi_{\text{norm}})$. $\sum_i$ represents a summation over all nuclear protons at positions $\mathbf{r}_i$.

The internal pair production probability involves the same matrix element M, and the same power of the nuclear radius R, and is

$$w_P=\frac{32}{9\pi[\Gamma(2\gamma+1)]^4}M^2\alpha^2(2R)^{4\gamma-4}I,$$

where

$$I=\int_1^{E-1}\frac{dx[x(E-x)-\gamma^2]}{\{(x^2-1)[(E-x)^2-1]\}^{\frac{1}{2}-\gamma}}\exp\left\{\pi\alpha Z\left[\frac{(E-x)}{[(E-x)^2-1]^{\frac{1}{2}}}-\frac{x}{[x^2-1]^{\frac{1}{2}}}\right]\right\}$$
$$\times\left|\Gamma\left(\gamma+i\alpha Z\frac{(E-x)}{[(E-x)^2-1]^{\frac{1}{2}}}\right)\right|^2\left|\Gamma\left(\gamma+\frac{i\alpha Zx}{[x^2-1]^{\frac{1}{2}}}\right)\right|^2.$$

The above formulae for w_K and w_P are exact. For each case treated I will have to be integrated numerically.

[1] S. M. Dancoff and P. Morrison, Phys. Rev. **55**, 122 (1939).
[2] G. Gamow, *Constitution of Atomic Nuclei and Radioactivity* (Oxford Univ. Press, 1931), p. 79.
[3] J. R. Oppenheimer and J. S. Schwinger, Phys. Rev. **56**, 1066 (1939).

Several useful approximations may be given. For Z large (>60) so that $\exp\{2\pi\alpha z\}\gg 1$

$$\left|\Gamma\left(\gamma+i\alpha Z\frac{W}{P}\right)\right|^2\rightarrow[3-2\gamma]2\pi\alpha Z\frac{W}{P}\exp\left\{-\pi\alpha Z\frac{W}{P}\right\},$$

where we have also made an expansion in powers of $(1-\gamma)$ which is good for all nuclei ($Z\lesssim 90$). Then

$$\frac{w_K}{w_P}=\frac{\Gamma(2\gamma+1)[(1-\gamma)+\sqrt{3}(1+\gamma)]^2}{24[3-2\gamma]}(Z\alpha)^{2\gamma}P^{2\gamma-2}\frac{W(W+\gamma)}{I'},$$

where

$$I'=\int_1^{E-1}\frac{x[E-x][x(E-x)-\gamma^2]}{\{(x^2-1)[(E-x)^2-1]\}^{1-\gamma}}\exp\left\{\frac{-2\pi\alpha Zx}{[x^2-1]^{\frac{1}{2}}}\right\}dx.$$

For low $Z(<20)$ we may put $\gamma=1$; then

$$w_K/w_P=2\pi(Z\alpha)^3P(W+1)/I''$$

and

$$I''=\int_1^{E-1}\{(x^2-1)[(E-x)^2-1]\}^{\frac{1}{2}}[x(E-x)-1]dx.$$

In the extreme relativistic case ($E\gg 1$) and low Z

$$w_K/w_P=60\pi(Z\alpha/E)^3.$$

For radium C′, $Z=84$ and $E=1.42$ Mev. The approximation for large Z gives $w_K/w_P\sim 170$. The positrons might be observable in a trochoid focusing spectrometer, particularly since the positron usually carries off most of the energy; or the annihilation radiation might be observed. One should note that two quantum emission does not play an important part in this case, since its probability is $\sim 10^{-7}$ of the internal conversion probability.[3]

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