Passage of Uranium Fission Fragments Through Matter*

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The ranges and rates of energy loss of the fission fragments of uranium are calculated on the basis of a model in which the charge of the fragment is obtained from its energy and its successive ionization potentials. The energy loss cross section for protons of the same velocity is then used to calculate the ranges of the two groups of fragments. For ($Z_1=42$, $A_1=100$) and ($Z_2=50$, $A_2=136$) these are found to be 2.42 cm and 2.08 cm, respectively, for a total assumed kinetic energy of 188 Mev and a final kinetic energy of the lighter fragment of 5 Mev (corresponding to ionization-chamber background). These are in fair agreement with the observed ranges of 2.2 cm and 1.5 cm. The experimental and theoretical range-energy relations are also in fair agreement. The validity of the model is discussed in detail, and it appears that it should be fairly good for fragments above 5 Mev. The initial charges of the fission fragments are found to be 17 and 13, respectively, and these are given as a function of the fragment energy in Table I. The density of ionization is found to decrease along the track, in marked contrast to the behavior for protons and alpha-particles.

INTRODUCTION

It is the purpose of this note to give a simple theoretical description of the slowing down of the fission fragments which are produced in the bombardment of uranium and certain other heavy elements. A typical reaction of this type is

$$ {}_0n^1+{}_{92}\mathrm{U}^{235}=X_1(Z_1, A_1)+X_2(Z_2, A_2). \quad (1)$$

The experimentally observed fragments seem to be clustered in two groups, corresponding roughly to $Z_1\sim42$, $A_1\sim100$, and $Z_2\sim50$, $A_2\sim136$. The theoretical[1] maximum energy liberated in the case of symmetrical fission is 200 ± 10 Mev, but for an unsymmetrical fission such as the above, this figure is reduced to around 180 Mev. Since some of the energy will be used in the internal excitation of the nuclear fragments, the available kinetic energy is not far different from the recent experimental value of 159–162 Mev obtained from ionization measurements by Kanner and Barschall,[2] or the heat of fission of 180 ± 5 Mev observed by Henderson.[3]

The ranges of the fission fragments have been measured in air and other substances by a number of investigators. Some of the more recent results[4] are listed below:

Joliot,[5] aluminum foils, maximum range of 3 cm air equivalent.

McMillen,[6] thin foils, 2.4 cm air equivalent.

Corson and Thornton,[7] cloud chamber, 3 cm maximum range.

Haxel,[8] combined use of ionization chamber and absorption in aluminum foils, 2 groups, 1.8 ± 0.24 cm and 1.5 ± 0.2 cm air equivalent.

Jenschke and Prankl,[9] ionization chamber, 2 groups, 2.0 cm and 1.5 cm. Background of 5–6 Mev.

Booth, Dunning, and Slack,[10] ionization chamber, 2 groups, 2.2 ± 0.1 cm and 1.5 cm. Background of 4 Mev.

It appears also from the chemical behavior of the most penetrating particles, studied by

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[1] N. Bohr and J. A. Wheeler, Phys. Rev. **56**, 426 (1939).

[2] M. H. Kanner and H. H. Barschall, Phys. Rev. **57**, 372 (1940).

[3] M. C. Henderson, Phys. Rev. **58**, 200A (1940).

[4] A survey article by L. A. Turner has appeared in Rev. Mod. Phys. **12**, 1 (1940). *Note added in proof.*—An account of experimental work by K. A. Petrzhak, Comptes rendus (Doklady) USSR **27**, 209 (1940) has just come to my attention, with results agreeing with other workers. A reference is made to a theoretical paper by A. Migdahl in a journal unfortunately not obtainable here. This contains an estimate of the initial charge of the fragment $7<Z<16$. A dissertation by E. Lisitzin, Helsingfors (1938), contains tables of the successive ionization potentials of all the elements which are more accurate than the values given by the statistical model. It is planned to use these later in an improved calculation.

[5] F. Joliot, J. de phys. et rad. (7), **10**, 159 (1939).

[6] E. McMillen, Phys. Rev. **55**, 510 (1939).

[7] D. R. Corson and R. L. Thornton, Phys. Rev. **55**, 509 (1939).

[8] O. Haxel, Zeits. f. Physik **112**, 681 (1939).

[9] W. Jenschke and F. Prankl, Physik. Zeits. **40**, 706 (1939); also Akad. Wiss. Wien (Sitzunber. Abt. IIa) **148**, 237 (1939).

[10] E. T. Booth, J. R. Dunning and F. Slack, Phys. Rev. **55**, 982 (1939).

Glasoe and Steigman,[11] that the fragments of the lighter mass groups have the longer range. In addition, the energy lost by the fragments in passage through various thicknesses of air and aluminum has been measured by Jenschke and Prankl,[9] and by Haxel.[8]

In order to show clearly the complicated processes involved in the slowing down of the fragments, let us trace out the "life history" of one of them. The capture of a neutron by a uranium nucleus provides the necessary activation energy, and the fission occurs. The fragment considered is accelerated in the Coulomb field of the other, and acquires the kinetic energy observed. The electrons of the original uranium atom are quite seriously disturbed by the fission process, and only some of them will be carried along by the fragments, so that the latter begin their journey with a fairly high charge (which will be estimated later). Due to this charge, the fragment ionizes and excites atoms which are at some distance from its path, and hence loses energy. The electrons torn off atoms in this way may either become free or be captured by the fragment, thus changing its net charge. In addition, the fragment will strike atoms more nearly head-on, when there will be a very complicated rearrangement of the electrons on both the struck atom and the fragment. This also will contribute to the energy loss, and of course, the charge of the fragment may be increased or decreased in such an encounter. In some of these collisions, there will be nuclear encounters which will give rise to the forked recoil tracks observed in the cloud chamber. As the fragment is slowed down, its average net charge will tend to decrease, and eventually it will become neutral, when only the close collisions will remain to bring it down to thermal velocities.

The problem outlined above is so complicated that a complete theoretical treatment is out of the question at present.[12] Nevertheless, to provide a basis for discussion, it seems worth while to calculate the energy loss and range of a fragment on the basis of a simple model of the mechanism of energy loss. The extent to which this can be justified will be discussed later in detail.

Model

It will be assumed that: (a) The energy loss of a fragment is due entirely to the effect of its charge, considered as a point charge, on the surrounding atoms, and that the other sources of energy loss mentioned above may be neglected. (b) For any velocity of the fragment, there will be an associated average charge, about whose value, the actual charge fluctuates but little. (c) The average charge $z(v)$ associated with a certain velocity v will be determined by energetic considerations; i.e., the fragment will be stripped down until the ionization potential of the next stage of ionization is greater than the kinetic energy of electrons bombarding the fragment with a velocity v. This assumption means essentially the neglect of the binding of the electrons and of any specific effects from the nuclei of the stopping material. (d) The rate of energy loss is just $(z(v))^2$ times as large as that for a proton of the same velocity. This assumption implies the validity of the Born approximation for the energy loss.

Method of Calculation

The successive ionization potentials for the elements $Z_1=42$ and $Z_2=50$ required in the use of assumption (c) were estimated by the Thomas-Fermi method, except that for the first few stages of ionization, semi-empirical values were used. (The statistical model, as is to be expected, gives too low ionization potentials for the first few ions, but then joins on smoothly to the observed values around $z=6$.)

The rate of energy loss for a proton may be written in the form predicted by the Born approximation[13]

$$dE/dx=4\pi Ne^4B/mv^2, \tag{2}$$

[11] G. N. Glasoe and J. Steigman, Phys. Rev. **55**, 982 (1939).

[12] G. Beck and P. Havas, Comptes rendus **208**, 1643 (1939) and P. Havas, J. de phys. et rad. (8), **1**, 146 (1940) have attempted a theoretical treatment assuming an exponential decrease of the charge on the fragments. The rate of recombination is estimated roughly for both radiative and nonradiative capture. This must necessarily be uncertain. In addition, the processes leading to an increase in the charge of the fragment have been ignored. However, the resulting picture of the course of the range agrees qualitatively with ours.

[13] M. S. Livingston and H. A. Bethe, Rev. Mod. Phys. **9**, 262 (1939).

where N is the number of atoms per unit volume, m is the electron mass, and v the velocity of the proton. The quantity B depends on the stopping atoms. In the familiar case of high (nonrelativistic) energies, it has the form

$$B=\sum_i f_i \log\left(\frac{mv^2}{\hbar\omega_i}\right), \quad (3)$$

where the f_i are the oscillator strengths for the various electronic transitions, and the $\hbar\omega_i$ are essentially the corresponding excitation energies. For the low velocities with which we are concerned, not only is B a very complicated function of the velocity according to the Born approximation, but this approximation is itself not entirely valid. In the case of air, however, semi-empirical values have been given by Bethe and Livingston[13] in the form of a plot of the energy loss cross section

$$\sigma=4\pi e^4 B/mv^2 \quad (4)$$

as a function of the proton energy. With these values, and hypothesis (c), the range traversed by a fragment of initial energy E_0 in being slowed down to an energy E may be calculated by

$$R(E_0, E)=\int_E^{E_0} \frac{dE}{Nz^2\sigma}. \quad (5)$$

TABLE I. *The relations between energy, charge, range for the two groups of fragments. The ranges traversed between any two energies must be obtained by taking differences of the corresponding ranges listed in the table. In addition, values of β/α giving the velocity in units of 2.19×10^8 cm/sec., and the loss of energy in Mev/cm (roughly proportional to the ionization density along the track) are given for the lighter fragment. The energies in parentheses were obtained semi-empirically instead of from the statistical model. As discussed in the text, the table has little meaning for energies below 5 Mev.*

Charge $z(v)$	β/α	Light Fragment ($Z=42$, $A=100$) Energy Mev	Range cm	d(Mev)/dx	Heavy Fragment ($Z=50$, $A=136$) Energy Mev	Range cm
1	0.74	(1.36)	0.000	0.63	(1.86)	0.000
2	1.03	(2.66)	1.882	3.05	(3.62)	2.540
3	1.50	(5.60)	2.824	7.22	(7.62)	3.821
4	1.71	(7.24)	3.051	11.4	(9.85)	4.127
5	2.07	(10.68)	3.401	20.4	(14.53)	4.512
6	2.23	(12.45)	3.527	29.6	(16.93)	4.603
7	2.55	16.17	3.616	39.9	20.69	4.713
8	2.81	19.70	3.697	51.1	26.31	4.873
9	3.08	23.70	3.825	62.0	31.91	4.985
10	3.53	31.17	3.972	68.5	36.64	5.063
11	4.03	40.36	4.098	69.2	43.04	5.155
12	4.41	48.40	4.223	71.0	56.55	5.336
13	4.77	56.70	4.359	73.5	68.98	5.504
14	5.17	66.18	4.494	76.1	80.60	5.657
15	5.52	75.80	4.640	79.8	92.69	5.810
16	5.91	86.95	4.786	82.6	105.6	5.970
17	6.30	98.45	4.930	86.0	120.2	6.142
18	6.66	110.4	5.083	89.5		
19	7.06	123.4	5.234	91.9		

RESULTS

The results are given in Table I. For convenience, the ranges are measured arbitrarily from the fragment energy corresponding to $z(v)=1$, so that ranges corresponding to arbitrary E_0, E may be obtained from the table by taking differences.

For the present, we will consider that the ranges observed in ionization chambers correspond to a final energy of 5 Mev for the lighter fragment, due to the presence of a background ionization caused by the natural alpha-particle activity of uranium and the nitrogen disintegration recoils produced by neutrons in the chamber. The heavier fragment will correspondingly be given a final energy at which its energy loss, and hence ionization, is the same as for the lighter fragment. The ranges calculated in this way are 2.42 cm and 2.08 cm for the fragments of mass 100 and 136, respectively, provided an initial energy of 108 Mev is assumed for the lighter fragment. The heavier particle will then have an energy of 79.5 Mev, since by momentum conservation $M_1E_1=M_2E_2$ holds during the fission process. For slightly lower initial energies, the ranges would be less by about 0.0123 and 0.0138 cm/Mev, respectively. For final energies below 5 Mev, however, the ranges increase rapidly, reaching values twice as large as the observed at $z(v)=1$, but as we shall see later, our assumptions[14] become particularly bad when the charge on the fragment is low, and the other sources of energy loss play an essential role. A better test of the theory is given by the measurements of the range-energy relation for the longer range group by Jenschke and Prankl.[9] Their results are

[14] In addition, the ionization potentials for the two fragments were assumed to be the same for $z\leq 6$. As a result, the excess in range of 0.34 cm which the lighter fragment has at 5 Mev is lost at the "end" of its range. Actually, this behavior is misleading since the lighter fragment ought to be given higher ionization potentials, and this would increase its charge. The difference in range ought really to be about 0.4 cm, agreeing well enough with the experiments. Also the lighter fragment has the longer range, agreeing with the observations of Glasoe and Steigman, reference 11.

compared with theory in Fig. 1 in which the energy of a fragment assumed to have 108 Mev initially is plotted as a function of the distance it has traversed. The experimental point at 108 Mev is said to be very uncertain. The agreement must be regarded as close as could be expected from such a crude theory, when one remembers how sensitive the energy loss is to a variation in the charge of the fragment.

Discussion of the Model

The assumption (d) that the energy loss of a fragment is related to that of a proton of the same velocity is not entirely valid because the charge of the fragment is much greater than unity over much of the range. The Born approximation which is used to derive Eq. (5) is not valid if the field of the passing fragment too greatly distorts the atoms of the stopping material. In order to discuss this, we employ the method of impact parameters[15] to interpret the logarithm in the Born approximation expression for the energy loss. One finds

$$\log mv^2/\hbar\omega = \log (b/\lambdabar), \tag{6}$$

where the distance $\lambdabar = \hbar/mv$, the de Broglie wave-length of an electron of velocity v plays the role of a minimum effective impact distance. One usually says that its appearance is a quantum-mechanical effect connected with the impossibility of forming wave packets of smaller size than $\hbar/mv$ for which the classical energy transfer will occur. The outer impact parameter $b = v/\omega$ is that distance from the track, beyond which the field of the passing particle no longer contains abundantly frequencies of the order of ω, and it will be called for that reason the adiabatic cut-off.

It is well known that if the quantity $n = ze^2/\hbar v$ is much greater than unity, the Born approximation is no longer applicable to describe the motion of an electron in the Coulomb field of a charge z. In our case, $n = z\alpha/\beta$ (where $\beta = v/c$, and $\alpha = 1/\text{“}137\text{”}$) varies from 2.58 to 1.43 over the range of the lighter fragment (down to 5 Mev). For $n \gg 1$, the energy loss due to the electrons in a cylinder of radius b may be computed classically, as Bohr did in 1913. The result is

$$\frac{dE}{dx} = \frac{4\pi N z^2 e^4}{mv^2} \log \left[1 + \left(\frac{mv^2 b}{ze^2} \right)^2 \right]^{\frac{1}{2}}. \tag{7}$$

For high energies, the logarithm reduces to $\log (b/B^*)$, where

$$B^* = ze^2/mv^2 \equiv n\lambdabar$$

is essentially the classical distance of closest approach. One may interpret the appearance of this as a minimum impact parameter instead of the $\lambdabar$ of the Born approximation by saying that the electrons originally closer to the track than B^* are swept out by the field of the fragment. In this way, less energy is transferred to them than would be if the charge of the fragment were

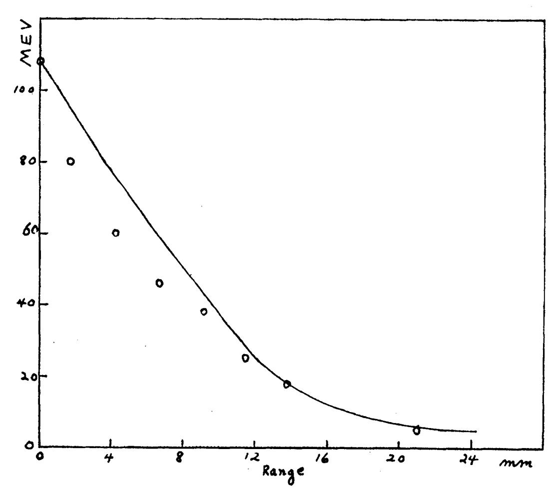

Fig. 1. The experimental points on the energy-range relation obtained by Jenschke and Prankl are compared with the theoretical curve.

not too high for the Born approximation to be fully applicable.

In analogy with the Born approximation result, and Bohr's classical treatment for a medium of harmonic oscillators, one would generalize Eq. (7) for the case of an atom by putting $b_i = v/\omega_i$ and

$$\frac{dE}{dx} = \frac{4\pi N z^2 e^4}{mv^2} \sum_i f_i \log \left[1 + \left(\frac{mv^3}{ze^2\omega_i} \right)^2 \right]^{\frac{1}{2}}. \tag{8}$$

[15] For a discussion of this method, see E. J. Williams, Det. Kgl. Dansk. Vid. Selskab. **13**, No. 4 (1935).

Calculations[16] of the ranges of the two groups have been made with this equation for aluminum, air, and hydrogen, with rough but plausible values for the f_i and ω_i. The results for air are naturally somewhat larger than those given by the Born approximation, about 15 percent, but the general situation is essentially as above. The ranges in hydrogen are roughly 4.8 times larger than in air, and 6.7 times larger than in aluminum (for equal numbers of atoms per unit volume). These figures, however, are not far different from the currently used conversion factors[17] for alpha-particles.

The derivation of Eq. (8), however, leaves out a process which may sometimes occur beyond the adiabatic cut-off, namely a kind of cold emission. The strong field of the rather slowly passing fragment will result in a lowering of the potential in which the atomic electrons move. For the case of hydrogen atoms, the electron moves in a potential

$$V(\mathbf{x}) = -e/|\mathbf{x}| - ze/|\mathbf{x}-\mathbf{r}|, \qquad (9)$$

when the fragment is at a distance $\mathbf{r}$ from the atom. This surface has a saddle point through, or over, which the electron may move from one atom to the other. This is at a distance $x_s = r/(1+\sqrt{z})$ from the hydrogen atom, and here $-eV(x_s)$ has the value $e^2(1+\sqrt{z})^2/r$. An electron with a binding energy of this order will be able to leave its own atom, temporarily at least, and be found on the fragment. If the collision were extremely slow, the electron would always have to find its way back to the hydrogen atom before complete separation occurred, as complete resonance is extremely unlikely. With the actual velocities, nonadiabatic transitions may occur, and the electron may be excited, ionized, or left on the fragment. Assuming an ionization potential of the order of $\frac{1}{2}\alpha^2 mc^2$ (13.5 ev) and allowing for the first-order depression of the energy level by an amount ze^2/r, one finds that this wandering of the electron is possible for atoms within a distance of the order of $(2+4\sqrt{z})a_0$ from the track. This distance is related to the adiabatic cut-off $b = v/\omega = 2(\beta/\alpha)a_0$ by a factor of the order $\alpha(1+2\sqrt{z})/\beta$ which has a value of 1.40 at the beginning of the range and 2.71 at 5 Mev. Thus some energy loss from processes beyond the adiabatic cut-off is being left out of account. One might attempt to include this (and thus obtain an upper limit for the energy loss) by increasing the outer impact parameter from b to x_s, but the other defects in the theory would make this a spurious refinement. One may hope, however, that since the effect of the large charge $z(v)$ is to increase both the inner and outer effective impact parameters beyond the values assumed in the Born approximation in about the same ratio that the assumption (d) is better than one would expect at first.

The hypothesis (c) would be essentially valid if the fragment were being slowed down in a rarified gas of free electrons. One could then consider that the electrons were bombarding the fragment with a kinetic energy $\frac{1}{2}mv^2$. The time between impacts would be great enough so that the ion would always have time to return to its lowest state before the next collision. Under these circumstances one could be certain that the fragment would not be stripped further than to the extent implied by assumption (c). If it were so initially, capture of electrons by inverse Auger processes (or radiative capture) would make up the deficiency. Sometimes, of course, an additional electron would be captured, resulting in a charge $z(v)-1$. The next collision, however, would be far more likely to remove an electron than to lead to a further capture process, especially so since the cross section for ionization is very large just above the threshold. Thus the charge would fluctuate for the most part between $z(v)$ and $z(v)-1$.

However, there is not really a free electron gas bombarding the fragment. In the first place, there are also the nuclei of the atoms which are actually present. These have a much higher energy than the electrons because of their greater mass. On energetic grounds alone, they could strip the fragment to a much greater extent than $z(v)$. It is well known, however, that the transfer of more energy by a heavy particle than is possible by an electron is very improbable. Thus the charge may now fluctuate both below and above $z(v)$, but probably by not much.

[16] For $n \sim 1$, as in our case, the interpolation formula given by F. Bloch, Ann. d. Physik 16, 285 (1933) should really be used. However, not much error is made by using Eq. (8).

[17] Reference 13, p. 271.

A further complication, both for assumptions (c) and (b), is that due to the atomic structure of the stopping medium, the bombarding particles do not come past the fragment separately, but in clusters of electrons and nuclei. Because of the cooperation of several of these particles, it may be possible to transfer more energy or charge to the fragment than would otherwise be possible. After a particularly violent head-on impact, the fragment may be left with practically any degree of ionization, and we must assume that these impacts are not too frequent, and that there are many more less violent impacts which will quickly restore, in accordance with hypothesis (c), the havoc wrought. The looseness of the electronic structure expressed by the approximate validity of the Hartree model for atoms, and the many essentially free electrons brought in from atoms off the track by the large charge on the fragment will help to make these two assumptions justified.

Unfortunately, the energy loss depends on the average of z^2, not on its average value squared, so that even fluctuations symmetrical about the average value can affect the range and energy loss.[18] Still, the agreement obtained in Fig. 1 indicates that the true effective charge is not too far different from $z(v)$.

With hypothesis (a), one assumes that the close collisions contribute a rate of energy loss which is small compared to that due to the charge $z(v)$. We shall now find that this does not remain true below 5–10 Mev. For a close collision at very low (thermal) energies, one could use the molecular potential curve to describe the motion of the two atoms. The steep rise in potential at distances of the order of 10^{-8} cm would give a scattering cross section $S\sim 10^{-16}$ cm^2, and an energy loss (assuming rigid spheres, and a stopping gas much lighter than the fragment)

$$dE/dx = NSM_a v^2. \tag{10}$$

This gives a fractional energy loss per cm of $2NS(M_a/M_f)\sim 3\times 10^3$ (the subscripts refer to atom and fragment), and if Eq. (10) could be used for the whole path down to thermal velocities, a range of less than a millimeter would result. The adiabatic potential curve used above, of course, does not rise indefinitely at 10^{-8} cm, since it must become just the nuclear Coulomb repulsion at very small distances of approach. Furthermore, it would lose meaning for fragments of any appreciable energy due to the closeness of many such curves belonging to different electronic configurations and the occurrence of nonadiabatic transitions between them. In this region, however, one would no longer treat the atoms as rigid, but could separate the energy lost into that due to the excitation of electrons, and that due to the recoil of the nuclei. A plausible upper limit for the first source of energy loss can be obtained from geometrical considerations. An electron passing through the fragment can lead to an energy transfer of at most about $\frac{1}{2}mv^2$, a heavy particle might give $2mv^2$. With an effective radius of action $r\sim 2\times 10^{-9}$ cm, the energy loss per cm would be $(11/2)\pi Nr^2mv^2$ for the case of nitrogen, which has a value of about 5 Mev/cm at the beginning of the range (108 Mev), and proportionately lower for smaller energies. Thus even a high[19] estimate for the energy loss due to the close impacts gives a small (although not really negligible) contribution compared to the effect of the charge as given by assumption (d). (Cf. Table I.)

Finally we consider the nuclear recoil energy loss. The potential energy for small distances of approach will be of the form

$$V(r) = (Z_a Z_f e^2/r) - V_0, \tag{11}$$

where V_0 represents the gain in binding energy of the electrons when a united atom in its ground state is formed. The statistical model gives for V_0 a value

$$V_0 \simeq 20.8[(Z_a+Z_f)^{7/3} - Z_a^{7/3} - Z_f^{7/3}] \text{ volts},$$

or about 50,000 volts for air. The energy transferred to the air nuclei due to $V(r)$ is approximately given by an equation of type (7) with

[18] These fluctuations might visibly affect the density of ionization in cloud-chamber pictures of the tracks of the fragments.

[19] That $r=2\times 10^{-9}$ cm is not too small an effective radius for the transfer of an energy $(11/2)\ mv^2$ to the fragment by a nitrogen atom may be confirmed by using the usual energy loss formulas to estimate the energy lost by an electron in passing through the fission fragment. One obtains essentially the numerical result given by the argument in the text. I am indebted to Professor R. Serber for this remark.

the upper impact parameter b defined through $V(b)\sim 0$,

$$\frac{dE}{dx}=\frac{4\pi N Z_a^2 Z_f^2 e^4}{M_a v^2}\log\left[1+\left(\frac{M_a v^2 b}{Z_a Z_f e^2}\right)^2\right]^{\frac{1}{2}}. \quad (12)$$

In air, this varies from ~ 0.3 percent to ~ 50 percent of the energy loss due to assumption (d) over the range of Fig. 1. Thus we see that below 5 Mev, it is necessary to take into account the recoil energy loss. An estimate of the range remaining at this energy gives 0.5 cm, but this may be far too large. Perhaps one should include the energy loss of Eq. (12) for $E>V_0$ and that of Eq. (10) for $E<V_0$ in using Eq. (5), but at present this would not be a justified refinement of the theory despite the improved agreement it would give both to the range and the range-energy relation.

Estimate of Initial Charge

How many electrons of the original uranium atom are carried along by each fragment at the start of its journey? It is obvious that in the limit of zero velocity of separation, both fragments would have their full complement of electrons, and that for very high velocities, none of the electrons would be carried along. In the intermediate case with which we are concerned, one would have to try to solve the wave equation for a uranium atom in which separation of the nucleus into the two fragments with a relative velocity $2v$ begins at a certain time $t=0$. This equation is too difficult to solve except in the above limiting cases, and we must therefore be content with a rough estimate of the residual charges on the fragments. A much simpler problem which will guide us is that of a hydrogen atom in which the proton is suddenly given the velocity v. In this case the solution is easily given, and one finds that if v is equal to the average speed of the electron in its orbit $=\alpha c$, the probability of ionization is 25 percent. For higher velocities, the probability rises rapidly. This suggests that in the more complicated case of uranium, we should consider that an electron is able to remain bound to one or the other fragment if and only if its average velocity in the uranium atom is greater than the velocity v of one of the fragments.[20] The average speed of an electron in a certain shell is of the order $\alpha Z^* c/n^*$ where n^* is the principal quantum number of the shell and Z^* is the effective nuclear charge for the electron. According to this criterion, the electrons of the K, L, M shells and all but the last four of the N shell will find their way to an orbit on one of the fragments. This would give an initial charge of 18 for a fragment (ignoring the difference of the two groups), and this is of the order of the initial charge required after equilibrium (according to hypothesis (c)) is established.[21] However, the above estimate is too rough to have much real significance.

It is a pleasure to express my appreciation to Professors E. Fermi, J. R. Oppenheimer, and E. P. Wigner for their helpful discussions and encouragement.

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[20] Havas (reference 9) uses a different criterion according to which the electron remains bound if, and only if, it has a velocity $>2\pi v$. This leads of course to a higher initial charge.

[21] Dr. M. Phillips has pointed out that our criterion is, in principle, identical with hypothesis (c), so that it is not surprising that the two initial charges agree approximately.