# On the Angular Distribution in the Reaction $F^{19}$ ($p$, $\alpha$)

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The angular distribution of the long range $\alpha$-particles shows marked lack of spherical symmetry, with asymmetry in the forward and backward directions. The distribution is constant with bombarding energy in the range 330 to 435 kev and contains large $\cos\theta$ and $\cos^2\theta$ terms with somewhat smaller and less certain amounts of the third and fourth powers. It is shown that the energy dependence implies broad levels and constant penetrabilities; the latter is found to agree with calculation. The experimental distributions can most simply be accounted for by the assumption that the chief contributions are from three levels of total angular momentum $J=0,1,2$. This leads to the experimental curves without arbitrary assumptions as to the values of the line breadths or detailed assumptions about the nuclear coupling scheme. Powers of $\cos\theta$ higher than the fourth in the angular distribution are not to be expected.

THE data obtained by Ellett and his co-workers on the angular distribution of the long range $\alpha$-particles from the proton bombardment of $F^{19}$ show a marked lack of spherical symmetry with asymmetry in the forward and backward directions. A distribution containing at least the first and second powers of $\cos\theta$ is required to fit the data at all well, and somewhat smaller and less certain amounts of $\cos^3\theta$ and $\cos^4\theta$ also seem to be present. More explicitly, the observed angular distribution is well represented by a fourth-degree equation of the form:

$$I(\theta)=1+0.65\cos\theta+0.11\cos^2\theta+0.44\cos^3\theta+0.24\cos^4\theta \quad (1)$$

or probably equally well by:

$$I(\theta)=1+0.66\cos\theta+0.25\cos^2\theta+0.41\cos^3\theta. \quad (2)$$

These curves are calculated to fit the data for the bombarding energy of 375 kev but they fit the distributions for bombarding energies of 330 and 435 kev almost as well, and in fact the quadratic equation:

$$I(\theta)=1+0.81\cos\theta+0.13\cos^2\theta \quad (3)$$

fits the data for the three bombarding energies to within less than ten percent.

On first sight these curves, drawn for the coordinate system in which the center of mass is at rest, present some surprising features, namely the pronounced asymmetry in the forward and backward directions, the large coefficients of the $\cos\theta$ and higher terms, and the marked constancy with bombarding energy. It is the purpose of this paper to show that none of these features is inconsistent with our present theory of nuclear reactions, and that it is possible to fit the experimental curves by making reasonable assumptions about the unknown quantities such as line breadths which occur in the theory.*

For any initial state of the system, the angular distribution of the final $\alpha$-particle wave function, in the coordinate system in which the center of mass is at rest is:

$$\left|\sum_{l,m} A_{li}^{m} Y_l^m(\theta,\phi)\right|^2, \quad (4)$$

where the $Y_l^m$ are the spherical harmonics and the $A_{li}^m$ are complex coefficients which are determined by the initial state $i$ and by the selection rules conserving the parity, total angular momentum $J$, and total magnetic quantum number $m$, as well as by the probabilities of capture and emission of the respective particles. The total angular momentum of $F^{19}$ in its ground state is $\frac{1}{2}$. The resultant angular distribution, obtained by summing (4) over the statistically independent initial states $i$, may then be found by summing over the possible different orientations of the spin $S_p$ of the incoming proton and the total angular momentum $I$ of the $F^{19}$ nucleus. We shall adopt the procedure of summing (4) over the four independent eigenfunctions of the operator $T=I+S_p$, the sum of the spin of the proton and the total angular momentum of the nucleus, indexing these states

* To calculate these unknown quantities explicitly, a detailed knowledge of the coupling scheme in the nucleus would be required. We shall fit these quantities to the experimental curves, and our calculation will then be carried through without any assumptions about the coupling.

by $i$. Furthermore, taking the parity of the ground state of $F^{19}$ as even, because $T$ is at most 1, and the final spin is 0, the disintegration into an $\alpha$-particle of orbital angular momentum $L=l$ can only arise from the capture of a proton of equal $L$ to form a compound state of $Ne^{20*}$ of $J=l$.

It is, in principle, possible for any number of levels of $J=l$ of $Ne^{20*}$ to contribute to $A_{li}{}^m$ in (4) and it is clear that a complicated picture of the reaction of this sort could readily lead to the distributions (1), (2) or (3). We shall assume simply that the process is one of resonance with two, or at most three, broad overlapping levels of the compound nucleus $Ne^{20*}$ of total angular momentum $J=0$, 1, 2, and therefore write:

$$A_{li}{}^m=\frac{\alpha_{li}{}^m(\Gamma_{plmi}\Gamma_{\alpha lm})^{\frac{1}{2}}e^{\omega_{lmi}}}{E-E_{rl}+\frac{1}{2}i\Gamma_l}. \qquad (5)$$

In (5) $\alpha_{li}{}^m$ is the amount of $\psi(T_i llm)$ in the initial wave function, where $\psi(TLJm)$ is an eigenfunction of $J$ expressed in terms of the initial eigenfunctions of $T$ and $L$. $\Gamma_l$ and $E_{rl}$ are the total width and resonance energy of the level $J=l$ of the compound nucleus, $\Gamma_{plmi}$ and $\Gamma_{\alpha lm}$ the decay constants from this state to $\psi_{li}{}^m$ and the final $Y_l{}^m$, respectively. The $\omega_{lmi}$ are determinate, though unknown phases, and finally we may write $\Gamma_{plmi}$ as $P_{pl}\gamma_{plmi}$ the product of $P_{pl}$, the penetrability of the barrier for the state whose wave function is $\psi_{li}{}^m$ and the "partial width without barrier," $\gamma_{plmi}$, which may also be regarded as the probability of forming the particular compound state once the particle has penetrated the nucleus.‡ It follows from (4) and (5) that the constancy of the distribution with energy will be highly unlikely unless the penetrabilities for the various $l$'s are constant with energy. This necessary condition is satisfied as seen from Table I which summarizes the calculated penetrabilities for the three different energies and for two different values of the radius of $F^{19}$.

TABLE I. *Calculated penetrabilities of* $F^{19}$.

| $E$ | $R\times10^{13}$ | $P_{pl=1}/P_{pl=0}$ | $P_{pl=2}/P_{pl=0}$ |
|---|---|---|---|
| 330 kev | 4.00 | 0.106 | 0.002 |
| 375 | 4.00 | 0.110 | 0.003 |
| 435 | 4.00 | 0.112 | 0.003 |
| 330 | 3.52 | 0.092 | 0.002 |
| 375 | 3.52 | 0.094 | 0.002 |
| 435 | 3.52 | 0.096 | 0.002 |

In addition, in order to explain particularly the constancy of the interference terms in (1), (2) and (3) with energy, it is necessary that the phases of the $A_{li}{}^m$ remain essentially constant in the energy range considered, or at most vary so that the phase differences remain constant. It is this requirement which makes necessary the assumption that the levels are broad. We wish to stress that this requirement and the absence of marked resonance structure in the yield curve for this reaction at these energies, means that the assumption of resonance with three levels implies nothing more than that these levels contribute larger terms to the cross section than other levels. The assumption is by no means necessary and is justified only by the fact that it leads to the observed distribution without arbitrary assumptions about the values of the line breadths. In addition, in order to simplify the calculation, we shall assume at once that the level widths $\Gamma_l$ in (5) are approximately equal.

We now note the following: The Clebsch-Gordan series gives $\alpha_{li}{}^0=0$ for $i$ corresponding to the initial state $T=1$, $m=0$. This, coupled with the selection rules, will mean that we obtain interference between $P$ and $D$ waves in the final distribution for incident states $T=1$, $m=1$ and $-1$, and interference between all three $S$, $P$, $D$ waves for the $T=0$ state. Or, alternatively, that there is interference between $P$ and $D$ when the $z$ components of the angular momenta of the incident proton and $F^{19}$ nucleus are parallel, and interference between the three waves when the $z$ components are antiparallel. Moreover, we can now drop the $i$ subscript in (5) and index the terms by their $m$ values, $m=0$ corresponding to an initial $T=0$ state and the other $m$ values to initial $T=1$ states. The $\Gamma_{\alpha lm}$ are independent of $m$ while the $\gamma_{plm}$ and $\omega_{lm}$

‡ The $\alpha_{li}{}^m$ are calculated with the aid of the Clebsch-Gordan series and depend on the order in which the eigenfunctions of the three operators $I$, $S_p$ and $L$ are combined. We have here chosen to combine first $I$ and $S_p$ but a different initial pair would lead to somewhat different numerical coefficients in (6) below and consequently require a different relation (7) to fit the observed distribution. The physical results are, of course, the same and this is simply a consequence of the fact that the different orders of combining the operators correspond to a different choice of the set of independent states of the initial system.

depend only on whether the initial state was of $T=1$ or 0. With our assumptions the denominator of (5) is approximately the same for all three levels and may be canceled from (4). The $\Gamma_{\alpha lm}$ will form the largest part of $\Gamma_l$ and our assumption of approximate equality for the $\Gamma_l$'s implies approximate equality for the $\Gamma_{\alpha lm}$'s as well, and these may also be canceled. Now insert in (5) the calculated values of the $\alpha_l^m$ and the penetrabilities for $E=330$ kev, $R=4\times10^{-13}$, put $\omega_{00}=0$, and equate the $m=-1$ to the $m=1$ terms. If we substitute and sum over $m$ in (4), the resultant angular distribution is:

$$\begin{aligned}I(\theta)=&\gamma_{00}+0.477\gamma_{11}+0.0125\gamma_{20}\\&-0.225(\gamma_{00}\gamma_{20})^{\frac{1}{2}}\cos\omega_{20}\\&+\cos\theta[1.956(\gamma_{00}\gamma_{10})^{\frac{1}{2}}\cos\omega_{10}\\&-0.220(\gamma_{10}\gamma_{20})^{\frac{1}{2}}\cos(\omega_{20}-\omega_{10})\\&+0.381(\gamma_{11}\gamma_{21})^{\frac{1}{2}}\cos(\omega_{21}-\omega_{11})]\\&+\cos^2\theta[0.954\gamma_{10}-0.477\gamma_{11}+0.075\gamma_{21}\\&-0.075\gamma_{20}+0.675(\gamma_{00}\gamma_{20})^{\frac{1}{2}}\cos\omega_{20}]\\&+\cos^3\theta[0.660(\gamma_{10}\gamma_{20})^{\frac{1}{2}}\cos(\omega_{20}-\omega_{10})\\&-0.381(\gamma_{11}\gamma_{21})^{\frac{1}{2}}\cos(\omega_{21}-\omega_{11})]\\&+\cos^4\theta[0.1125\gamma_{20}-0.075\gamma_{21}].\qquad(6)\end{aligned}$$

All unnecessary subscripts have been omitted.

To fit the simple quadratic distribution (3) we need only assume the existence of two overlapping levels of total angular momentum $J=0$ and 1. Dropping the terms in $\gamma_{20}$ and $\gamma_{21}$ in (6) we find that with

$$\gamma_{11}=\gamma_{10}=\gamma_{00}/3, \qquad (7)$$

the resultant angular distribution will be

$$I(\theta)=1+0.97\cos\omega_{10}\cos\theta+0.13\cos^2\theta.$$

The relation (7) is satisfactory because it is not improbable and is what might be expected from a nucleus in which Russell-Saunders coupling is not valid and the states of equal $J$ are well mixed. Clearly (6) can be made to fit the observed distribution if we assume other sets of overlapping levels. These, however, will in general require rather more arbitrary assumptions about the values of the $\gamma_{lm}$'s. In particular, the assumption of $S$ and $D$ resonance will lead to a distribution with a negligible $\cos\theta$ term, and the assumption of $P$ and $D$ resonance requires juggling of the phases in order to make the $\cos\theta$ term larger than the $\cos^3\theta$ term. In addition, with either of these assumptions, as well as for the assumption of the overlapping of all three levels, or $S$, $P$, $D$ resonance, the $\cos^4\theta$ term cannot be fitted without always assuming the $\gamma_{2m}$'s rather larger than the other $\gamma_{lm}$'s.

Because of the numerous assumptions involved, it is not possible to say any more than that the results of this paper suggest the following conclusions: The possibility of fitting the curve so well with the distribution (3) and of deriving this distribution without making arbitrary assumptions as to the values of the level widths seems to indicate that the observed distribution arises chiefly from a resonance, in the sense indicated, with two broad overlapping levels of $J=0$ and 1. The difficulties encountered in attempting to fit the $\cos^4\theta$ term might indicate that this term and the $\cos^3\theta$ term are actually somewhat smaller than observed, but it is quite possible that these terms arise from variation in the level widths among these three levels or even in a more complicated way from the contributions of other more distant levels. It should be noted that the possibility of large $P$ scattering arises from the fairly large value of the penetrability, combined with the normalization factors of the spherical harmonics and the weights corresponding to the more numerous possible orientations of the particles of higher orbital angular momentum in the incident beam. These factors are not quite sufficient to compensate for the decrease of penetrability for the $D$ wave, and the increasingly rapid rise of the potential barrier for waves of higher $L$ make it unnecessary to consider these waves of higher $L$ and similarly they should not appear noticeably in the angular distribution.

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