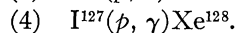
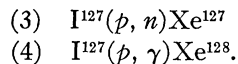


activity makes it seem that a xenon isotope is responsible. Furthermore, the x-rays accompanying the internal conversion were shown to be characteristic of xenon. See Fig. 6. A more complete study of the absorption in silver showed the ionization due to the x-rays was about equal to that of the unconverted gamma-rays.

The possible ways of producing xenon are the following:



Xe^{128} is stable, and the gamma-rays may arise from an excited state in it. Xe^{127} is not found in nature, nor has it been reported as a radioactive isotope. We have bombarded lead iodide and also sodium iodide for about 30 hours and obtained an activity of 34 ± 2 days half-life (see Fig. 7), consisting of electrons, x-rays and probably some gamma-rays. The range of the electrons is about 350 mg/cm² of aluminum. By means of a counter kindly supplied by Mr. J. G. Fox, they were found to be negative. A softer (negative) group also appears in the

complete absorption curve (Fig. 8), with range about 35 mg/cm² of aluminum. These, however, are undoubtedly secondaries produced by the harder radiation in the material of the source and the absorber, since the same group is found by absorbing in aluminum the radiation that comes through various thicknesses of lead. The greater number of these secondaries produced in lead than in aluminum makes the total radiation received through a given mass of lead greater than that through the same mass of aluminum, until the total absorber thickness is about 0.9 gram/cm². This effect is shown in Fig. 9.

A study of the x-radiation with critical absorbers in an attempt to ascertain whether the x-ray is from iodine or xenon gave an inconclusive result. From the absorption coefficient in Pb, its energy is found to be about 40–45 kev.

Although the number of nuclei with the 75-second half-life made in a given bombardment is of the same order of magnitude as the number decaying with the 34-day period, no genetic relationship between the two processes has as yet been established.

Internal Conversion of Gamma-Radiation in the *L* Shell

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The nonrelativistic calculations of the internal conversion of electric multipole radiation which have been made for the *K* shell are extended to the *L* shell. Numerical values of the conversion coefficient for both the *K* and *L* shells are given and curves showing the ratio of the *K* conversion to the *L* conversion for different energies and atomic number are presented. This latter ratio is quite sensitive to the multipole order and varies between ~ 0.1 and 10 in the range in which the calculations are valid. A simple relativistic formula for the conversion of magnetic multipole radiation is given. In Section 5 we summarize selection rules and give applications of the formulas to experiments.

1. INTRODUCTION

THE increasing number of cases in the lighter elements in which the internal conversion of gamma-radiation has been observed makes a study of this phenomenon more important. Calculations of the ratio of the number of conversion electrons to the number of gamma-quanta N_e/N_γ

have already been carried through for electric and magnetic multipole radiation for electrons of the *K* shell.¹ In this paper we shall extend these calculations with the same restrictions on their validity to electrons of the *L* shell. This has seemed worth while because experimentally it is much easier to obtain the ratio of the conversion in the *K* shell to that in the *L* shell N_K/N_L than to measure either N_K/N_q or N_L/N_q separately. Consequently, with calculations of both N_K/N_q and N_L/N_q available, an easier and more accurate comparison of experiment and theory can be made. In order to facilitate this comparison we have made fairly extensive numerical calculations of both quantities N_K/N_q and N_L/N_q and of the ratio N_K/N_L .

First we present the calculations for conversion of electric multipole radiation; in Section 4 we consider the conversion of magnetic multipole radiation.

2. CONVERSION OF ELECTRIC MULTIPOLE RADIATION IN THE *L* SHELL

In many of the artificially radioactive gamma-emitters nonrelativistic calculations of the conversion of electric multipole radiation are appropriate; namely, where the energy of the gamma-ray is small so that for the ejected electron $v \ll c$ and where Z is less than ~ 50 . We present such calculations following rather closely the method of Dancoff and Morrison. Therefore we shall give only the most important steps and the results, referring to their paper for details.

We are concerned with the ratio N_e/N_q where N_e is the number of electrons per second ejected from a given state or shell by the gamma-radiation from the nucleus and N_q is the number of gamma-quanta per second which escape from the atom. In the nonrelativistic approximation in which the wave-length of the gamma-ray is large compared with atomic dimensions, we have²

$$\frac{N_e}{N_q} = 2\pi\alpha \frac{l}{l+1} \left[\frac{(2l)!}{2^l l!} \right]^2 W^{-2l-1} \left| \int dv \psi_f^* \psi_0 Y_l^m r^{-l-1} \right|^2 \quad (1)$$

for a single electron initially in the state ψ_0 . For simplicity in writing we use the natural system of units in which the units of mass, action and velocity are, respectively, m , \hbar and c . Thus in Eq. (1), W is the energy of the gamma-ray in units mc^2 and $2\pi/W$ is the wave-length of the gamma-ray in units of the Compton wave-length. Equation (1) holds for an electric 2^l -pole, l being the change of angular momentum of the nucleus in the gamma-transition. We use α to represent the fine-structure constant $\sim 1/137$, Y_l^m for the normalized surface harmonic and ψ_f for the final electronic state in the continuum, normalized to unit flux at infinity. We have

$$\psi_f^* = Y_{l'-m'} \frac{|\Gamma(l'+1+in)|}{(2l'+1)!} 2p^{\frac{1}{2}} e^{\pi n/2} (2pr)^{l'} e^{-ipr} F(l'+1+in; 2l'+2; 2ipr),$$

where p is the momentum of the ejected electron (in units mc) and $n = \gamma/p = \alpha Z/p$.

For the *L* shell we have the 2*s*-function

$$\psi_0 = (\gamma^3/8\pi)^{\frac{1}{2}} (1 - \gamma r/2) e^{-\gamma r/2}$$

and the three 2*p*-functions

$$\psi_0 = (\gamma^5/24)^{\frac{1}{2}} r e^{-\gamma r/2} Y_1^0, \pm 1.$$

The angular integrations in Eq. (1) require $l' = l$, $m' = m$ if ψ_0 is the 2*s*-function and $l' = l \pm 1$, $m' = m$, $m \pm 1$ for the 2*p*-functions. The radial integrations can be performed in series and we find for two

¹ M. H. Hebb and G. E. Uhlenbeck, *Physica* **7**, 605 (1938); S. M. Dancoff and P. Morrison, *Phys. Rev.* **55**, 122 (1939).

² Equation (1) can be obtained most easily by taking for the perturbation acting on the electron the potential of a static multipole $V = \text{const. } r^{-l-1} Y_l^m$ multiplied by e^{-iWt} . It is also in agreement with the method of Dancoff and Morrison as can be shown by transforming the potentials from their primed gauge for which $\text{div } \mathbf{A}' = \phi' = 0$ to the conventional gauge. In the conventional gauge the only important potential in the nonrelativistic approximation is the scalar potential ϕ .

2s-electrons

$$\frac{N_e}{N_q} = 2\alpha\gamma^3 \frac{l}{l+1} \frac{|\Gamma(l+1+in)|^2}{(2l+1)^2(l!)^2} e^{\pi n} (\rho/W)^{2l+1} \left| \frac{1}{\rho^2} F_2^l - \frac{\gamma}{\rho^3} F_3^l \right|^2, \tag{2}$$

where

$$\rho = (\gamma/2) + i\rho \quad \text{and} \quad F_q^l = F(q, l+1+in; 2l+2; 4/(2-in)).$$

Similarly for the six 2p-electrons, we find two terms. The first of these for which $l' = l-1$ is

$$\frac{N_e}{N_q} = \frac{\alpha\gamma^3}{2} \frac{l^2 |\Gamma(l+in)|^2}{(l+1)(2l+1)(l-1)!^2} n^2 e^{\pi n} (\rho/W)^{2l+1} \left| \frac{1}{\rho^2} F_2^{l-1} \right|^2 \tag{3}$$

and the second where $l' = l+1$ is

$$\frac{N_e}{N_q} = 18\alpha \frac{\gamma^5 l |\Gamma(l+2+in)|^2}{(2l+3)^2(2l+1)^2(l+1)!^2} e^{\pi n} (\rho/W)^{2l+1} \rho^{-4} |F_4^{l+1}|^2. \tag{4}$$

The sum of the expressions (2), (3) and (4) then gives the quantity N_L/N_q for the eight electrons of the complete L shell.

To evaluate the hypergeometric functions we apply the Gauss relations between contiguous hypergeometric functions³ and the formula for the analytic continuation of the hypergeometric series.⁴ Then

$$\begin{aligned} \gamma^{2l+2} \frac{N_L}{N_q} &= 128\pi\alpha \frac{l}{l+1} \left[\frac{(2l)!}{2^l l!} \right]^2 n^{2l+6} \{ [l^2+n^2][l-1+n^2] \cdots [1+n^2][1-e^{-2\pi n}] \}^{-1} \\ &\quad \cdot (4+n^2)^{-3} \left\{ A_l^2 + \frac{9(l+1)}{4(2l+1)} \frac{n^2 A_l^{+2}}{[(l+1)^2+n^2]} + \frac{16ln^2}{2l+1} \frac{[l^2+n^2]}{(4+n^2)^2} A_l^{-2} \right\}, \tag{5} \end{aligned}$$

TABLE I. Values of $\log_{10} [\gamma^{2l+2}(N_e/N_q)]$ for electric 2^l -pole radiation. W =energy of gamma-ray in units mc^2 , $\gamma=Z/137$.

W/γ^2	$l=1$	$l=2$	K SHELL $l=3$	$l=4$	$l=5$	$l=1$	$l=2$	L SHELL $l=3$	$l=4$	$l=5$
0.125						1.67	2.11	3.83	4.95	5.71
0.25						2.80	0.69	1.98	3.02	3.89
0.5	2.73	2.67	2.20	3.47	4.58	3.94	1.07	0.14	0.96	1.65
	2.25	2.33	2.27	2.17	2.06	3.42	2.22	1.06	1.72	0.28
1.0	3.90	3.97	3.93	3.86	3.79	3.05	3.62	2.30	2.84	1.29
	3.62	3.65	3.59	3.50	3.41	4.76	3.18	3.72	2.16	2.52
1.5	3.39	3.38	3.28	3.16	3.04	4.52	4.82	3.25	3.60	3.89
	3.19	3.14	3.01	4.85	4.70	4.32	4.52	4.85	3.13	3.35
2.0	3.02	4.93	4.76	4.57	4.38	4.14	4.27	4.51	4.72	4.88
	4.87	4.75	4.54	4.32	4.09	5.98	4.05	4.21	4.37	4.47
2.5	4.73	4.57	4.34	4.09	5.83	5.84	5.85	5.95	4.05	4.11
	4.60	4.42	4.15	5.87	5.58	5.71	5.67	5.71	5.76	5.78
3.0	4.49	4.27	5.98	5.67	5.36	5.60	5.51	5.50	5.50	5.47
	4.38	4.14	5.82	5.48	5.14	5.49	5.36	5.30	5.26	5.20
3.5	4.28	4.02	5.67	5.31	6.95	5.39	5.23	5.12	5.04	6.94
4.0	4.10	5.79	5.40	6.99	6.58	5.21	6.98	6.80	6.64	6.48
4.5	5.94	5.59	5.16	6.71	6.26	5.05	6.77	6.52	6.30	6.08
5.0	5.80	5.41	6.94	6.45	7.96	6.90	6.58	6.27	7.99	7.71
5.5	5.67	5.24	6.74	6.22	7.69	6.77	6.40	6.04	7.71	7.39
6.0	5.55	5.09	6.56	6.00	7.44	6.65	6.24	7.84	7.46	7.09
6.5	5.44	6.95	6.39	7.80	7.21	6.54	6.10	7.65	7.23	8.82
7.0	5.34	6.82	6.23	7.62	7.00	6.44	7.96	7.48	7.02	8.57
7.5	5.24	6.70	6.08	7.44	6.80	6.34	7.84	7.32	6.82	8.34
8.0	5.16	6.59	7.94	7.28	8.61	6.25	7.72	7.17	8.64	8.12
8.5	5.07	6.48	7.81	7.13	8.43	6.17	7.61	7.03	8.47	9.92
9.0	6.99	6.38	7.69	8.98	8.27	6.09	7.51	8.90	8.31	9.73
9.5	6.92	6.28	7.57	8.84	8.11	6.01	7.41	8.77	8.15	9.55
10.0	6.84	6.19	7.46	8.71	9.96	7.94	7.31	8.65	8.01	9.38

³ Gauss, *Werke* 3, 130.

⁴ Whittaker and Watson, *Modern Analysis*(Cambridge, 4th ed.), Sec. 14.51.

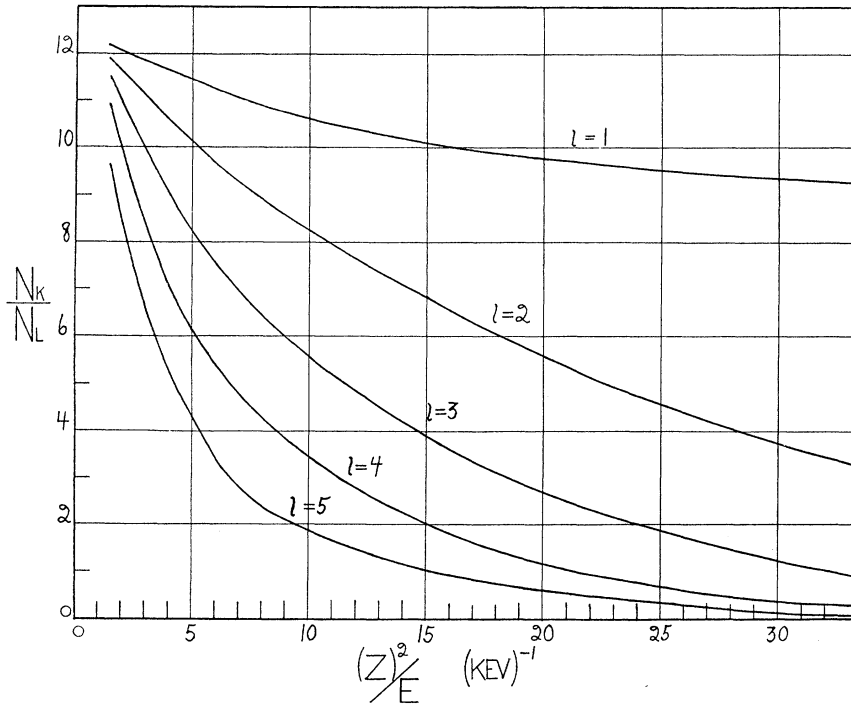


FIG. 1. Curves for N_K/N_L as a function of Z^2/E .

where

$$A_l = 2(l+1)[4 + (l+3)n^2](4+n^2)^{-1}e^{-2n \tan^{-1}(2/n)} - B_l,$$

$$A_{l+} = \frac{1}{3}[4(3l^2+9l+4) + n^2(2l^3+15l^2+31l+16)](4+n^2)^{-1}e^{-2n \tan^{-1}(2/n)} - B_{l+},$$

$$A_{l-} = (l+1)e^{-2n \tan^{-1}(2/n)} + B_{l-}.$$

The B 's are conveniently specified by giving the recurrence formulas which they satisfy.

$$(l+1)[4+n^2(l+3)]B_{l+1} = (l+2)[4+n^2(l+4)]B_l + \frac{l2^{4l+1}}{(2l+1)!} \frac{[l^2+n^2] \cdots [1+n^2][4-n^2(l-2)]}{(4+n^2)^l},$$

$$[4(3l^2+9l+4) + n^2(2l^3+15l^2+31l+16)]B_{l+1+} = [4(3l^2+15l+16) + n^2(2l^3+21l^2+67l+64)]B_{l+} + \frac{l4^{2l+2}}{(2l+1)!} \frac{[(l+1)^2+n^2] \cdots [1+n^2]}{(4+n^2)^l},$$

$$(l+1)B_{l+1-} = (l+2)B_{l-} - \frac{(l-2)}{(2l-1)!} 2^{4l-3} \frac{[(l-1)^2+n^2] \cdots [1+n^2]}{(4+n^2)^{l-1}}.$$

The first few values are

$$B_1 = 0, \quad B_{1+} = 0, \quad B_{1-} = 0,$$

$$B_2 = \frac{2}{3}, \quad B_{2+} = \frac{2}{3}, \quad B_{2-} = 1,$$

$$B_3 = 16(6+n^2)/15(4+n^2), \quad B_{3+} = 2(68+13n^2)/15(4+n^2), \quad B_{3-} = \frac{4}{3}.$$

3. NUMERICAL RESULTS

In Table I are shown the results of the calculations. The values for the K shell given in the left half of the table were calculated from the formulas previously obtained by Hebb and Uhlenbeck and by Dancoff and Morrison. The right half of the table refers to the L shell and was computed with the aid of Eq. (5). We have tabulated the quantity $\log_{10} [\gamma^{2l+2}(N_e/N_q)]$ since this depends on W and Z only in the combination W/γ^2 . The energy of the gamma-ray is related to the momentum of the electron and the parameter n by

$$W = \frac{1}{2}(p^2 + \gamma^2), \quad W/\gamma^2 = (1+n^2)/2n^2$$

for the K shell and by

$$W = \frac{1}{2}(p^2 + \gamma^2/4), \quad W/\gamma^2 = (4+n^2)/8n^2$$

for the L shell. If E represents the energy of the gamma-ray in kev, then $W = E/511$ and $W/\gamma^2 = 36.7E/Z^2$.

The influence of screening is easily taken into account in Eq. (5) and the table. One has simply to replace Z by $Z_{\text{eff}} = Z - \sigma$ where σ is a screening constant. According to Slater⁵ one should take $\sigma = 0.30$ for the K shell and $\sigma = 4.15$ for the L shell. With neglect of screening it is possible to express N_K/N_L in terms of one parameter W/γ^2 or n , but when screening is taken into account this is no longer true, and N_K/N_L depends essentially on both W/γ^2 and Z . However, the dependence on Z is not strong, and it is still feasible to construct approximate curves giving N_K/N_L directly. These are shown in Fig. 1 plotted against Z^2/E . They are strictly correct only for $Z = 35$; for higher Z the curves should be lowered somewhat and for smaller Z raised compared to the positions shown. With $25 < Z < 50$ the departures are ~ 10 to 20 percent.

4. CONVERSION OF MAGNETIC MULTIPOLE RADIATION IN THE L SHELL

Since the conversion of magnetic multipole radiation by s electrons depends essentially on the electron spin,¹ its calculation must be relativistic. In the lighter elements for gamma-ray energies not too near the threshold, the binding of the electron will not play an important part and a good estimate of the magnetic conversion may be obtained by using the Born approximation in evaluating the matrix elements. We have calculated a simple relativistic formula giving the conversion of magnetic multipole radiation in the L shell similar to that obtained by Dancoff and Morrison for the K shell.

The number of electrons ejected per second from the L shell into a solid angle $d\Omega$ is

$$N_e d\Omega = \frac{2\pi e^2}{\hbar} d\Omega |H|^2,$$

where

$$|H|^2 = \sum_{\lambda=0}^{\lambda=1} \sum_{\mu=-\lambda}^{\mu=+\lambda} \sum_{\text{spin}} \left| \int dv \psi_f^* (\boldsymbol{\alpha} \cdot \mathbf{A}') \psi_{\lambda}^{\mu} \right|^2.$$

$\boldsymbol{\alpha}$ is the vector whose components are the first three Dirac velocity matrices, \mathbf{A}' is the vector potential of a magnetic multipole in the gauge $\text{div } \mathbf{A}' = 0$, and ψ_f is normalized to unit energy. The sums are taken over the spin and orbital states of the L shell. To evaluate the matrix elements we use the Born approximation, neglecting the influence of the Coulomb field and taking a plane wave for the space-dependent part of ψ_f . Doing the sums and integrals and dividing by the rate of radiation, we have for eight electrons in the L shell:

$$\frac{N_e}{N_q} = \frac{Z^3 \alpha^4}{4W} \left(\frac{W+2}{W} \right)^{l+3} \left\{ 1 + \frac{(Z\alpha)^2}{4} \left(\frac{W+2}{W} \right) \left[\frac{l+1}{2l+1} + \frac{l(2l+1)}{4} \left(\frac{2l-1}{2l+1} - \frac{W}{W+2} \right)^2 \right] \right\}. \quad (6)$$

⁵ J. C. Slater, Phys. Rev. **36**, 57 (1930).

The first term in Eq. (6) comes from the 2s electrons and is just one-eighth of the conversion for the 1s electrons of the K shell. As pointed out by Dancoff and Morrison, conversion of magnetic radiation by s electrons is essentially relativistic. On the other hand, this is not so for p electrons for which nonrelativistic calculations give a finite conversion. The second term in Eq. (6) is the contribution from the $2p$ electrons. The ratio of the magnetic K conversion to the magnetic L conversion is readily obtained, since the formula for the K shell¹

$$\frac{2Z^3\alpha^4}{W} \left(\frac{W+2}{W} \right)^{l+\frac{1}{2}}$$

is contained as a factor in Eq. (6). Like the corresponding ratio for electric multipole radiation, it is sensitive to the multipole order, but for a given case it is definitely larger than that for the electric multipoles.

5. APPLICATION TO EXPERIMENTS

A gamma-ray of energy W is emitted in a nuclear transition between states of total angular momentum J and J' and energy difference W . From two constants of the motion, the parity and the total angular momentum of the system nucleus+gamma-ray, we obtain the selection rules.¹ The gamma-ray will be predominantly multipole radiation of the lower order l in the range $|J-J'| \leq l \leq |J+J'|$ permitted by the parity and not reduced by special arguments of symmetry. If it is permitted by the parity, we need consider only electric multipole radiation of order $l=|J-J'|$; if it is not permitted by the parity, the gamma-ray will, in general, be a mixture of magnetic 2^l -pole and electric 2^{l+1} -pole radiation. It would seem that the occurrence of these two cases should be equally probable.

The type of radiation involved may be classified by fitting the experimentally determined values of the total internal conversion coefficient, the K to L conversion ratio, and the lifetime, each of which is sensitive to l . Since the dependence on l of the K to L ratio for electric and magnetic multipoles is different, it provides a method of distinguishing between them. A mixture of the two will appear in a larger K to L ratio than is consistent with the lifetime for electric multipole radiation.

In two cases sufficient information is available to assign the conversion to electric multipole radiation. In Zn⁶⁷ the observed total internal conversion coefficient of ~ 0.5 to 1.0 ,⁶ the K to L ratio of 8,⁷ and the short lifetime are in good agreement with the calculated values for electric quadrupole radiation. The 6.4-hour Cd activity has a K to L ratio of one. This could be fitted to electric multipole radiation for $l=4$ if the conversion electrons came from a short-lived product of the disintegration. Experiments have shown that the gamma-ray and the conversion electrons actually come from a 40-sec. isomer of Ag.⁸ The 170-kev internally converted gamma-ray of the 1.2-day Te has a K to L ratio of 2.⁶ This can be interpreted consistently if the gamma-ray is $\frac{1}{3}$ magnetic multipole radiation, $l=4$, and $\frac{2}{3}$ electric multipole radiation, $l=5$.

From the preceding applications it is clear that the ratio of the K conversion to the L conversion N_K/N_L will be useful in determining the multipole order of gamma-rays of energy $\ll mc^2$. For gamma-rays of energy $\approx mc^2$ the K to L ratio for both electric and magnetic multipoles approaches the same limit ~ 10 for all multipole orders.

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⁶ The authors are indebted to Dr. A. C. Helmholtz for this information.

⁷ G. E. Valley and R. L. McCreary, Phys. Rev. **56**, 863 (1939).

⁸ L. W. Alvarez, A. C. Helmholtz and E. Nelson, Phys. Rev. **57**, 660 (1940).