hypothesis is entirely too simple a picture. The rapid fluctuation in the curve just at the critical temperature is due to the previously discussed dip in the curve of Fig. 3. The same remarks as to its genuineness would apply. Just beyond the critical temperature, the anisotropy begins to decrease with increasing temperature just as it does in the neighborhood of room temperature. A qualitative picture of the temperature dependence of this anisotropy can be obtained by considering the behavior of the lower line of Fig. 1, as temperature is increased. The curve rotates counterclockwise about the value of $F(l,m,n)$, which corresponds to the [110] direction in the crystal lattice, until the temperature at which disorder begins to set in is reached, i.e., 150°C. As disordering proceeds, the rotation becomes clockwise and, in addition, the whole curve shifts upward until finally at the critical temperature it has risen considerably above its position at room temperature and possesses a much greater slope. Curve $A$, in Fig. 1, is a plot at a temperature just below the critical temperature, where the anisotropy is a maximum.

In conclusion, the writer gratefully acknowledges his indebtedness to Professor E. P. T. Tyndall for much helpful advice and many valuable suggestions throughout the work; to the Physics Department of the State University of Iowa for the facilities generously placed at his disposal; and to others who assisted him in this work.

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# The Internal Friction of Single Metal Crystals*

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The internal friction of crystalline copper, tin, lead, and zinc has been measured by the composite piezoelectric oscillator method. It is found that the decrement of an unannealed crystal may be as large as that of the polycrystalline material, that annealing reduces the decrement to a value of the order $10^{-4}$ to $10^{-5}$, and that both Young's modulus and the decrement vary with the vibrational strain amplitude at strain amplitudes as low as $10^{-6}$. In the case of zinc crystals, a detailed study has been made of the way in which the elastic modulus and internal friction depend on the previous history of the specimen, on the vibration frequency and amplitude, and on the orientation of the vibration axis with respect to the crystal slip planes. The results suggest that the mechanism involved is a propagated "dislocation" of the sort proposed by Taylor, Polanyi and Orowan to account for macroscopic plastic flow, and that the application of a stress is accompanied by a plastic strain, together with an associated strain hardening in consequence of which the stress-strain relation on removal of the applied stress is nearly elastic.

## INTRODUCTION

INTERNAL friction is a property of matter in virtue of which organized energy of imperfectly elastic stress is rendered irrecoverable, or unavailable for mechanical work. Thus a specimen body which is carried adiabatically through a complete stress cycle can be restored to its initial state only by the removal of a certain amount of heat, and this heat, called the "energy dissipated per cycle," affords a measure of the internal friction.

The "coefficient of internal friction," $\xi$, of a substance is defined by the formula[1]

$$W^d = \xi S^2, \qquad (1)$$

where $W^d$ is the energy dissipated per cycle per unit volume, and $S$ is the stress amplitude. The internal friction of solid materials is usually studied by observing the behavior of a properly

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[1] A. L. Kimball and D. E. Lovell, Phys. Rev. 30, 948 (1927).

shaped specimen body which is vibrating in one of its normal modes. Such experiments show that the value of $\xi$ depends on the type of stress; on the frequency, and possibly the amplitude, of vibration; and, in ferromagnetic substances, on the magnetization.[2]

Considerable progress has been made toward an understanding of the processes responsible for the dissipation of vibrational energy in solids. Thus an adequate account has been given of the magnetization-dependent part of the internal friction in ferromagnetic materials,[3] and of the greater part of the internal friction in nonferromagnetic polycrystalline metals. The latter is largely the work of Zener and his collaborators.[4] According to Zener's theory, microscopic stress inhomogeneities due to occluded impurities, cold working, or simply microcrystalline anisotropy, give rise to local temperature fluctuations in the vibrating material, and hence to heat flow, with a consequent irreversibility in the stress-strain relation. However, the experiments reveal that this is not the only dissipative mechanism operative in the medium, but that a supplementary process must be invoked to account for a residual internal friction which, though small, is still many times larger than that of crystalline quartz.[5] The present study of the internal friction in large single crystals of metals contains a clue to the nature of this process. For it is found that both plastic flow and strain hardening occur in these crystals at vibration frequencies as high as 70 kilocycles per second, and vibrational strain amplitudes as low as $10^{-6}$ cm/cm.

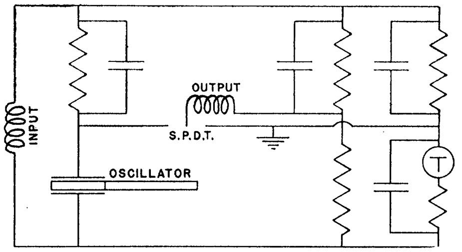

FIG. 1. Diagram of the alternating-current bridge, showing the vacuum thermocouple, $T$, in one arm of the Wagner ground connection.

## EXPERIMENTAL METHOD

It has been remarked that the value of the internal friction coefficient depends in the first instance upon the nature of the stress to which the medium is subjected. The type of stress here dealt with is the stretch associated with the longitudinal vibration of a slender rod of specimen material, and the experimental method permits the measurement of both $\xi$ and the Young's modulus, $E$. This method is essentially that used by Cooke and Brown,[2] so modified as to permit the investigation of nonhomogeneous specimens. The nonhomogeneity results from the fact that, in the materials here studied, the values of both $\xi$ and $E$ *depend on the strain amplitude*, and the latter varies sinusoidally along the vibrating rod. The method applicable to homogeneous materials will be described first, and then its extension to meet the demands of the present problem.

### Homogeneous specimen

The specimen, in the form of a right circular cylinder 5.38 mm in diameter and a few cm long, forms one part of a composite piezoelectric oscillator constructed by cementing to one end of the specimen an $X$ cut cylinder of crystalline quartz of identical cross section. The oscillator forms one arm of an alternating-current bridge (Fig. 1), which is excited by a very stable vacuum tube oscillator whose frequency is variable between 30 kc and 80 kc. The electric impedance of the composite oscillator varies critically with frequency in the neighborhood of certain "resonance frequencies," and the elastic and dissipative properties of the specimen material are deduced from the observed nature of this variation.

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[2] *Normal:* A. L. Kimball and D. E. Lovell, Phys. Rev. **30**, 948 (1927); R. L. Wegel and H. Walther, Physics **6**, 141 (1935); F. Forster and W. Koster, Zeits. f. Metalkunde **29**, 116 (1937), Naturwiss. **25**, 436 (1937). *Ferromagnetic:* In iron, W. T. Cooke and W. F. Brown, Phys. Rev. **50**, 1158 and 1165 (1936); in nickel, S. Siegel and S. L. Quimby, Phys. Rev. **49**, 663 (1936); in crystalline nickel, J. R. Zacharias, Phys. Rev. **44**, 116 (1933); in permalloy, S. Siegel and S. Rosin, Phys. Rev. **49**, 863 (1936). *Of crystalline quartz:* K. S. Van Dyke, Proc. I. R. E. **23**, 386 (1935). *During the lambda-point transition in ammonium chloride;* A. W. Lawson, Phys. Rev. **57**, 417 (1940).

[3] W. F. Brown, reference 2.

[4] C. Zener, Phys. Rev. **52**, 230 (1937); **53**, 90 (1938); **53**, 100 (1938); **53**, 582 (1938); **53**, 1010 (1938); Am. Inst. Mining and Metallurgical Eng., Technical Publication No. 1146.

[5] K. S. Van Dyke, reference 2.

It can be shown that, near a resonance frequency, the composite oscillator is electrically equivalent to a fixed capacity shunted by a series resonant electric circuit. If the vector impedance of the latter be denoted by $R+iX$, then the resonance frequencies, $f_0$, are, by definition, those for which $X=0$, and these are the solutions for $f_0$ of the equation

$$M_1f_1 \tan (\pi f_0/f_1)+M_2f_2 \tan (\pi f_0/f_2)=0, \quad (2)$$

where

$$\begin{aligned} M_i&=\text{mass of a cylinder,}\\ f_i&=(E_i/\rho_i)^{\frac{1}{2}}(2L_i)^{-1}, \quad (3)\\ E_i&=\text{Young's modulus,}\\ \rho_i&=\text{density of a cylinder,}\\ L_i&=\text{length of a cylinder,}\end{aligned}$$

and the subscripts 1 and 2 refer to the specimen and quartz cylinders, respectively. In consequence of the piezoelectric stress which accompanies the electric field in the quartz, a stationary state of forced longitudinal vibration is established in the composite oscillator, and the amplitude of this elastic vibration is a maximum at a resonance frequency.

The inductance, $\mathcal{L}$, and the capacity, $C$, of the equivalent series resonant circuit are given by the formulae

$$\mathcal{L}=\tfrac{1}{8}K(M_1+M_2) \quad (4)$$

and

$$C=2/[\pi^2f_0^2K(M_1+M_2)], \quad (5)$$

where $K$ is a quantity whose value depends only on the appropriate piezoelectric constant of the quartz and the geometry of the electrodes.[6] The resistance, $R$, of this circuit, which is also the resistance of the composite oscillator as measured with the bridge, is conveniently evaluated with the aid of two auxiliary quantities, $\Delta_1$ and $\Delta_2$, associated with the specimen and quartz cylinders respectively and defined by the formula

$$\Delta_i=W_i^d/2W_i^v, \quad (6)$$

where $W_i^d$ denotes the energy dissipated per cycle, and $W_i^v$ the total vibrational energy, in a cylinder. The $\Delta_i$'s are called the "decrements" of the cylinders, and it can easily be shown that if, and only if, $\xi$ is independent of the stress amplitude then

$$\Delta_i=\xi_iE_i, \quad (7)$$

and hence that, in this event, $\Delta_1$ is a measure of a property both of the specimen and the specimen material. The resistance is given by the formula

$$R=\tfrac{1}{4}K(M_1\Delta_1+M_2\Delta_2)f_0, \quad (8)$$

and it follows from Eqs. (4) and (8) that

$$R/2f_0\mathcal{L}=(M_1\Delta_1+M_2\Delta_2)/(M_1+M_2). \quad (9)$$

It will be noted that the left-hand member of this equation is, by definition, the electric decrement, $\Delta_e$, of the equivalent series resonant circuit, and so is a quantity which, like $R$, is directly measureable with the bridge. Thus

$$\Delta_e=R/2f_0\mathcal{L}, \quad (10)$$

and the value of $\mathcal{L}$, if desired, follows immediately from measurements of $R$ and $\Delta_e$.

The foregoing formulae, with $M_1$ set equal to zero, describe the behavior of the quartz cylinder alone, without the specimen attached. Thus, by Eqs. (2), (9) and (10), $f_2$ is the frequency corresponding to zero reactance, and $\Delta_2$ the decrement, of the single quartz crystal oscillator. When these have been measured, the only additional data necessary to obtain a complete description of the behavior of the composite oscillator are $f_0$ and $\Delta_e$. For $f_1$ then follows from Eq. (2), $E_1$ from (3), $\Delta_1$ from (9) and (10), and $\xi_1$ from (7).

### Inhomogeneous specimen

It has been remarked that the inhomogeneity of the present specimens arises from the fact that the values of both $\xi$ and $E$ depend upon the local vibrational strain amplitude, which, in turn, varies sinusoidally along the rod. The following treatment is based upon two important considerations: first, that while, over the hundred-fold range of variation of strain amplitude here explored, the variation of $\xi$ may be a hundred-fold, that of $E$ is only of the order 0.04 percent; and second, that the lengths of the quartz and specimen rods are so adjusted that at resonance there are, with a discrepancy not greater than one percent, integral numbers of half-waves of vibra-

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[6] $K=1/(b^2\epsilon^2)$, in an example with idealized geometry, where $\epsilon$ is the piezoelectric constant and $b$ is the width of an electrode on the quartz. The value of $K$ for the present experimental arrangement is 10,300 at 38 kc, where $R$ is in ohms, $\mathcal{L}$ in henries, and $C$ in farads.

tion in the quartz and specimen rods. In other words, under the conditions of measurement both the quartz and specimen cylinders are vibrating almost exactly in one of their respective normal modes.

The first question which will be considered concerns the measurement of the equivalent ioductance $\mathcal{L}$. When the specimen is homogenenus, $\mathcal{L}$ is calculated with Eq. (10), from the measured resistance at resonance and electric decrement. The latter quantity is evaluated by observing the frequency departure from resonance, $\delta f$, at which the scalar impedance is increased in the ratio $\sqrt{2} : 1$. Then $\Delta_e = 2\pi\delta f/f_0$, but this relation no longer holds when $\Delta_1$, and hence $R$, depends on the amplitude of vibration. Fortunately the materials here studied are of such nature that it is possible to find a range of vibration amplitudes sufficiently small that the variation of $\xi$ with amplitude over the range is negligible. It remains only to note, (Eq. (4)), that $\mathcal{L}$ depends on the properties of the specimen only through the mass, and hence that a single valid measurement of $\mathcal{L}$ suffices at all amplitudes. This conclusion has been verified experimentally.

The second question concerns the relation of $\xi_1$ to $\Delta_1$, and of $\Delta_1$ to $R$. $\Delta_1$ remains a property of the specimen, but is no longer a property of the material. However, a quantity, $\Delta$, which is a property of the material can be defined by the formula

$$\Delta = \xi E. \tag{11}$$

It then follows from Eqs. (1) and (6) that

$$\Delta_1 = \int_0^{L_1} \Delta U_1^2 dx \div \int_0^{L_1} U_1^2 dx, \tag{12}$$

where $U_1(=S_1/E_1)$ denotes the local strain amplitude, $dx$ an element of length on the cylinder axis, and the vibrational energy density in the specimen has been equated to $\frac{1}{2}E_1U_1^2$. In accordance with prevalent practice, $\Delta$ rather than $\xi$ will hereafter be used to describe the dissipative property of a substance.

The resistance of the oscillator is given by the formula (8), whether the specimen is homogeneous or not. This assertion can best be understood by observing that energy dissipation of whatever sort which occurs in the specimen, is manifested only through a dissipative stress acting on the quartz at the quartz-specimen interface. By dissipative stress is meant one in phase with the particle velocity. Accordingly, the question may be phrased thus: If one end of a quartz oscillator be subjected to a stress which dissipates energy at the rate (Eq. (6)) $W_1^d = 2\Delta_1 W_1^v$ per cycle, will the resistance of the oscillator be increased by the amount $\frac{1}{4}KM_1\Delta_1$ in consequence of this stress? The affirmative result then follows by direct calculation.

The third question concerns the meaning of the quantity $f_1$ which appears in Eq. (2), and its relation to the elasticity of the specimen. Here $f_1$ is defined as the solution of Eq. (2) which corresponds to a given (observed) value of $f_0$, and is also, by Eq. (3), the fundamental frequency of free longitudinal vibration of a *homogeneous* specimen. It will now be shown that, under the aforementioned conditions of measurement, $f_1$ is very nearly the fundamental frequency of free vibration of an elastically inhomogeneous specimen.

Equation (2) is usually obtained by a straightforward mathematical development in which the behavior of the composite oscillator is regarded as a problem in the theory of elasticity. The same result may be derived more easily from the following significant observation concerning the factors which determine the resonant frequency of a composite oscillator, *viz.:* The fundamental resonant frequency of a composite oscillator composed of lengths $L_1$ of homogeneous specimen material and $L_2$ of quartz is the common fundamental resonant frequency of lengths $L_1^0$ of specimen material and $L_2^0$ of quartz so chosen that, when both rods are vibrating in their respective fundamental modes, the stress and displacement amplitudes at distances $L_1$ from one end of $L_1^0$ and $L_2$ from one end of $L_2^0$ are respectively equal to each other and to the stress and displacement amplitude at the quartz-specimen interface of the composite oscillator. For under these circumstances lengths $L_1$ and $L_2$ of the rods, resonating at the common frequency $f_0$, can be imagined to be cut off and cemented together without disturbing the vibrational state, since the boundary conditions of continuity of stress and displacement at the interface are met.

The continuity of stress at the interface yields the equation

$$(E_1\pi u_1^0/L_1)\sin(\pi L_1/L_1^0) + (E_2\pi u_2^0/L_2)\sin(\pi L_2/L_2^0) = 0, \quad (13)$$

and the continuity of displacement the equation

$$u_1^0\cos(\pi L_1/L_1^0) = u_2^0\cos(\pi L_2/L_2^0), \quad (14)$$

in which $u_i^0$ denotes the amplitude of vibration of a rod. Accordingly

$$(E_1/L_1^0)\tan(\pi L_1/L_1^0) + (E_2/L_2^0)\tan(\pi L_2/L_2^0) = 0. \quad (15)$$

Now $E_i = \rho_i V_i^2$, where $V_i = 2f_0L_i^0 = 2f_iL_i$, hence

$$f_0/f_i = L_i/L_i^0, \quad (16)$$

and Eq. (2) follows immediately from Eq. (15). This form of development is particularly suited to reveal the effect of specimen inhomogeneity on the significance of $f_1$.

It has been remarked before that the type of inhomogeneity encountered in the present research is a small variation of $E_1$ with the local strain amplitude, and also that, under the conditions of measurement, the specimen is vibrating very nearly in a normal mode. Thus the space variation of the particle displacement near the interface, where the strain is nearly zero, is practically unaltered by the inhomogeneity and Eq. (15) holds, with $E_1$ interpreted as the Young's modulus for zero strain. Now it is shown in the next paragraph, and is also intuitionally evident, that the resonant frequency of a rod depends chiefly on its elasticity at the displacement *nodes* of vibration and the mass per unit length at the displacement *loops*. Hence Eq. (16) likewise holds very nearly, with $f_0$ and $f_1$ interpreted as the resonant frequencies of inhomogeneous rods of lengths $L_1^0$ and $L_1$, respectively.[7] However, $E_1$ can no longer be set equal to $4\rho_1f_1^2L_1^2$ exactly, and consequently the value of $M_1$ in Eq. (2) is uncertain to the extent of the entire variation in $E_1$, i.e. about 0.04 percent. Nevertheless, the experimental conditions that $L_1 = L_1^0$ and $L_2 = L_2^0$ very nearly imply that the resulting error in the calculated value of $f_1$ is negligible. For under these conditions

$$\tan(\pi f_0/f_1) = \pi(f_0-f_1)/f_1,$$
$$\tan(\pi f_0/f_2) = \pi(f_0-f_2)/f_2,$$

and $f_1$ is given, from Eq. (2), by the formula

$$f_1 = f_0 + (M_2/M_1)(f_0-f_2). \quad (17)$$

In a typical example, $f_0 = 40$ kilocycles and $(f_0-f_2)$ 200 cycles. Hence an uncertainty of 0.04 percent in $M_1$ produces an uncertainty of 0.08 cycles in the *absolute* value of $f_1$, and an uncertainty of 0.04 percent in the calculated value of the *change* in $f_1$ associated with a change in the amplitude of vibration. It is only the latter quantity that is of special interest in the present investigation.

It remains to relate $f_1$ to the elasticity of the inhomogeneous rod. This is accomplished by applying the usual approximation method[8] to integrate the differential equation $\partial(E\partial u/\partial x)/\partial x = \rho\partial^2u/\partial t^2$, with the result that

$$\delta f_1/f_1 = (1/L_1)\int_0^{L_1}(\delta E_1/E_1)\sin^2(\pi x/L_1)dx, \quad (18)$$

where $f_1$ is the resonant frequency of a homogeneous rod of elasticity $E_1$, and $\delta f_1$ is the change in $f_1$ associated with the change $\delta E_1$ in $E_1$.

## Measurement of the maximum strain amplitude

The calculation of the maximum strain amplitude in the specimen is based on the observation that the electric decrement of the equivalent electric circuit, as measured on the bridge, is equal to the ratio of the energy dissipated per cycle in the composite oscillator, $W_0^d$, to twice the total vibrational energy at resonance, $W_0^v$. Then

$$W_0^d = 10^7\mathcal{E}^2/Rf_0 = 2\Delta_eW_0^v = RW_0^v/f_0\mathfrak{L},$$

where $\mathcal{E}$ denotes the root-mean-squared voltage applied to the quartz. $W^v{}_0$ is readily evaluated in terms of the maximum strain amplitude in the specimen, $U_1^0$, and it follows that

$$U_1^0 = 6.325\times10^3(\mathcal{E}/R)(\mathfrak{L}/V_1E_1)^{\frac{1}{2}} \times[1+(m/n)(E_2\rho_2/E_1\rho_1)^{\frac{1}{2}}]^{-\frac{1}{2}}, \quad (19)$$

where $m$ and $n$ denote the number of half-waves

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[7] The error is of the order $(\delta f/f)^2$, where $\delta f$ is the change in resonant frequency due to the inhomogeneity.

[8] See, for example, J. C. Slater, *Introduction to Theoretical Physics* (McGraw-Hill, 1933), p. 154.

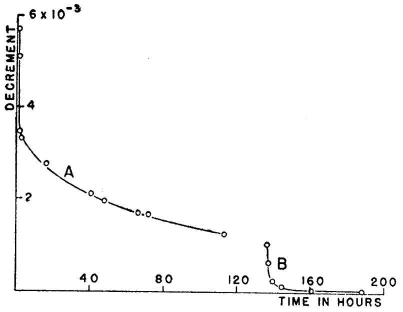

FIG. 2. Showing the change in the decrement of an isolated zinc crystal with time; Part A, before annealing; Part B, after annealing.

of vibration in the quartz and specimen, respectively.

## Experimental details

The quartz is etched in the manner described by Van Dyke[5] to minimize $\Delta_2$, and cemented to the specimen with a very thin film of beeswax and rosin. The composite oscillator rests horizontally on two fine silk threads placed at displacement nodes of vibration, with the quartz crystal between, but not touching, parallel plate electrodes. The environment of the oscillator is evacuated and temperature stabilized to 0.01°C at 26°C.

The frequency of the voltage applied to the oscillator is measured by comparison with the output of a piezoelectric clock.[9] The absolute value of the frequency is known only to one part in $10^5$, but frequency *changes* can be measured with an accuracy better than 0.01 cycle.

The voltage, $\mathcal{E}$, applied to the quartz is measured with the aid of a calibrated vacuum thermocouple placed in one arm of the Wagner ground connection, as shown in Fig. 1. The bridge detector is sufficiently sensitive to permit the use of the bridge when $\mathcal{E}$ is as small as 0.001 volt.

## SPECIMEN MATERIAL

The copper crystals are grown from commercial copper wire. The analyses of the remaining materials, as supplied by the makers, are as follows: tin (Eimer and Amend Co.)—percent impurity, Fe 0.004, Zn 0.004, Pb 0.001, As 0.00001; lead (Eimer and Amend Co. "test lead") silver, gold and bismuth free; zinc (New Jersey Zinc Co.)—percent impurity, Fe 0.0004 to 0.0009, Pb less than 0.0002, Cd less than 0.00005.

The copper crystals are grown in a graphite crucible, in the furnace described by Quimby.[9]

The tin, lead and zinc crystals are grown by Miller's modification of Bridgman's method.[10] The principal features of this method are the following: The material is melted in a Pyrex glass flask, and thoroughly degassed in a vacuum. It is then cast in cylindrical molds, made of Pyrex glass tubing, which have been thoroughly cleaned and coated inside with colloidal graphite to prevent the material from sticking to the walls. The loaded molds are packed in Sil-O-Cel, in a large test tube, and lowered through an electric furnace at the rate of a quarter of an inch per hour.

## RESULTS

The internal friction of well-annealed single crystals is considerably less than that of the corresponding polycrystalline materials as reported by other observers.[2] Thus the values of $\Delta_1$ at small amplitudes of vibration are, for copper—$3.6\times10^{-5}$; for tin—$6.9\times10^{-5}$; for lead—$2.8\times10^{-4}$; and for zinc—$1.0\times10^{-5}$ to $2.2\times10^{-5}$.[11] The decrements of unannealed single crystals may be as large as those of polycrystalline specimens. For example, the decrement of a freshly prepared copper crystal may decrease from $3\times10^{-3}$ to the value given above, i.e., almost a hundred-fold, after it is annealed for two hours in hydrogen at 500°C.

But the most significant dissipative characteristic of these crystals is an associated variation of the decrement and elastic modulus with the vibrational strain amplitude. It will later be suggested that this phenomenon is due to

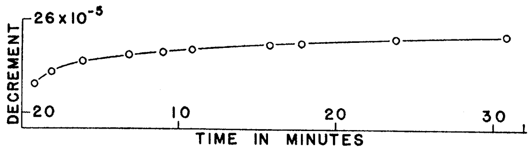

FIG. 3. Showing the change in the decrement of a continuously oscillating zinc crystal with time.

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[9] See S. L. Quimby, Phys. Rev. **39**, 345 (1932).

[10] P. W. Bridgman, Proc. Am. Acad. **60**, 305 (1925); R. F. Miller, Trans. Inst. of Metals Div. Am. Inst. Min. Eng. **122**, 176 (1936).

[11] A preliminary report of these measurements was given in Phys. Rev. **54**, 389 (1938).

sporadic displacements of atoms or groups of atoms parallel to the slip planes of the crystal. If this hypothesis is correct, the effect should be closely correlated with the relative orientation of the vibration axis and the slip planes. Now the crystalline structure of zinc is hexagonal, with a single slip plane perpendicular to the hexagonal axis. Accordingly this substance is particularly suited for the present investigation, and the remainder of this research is concerned with a study of the phenomenon as it appears in single crystals of zinc.

Definitive observations are not easily made, for the reason that almost anything that is done to the crystal leaves an almost indelible imprint on its subsequent behavior. The nature of the difficulties encountered is illustrated by the curves of Figs. 2 to 5.

The first portion of the curve of Fig. 2 shows the variation of $\Delta_1$ with time at a very small strain amplitude, for a crystal which was mounted, without annealing, immediately after removal from the crucible. During the indicated time the oscillator remained isolated at the temperature 26°C. After 4 days the oscillator was removed from the apparatus, disassembled, and the zinc crystal was annealed for 1 hr. at 250°C. The remainder of the curve shows the variation of $\Delta_1$ with time after reassembly and remounting.

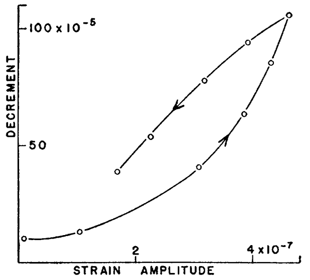

FIG. 4. The decrements measured when the strain amplitude is increased by steps are smaller than those when the strain amplitude is subsequently decreased.

Observations made with increasing strain amplitude cannot be repeated in reverse order (Fig. 4), nor can they be repeated at all until after an elapsed time which varies from several hours to several days. Curve $B$ of Fig. 5 was obtained 4 hr. after curve $A$.

The curves of Fig. 6 show the variation of $\Delta_1$ with maximum strain amplitude for an array of zinc crystals whose cylinder axes make various angles, $\theta$, with the hexagonal axis of the crystal lattice. The procedure by which the data here plotted were obtained was as follows: After the crystals were removed from the molds and sawed to the proper length with a fine jeweler's saw, they were annealed for 2 hr. at 250°C in a vacuum, and then mounted. The measurements were made one or two days later, always after 10 minutes steady oscillation at each amplitude, and always on *increasing* amplitude. The vibration frequency for all crystals was about 38 kc per sec.

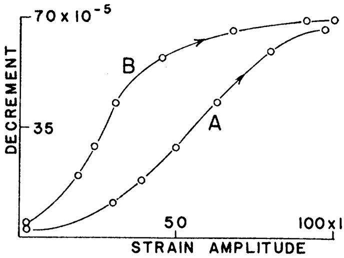

FIG. 5. Observations cannot be repeated until a considerable time has elapsed. The data for curve $B$ were obtained 4 hours after those for curve $A$.

The decrement of the crystals is approximately inversely proportional to the vibration frequency, as is indicated by the typical set of data plotted in Fig. 7. These observations were made on the same crystal when vibrating, in the first instance, at 38 kc with one half-wave in the specimen, and in the second, at 76 kc with two half-waves in the specimen.

The associated variation of the resonance frequency, $f_1$, with maximum strain amplitude in the four crystals is shown in Fig. 8. A comparison of the data given in Figs. 6 and 8 reveals the significant fact that the ratio $r = \Delta_1 \div (\delta f_1/f_1)$ *is a constant independent of the strain amplitude.* The experimental confirmation of this is exhibited in Fig. 9, and the values of $r$ for the different crystals are recorded in Table I.

The value of Young's modulus for an arbitrary direction in a hexagonal crystal is given in terms of the principal elastic moduli by the formula[12]

$$1/E = s_{11}\sin^4\theta + s_{33}\cos^4\theta + (2s_{13}+s_{44})\sin^2\theta\cos^2\theta.$$

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[12] W. Voigt, *Lehrbuch der Kristallphysik*, p. 746.

Accordingly, the present measurements permit the evaluation of the principal adiabatic moduli $s_{11}$ and $s_{33}$, and of the combination $(2s_{13}+s_{44})$. The corresponding isothermal quantities can be calculated with the aid of the thermal expansion coefficients measured by Gruneisen and Goens.[13] The results of the present measurements, together with those reported by other observers, are presented for comparison in Table II.

Before proceeding to a discussion of the foregoing data, which deal with the properties $\Delta_1$ and $\delta f_1$ of the specimen *rod*, it is well to point out the way in which these can, if desired, be correlated with the properties of the specimen *material*. Such correlation, which is based on Eqs. (12) and (18), is greatly facilitated by the experimental close matching of the frequencies $f_0$ and $f_1$. In this event the variation of the local strain amplitude, $U_1$, along the rod is given very nearly by the formula

$$U_1 = U_1^0 \sin(\pi x/L_1), \tag{20}$$

and Eq. (12) may then be rewritten in the form

$$\Delta_1 = (2/L_1)\int_0^{L_1} \Delta \sin^2(\pi x/L_1)dx. \tag{21}$$

The variation of the decrement of the *material*, $\Delta$, with *local* strain amplitude might now be obtained with the aid of Eqs. (20) and (21), by expanding $\Delta_1$ in a power series in $U_1^0$, whose coefficients are determined by the curves of Fig. 6, and $\Delta$ in a power series in $U_1$, whose coefficients can then be obtained by equating the coefficients of like powers of $U_1^0$ in Eq. (21).

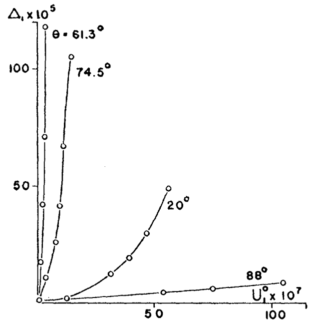

FIG. 6. Showing the variation of the decrement, $\Delta_1$, with strain amplitude, $U_1^0$, in zinc crystals whose cylinder axes make various angles, $\theta$, with the hexagonal axis.

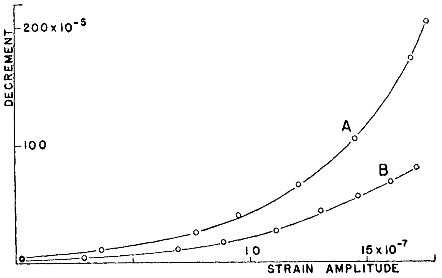

FIG. 7. The decrement is very nearly inversely proportional to the vibration frequency at all strain amplitudes. The vibration frequency for the data of curve *A* is 38 kc and of curve *B* is 76 kc.

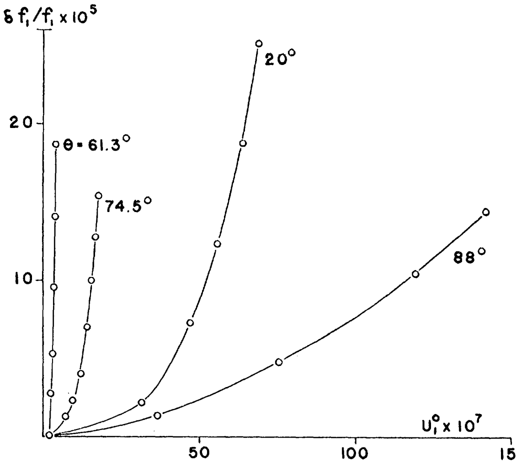

FIG. 8. Showing the fractional variation of the fundamental resonant frequency, $\delta f_1/f_1$, with the maximum strain amplitude, $U_1^0$, in zinc crystals whose cylinder axes make various angles, $\theta$, with the hexagonal axis.

A similar procedure, applied to Eq. (18), would reveal the relation between $E_1$ and $U_1$.

A detailed calculation of the sort just outlined is not considered important in connection with the present data, but the similarity in form of Eqs. (18) and (21) is extremely significant. For this result, together with the observed mutual independence of the ratio $r$ and $U_1^0$, implies that the relation between $2\Delta$ and $\delta E_1/E_1$ is exactly the same as the relation between $\Delta_1$ and $\delta f_1/f_1$, i.e., that

$$2\Delta \div (\delta E_1/E_1) = r. \tag{22}$$

## DISCUSSION

The fact, revealed by data of the sort plotted in Fig. 7, that the decrement of single metal crystals varies inversely with the frequency of

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[13] W. Voigt, reference 13. E. Gruneisen and E. Goens, Zeits. f. Physik **29**, 141 (1924).

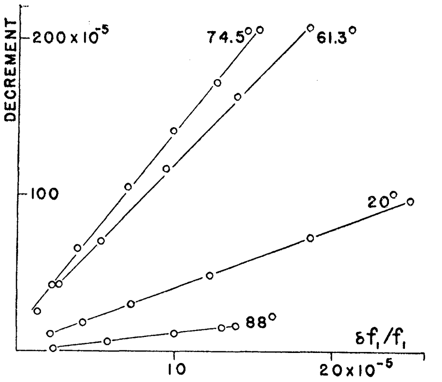

FIG. 9. Combination of the data plotted in Figs. 6 and 8 shows that the ratio of the decrement to the fractional change in resonant frequency is independent of the strain amplitude.

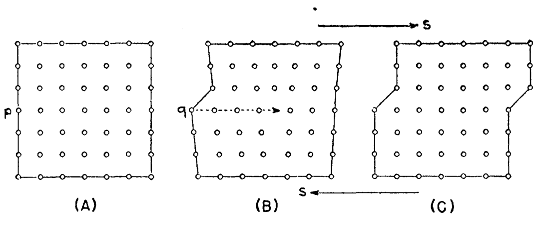

FIG. 10. Illustrating the formation and propagation of a dislocation (after Taylor).

vibration suggests that the origin of the internal friction here studied is the same as that of the observed macroscopic plastic flow. The latter phenomenon has been ascribed by Taylor, Polanyi, Orowan and others to the propagation through the lattice of a specific type of "dislocation" in the atomic arrangement.[14] The nature of the process is illustrated in Fig. 10, which is similar to a figure in one of Taylor's papers. Fig. 10(A) represents a cross section through the lattice of a perfect crystal, the slip plane being supposed parallel to a horizontal row of atoms in the figure. A dislocation occurs when, in consequence of a weakening of the interatomic binding force, and atom, for example at $p$, "jumps" an atomic distance parallel to the slip plane into a new equilibrium position at $q$. The action of an applied shearing stress, $S$, on the atoms above and below the slip plane causes the dislocation thus formed to be propagated through the lattice in the manner indicated in Fig. 10(B). Thus the final configuration of the atomic group is as shown in Fig. 10(C).

TABLE I. *Values of the ratio* $r = \Delta_1 \div (\delta f_1/f_1)$ *for different zinc crystals.*

| $\theta°$ | $r$ |
|---|---|
| 20.0 | 2.8 |
| 61.2 | 10.3 |
| 61.5 | 11.0 |
| 74.5 | 9.0 |
| 88.0 | 1.1 |

TABLE II. *The principal isothermal elastic moduli of crystalline zinc, as measured by various observers.*[1]

| OBSERVER | MODULUS ×10¹³ CM²/DYNE $s_{11}$ | $s_{33}$ | $2s_{13}+s_{44}$ |
|---|---|---|---|
| Read | 8.35 | 28.5 | 11.3 |
| Goens | 8.4 | 28.7 | 11.0 |
| Tyndall and Hanson | 7.70 | 27.66 | 10.54 |

[1] E. Goens, Ann. d. Physik **16**, 793 (1933); E. P. T. Tyndall, Phys. Rev. **47**, 398 (1935).

The dislocations are assumed to originate at, and their propagation to be arrested by, microscopic flaws in the crystal due to occluded impurity, to surfaces of misfit, or to a mosaic superstructure. They occur spontaneously as a result of thermal agitation, and are produced by an applied stress. The propagation of each dislocation is accompanied by a dissipation of energy, associated with the atomic jumps, and a local slip to the amount of one atomic distance. The net contribution of such processes to the internal friction and inelastic strain depends on the number density of dislocations present in the crystal.

The mechanism here described is adequate to account qualitatively for the several phenomena revealed in Figs. 2 to 7. Thus, since the presence of the dislocations implies an internal potential energy in excess of that characteristic of the perfect lattice, the number density of dislocations decreases in time at a rate which depends on the material and the temperature (Fig. 2). Again, the number density present in a vibrating crystal is fixed when the rate of production of dislocations by the applied stress is equal to the natural rate of decrease, and the establishment of this equilibrium number requires a finite time (Fig. 3). Similarly, a finite time is required to establish the new equilibrium value

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[14] G. I. Taylor, Proc. Roy. Soc. **A145**, 362, 388, 405 (1934); M. Polanyi, Zeits. f. Physik **89**, 660 (1934); E. Orowan, Zeits. f. Physik **89**, 605, 614, 634 (1934); **97**, 573 (1935); **98**, 382 (1936). For a complete bibliography see R. Houwink, *Elasticity, Plasticity, and Structure of Matter* (Cambridge, 1937).

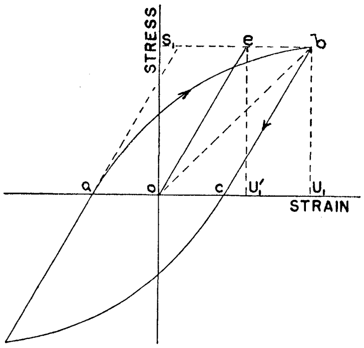

FIG. 11. Suggested form of the stress-strain diagram for single metal crystals.

associated with a decreased vibrational stress amplitude (Figs. 4 and 5). Lastly, the number density of dislocations increases with the amplitude of the component shearing stress parallel to the slip plane (Fig. 6). It remains to consider the variation of Young's modulus with the vibration amplitude.

The variation of $E_1$ with $U_1$, small though it is, nevertheless definitely indicates that the stress-strain relation is not that characteristic of *pure* plastic flow. A substance is plastic if, for a given strain, the stress decreases in time at a rate proportional to its own value. Thus the time variation of the stress in a plastic rod is given by the equation[15]

$$dS/dt = E(ds/dt) - pS, \tag{23}$$

where $s$ is the strain and $p$ is a constant which is, in the present experimental arrangement, related to $\Delta$ by the formula $p = 2f_1\Delta$. Hence the frequency variation of $\Delta$ is as stated at the beginning of this section. But it also follows from Eq. (23) that the stress-strain diagram is an ellipse, the slope of whose major axis, which is the effective value of Young's modulus, *does not vary with the strain amplitude*. It is evident, therefore, that the circumstances of pure plastic flow do not subsist over the entire cycle, and that the description offered by the ideal stress-strain ellipse must be modified.

It is suggested that plastic flow, associated with the propagation of dislocations, accompanies the *application* of a stress but not its *removal*, i.e., that the application of a stress produces what may properly be called "strain-hardening" in the material. The nature of the suggested stress-strain diagrams at very low and at larger amplitudes are indicated in Fig. 11. At very low amplitudes the strain is elastic, and the half-loop is the line *oe*. At larger amplitudes a plastic strain of amount $2(U_1 - U_1')$ takes place over the portion *ab* of the cycle, but, in consequence of the concurrent strain hardening, the portion *bc* is nearly elastic.

It must be remembered that Fig. 11 is not drawn to scale. Actually the plastic strain, shown by the intercept *ac*, is not larger than one-thousandth the strain amplitude, shown by the abscissa $U_1$. The stress-strain loop hugs its major axis so closely that the effective Young's modulus is always the slope of this axis. Accordingly, the value of $E_1$ at low amplitudes is $S_1/U_1'$, and at larger amplitudes is $S_1/U_1$, and it follows that

$$\delta E_1/E_1 = ac/2U_1, \tag{24}$$

where $\delta E_1$ is the change in $E_1$ with amplitude.

Considerable support is lent the present hypothesis by the fact that it is not only in accord with the observed invariance of the ratio $r$ (Eq. (22)) to change in $U_1$, but it leads to a value of this ratio which is in good agreement with observation. The area of the stress-strain diagram, which is the energy dissipated per unit volume per cycle, $W^d$, lies between $S_1 \cdot ac$ and $2S_1 \cdot ac$, and it follows immediately from Eqs. (1), (11), (22) and (24) that the value of $r$ lies between 4 and 8, independently of $U_1$, and in agreement with the data of Table I. Larger values of $r$ may be assumed to be the result of a state of affairs depicted by a downward flexure of the line *bc* in Fig. 11, and smaller values of that depicted by an upward flexure.

The origin of the variation of $r$ with the angle $\theta$, and the temperature dependence of the several phenomena, remain subjects for future investigation. In conclusion, the writer desires to acknowledge his indebtedness to the officers of the New Jersey Zinc Company, for their courtesy in supplying the extremely pure material from which the zinc crystals were grown; to Dr. W. F. Brown and Dr. Clarence Zener, for many helpful conversations; and to Dr. S. L. Quimby, for his guidance during the progress of the work.

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[15] R. Houwink, reference 14, p. 7.