Spin-Orbit Coupling in He^5

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Experimental results on the splitting of the ground state of He^5 are compared with the structures which would be expected on the basis of (*a*) Thomas relativistic spin-orbit coupling, and (*b*) the tensor spin-orbit interaction of mesotron theory. It is found that the Thomas coupling, as customarily formulated, gives an inverted doublet which is several orders of magnitude too small. The tensor force gives a result of the same order of magnitude as the experimental splitting and of the "normal" structure; these latter results can be established, however, only for a nonsingular behavior of the potential function. We discuss a modified treatment of the relativistic spin-orbit coupling, suggested by Breit, which would account for the existence of widely spaced spin-orbit multiplets.

I

RECENT experiments of Staub[1] on the anomalous scattering of neutrons in helium have set the energy of the resonant level at about 1 Mev and have determined the structure to be that of a doublet with a separation of 0.25 ± 0.1 Mev. In a Hartree model, one would suppose the ground state of He^5 to consist of a P neutron outside an alpha-particle core; such a configuration would, if spin-orbit forces were present, give rise to a doublet with J values $\frac{1}{2}$ and $\frac{3}{2}$. The relative intensities of the observed peaks agree fairly well with the ratio to be expected for such a doublet (2.5 to 1 in the forward direction); for this, however, it is necessary to assume that one has the "inverted" order, the $\frac{3}{2}$ level lying lower.

It does not seem possible to fit the observations by other assumptions regarding the low-lying states of He^5. Tyrrell[2] has made calculations which indicate that a state with the extra neutron in a higher S orbit would lie very close to the above P state. If one were to assume that the observations do not indicate a spin-orbit splitting, but rather resonance for both P and S states, the P state lying slightly lower, one would obtain an intensity ratio of 9 to 1 in the forward direction, far outside the range of experimental error.

One is then led to try to explain the magnitude and direction of the splitting on the basis of the effect of proposed nuclear spin-orbit forces on a ^2P state.

II

The Thomas relativistic spin-orbit force yields directly an inverted order of the doublet.[3] If the corresponding perturbing term in the Hamiltonian is taken as $\hbar\boldsymbol{\sigma}\cdot[\dot{\mathbf{v}}\times\mathbf{v}]/2c^2$, the magnitude of the splitting can be estimated very simply. If we adopt the Hartree model the effect is due entirely to the precession of the P neutron. We consider it to be acted upon by a central force represented by a "square well" potential of depth V_0 and radius a. The acceleration is then $-\hat{\mathbf{1}}(1/M)(\partial V/\partial r)$,[4] and the term becomes

$$-\frac{\hbar^2}{M^2c^2}\left(\frac{1}{r}\frac{\partial V}{\partial r}\right)(\boldsymbol{\sigma}\cdot\mathbf{L}), \tag{1}$$

where $\hbar\mathbf{L}=M[\mathbf{r}\times\mathbf{v}]$. The factor $(\boldsymbol{\sigma}\cdot\mathbf{L})$ introduces a coefficient $+1$ for $J=\frac{3}{2}$ and -2 for $J=\frac{1}{2}$. The splitting is

$$-\frac{3}{4}\frac{\hbar^2}{M^2c^2}\int r^2dr\left\{\frac{1}{r}\frac{\partial V}{\partial r}P^2(r)\right\}. \tag{2}$$

Here $P(r)$ is the radial part of the neutron's

[1] The author is indebted to Dr. Staub for communication of his results before publication. H. Staub and H. Tatel, Phys. Rev. **57**, 936 (1940). *Note added in proof.*—A more complete analysis of Dr. Staub's indicates that by taking into account the interference of s and p scattering, the data can be fitted rather better on the assumption of a normal than an inverted doublet, with the splitting still about 250 kev. This is indirectly confirmed by scattering of 2-Mev neutrons where it is reported that there exists a preponderance of small-angle scattering (low energy recoils). This would favor a normal doublet and, for 180-degree scattering as in Staub's case, destructive interference between s and p scattering. Thus, at present, the evidence, though still inconclusive, seems to indicate a normal doublet such as would be expected, as this paper shows, from known nuclear forces.

[2] W. A. Tyrrell, Jr. Phys. Rev. **56**, 250 (1939).

[3] D. R. Inglis, Phys. Rev. **50**, 183 (1936).

[4] We will use throughout $\hat{\mathbf{1}}=\mathbf{r}/r$.

wave function; we take $P(r)\sim re^{-r/\lambda}$ and obtain

$$-\frac{\hbar^2}{M^2c^2}\frac{a^3}{\lambda^5}V_0e^{-2a/\lambda}. \quad (3)$$

A relation between V_0 and a is obtained by setting up the condition for a resonant state of the P neutron near ground. This is

$$V_0=(\pi^2/2)\hbar^2/Ma^2. \quad (4)$$

A fairly good estimate of a, the radius of the potential trough, can be obtained from Staub's experimental data. The cross section for the spherically symmetric nonresonance scattering should approximately satisfy the relation $\sigma_S=4\pi a^2$. This gives $a=4.6\times10^{-13}$ cm. This formula for the S wave cross section is derived for a high repulsive barrier such as is customarily used in calculating "potential" scattering. Although there is in this case a Fermi repulsion at short distances between the incident neutron and the neutrons in the α-particle, the model is probably not sufficiently trustworthy to establish accurately the value of a. On the other hand, the value obtained is consistent with the expectation that the radius of the trough in He^5 should be considerably larger than the radius of the α-particle; for if the incident neutron can remain bound for any length of time it must do so by transferring most of its kinetic energy to the other particles, which consequently take up orbits of larger radii.† The quantity λ is more uncertain, but it should not be appreciably smaller than a. With $a=\lambda=4.6\times10^{-13}$ cm, the splitting is

$$-\frac{\pi^2}{2}\frac{\hbar^4}{M^3c^2}\frac{a}{\lambda^5}e^{-2a/\lambda}\sim2.8\text{ kev.} \quad (5)$$

This result is too small by two orders of magnitude to fit the observed splitting; it seems very improbable that the uncertainties involved in the estimate could be responsible for so large a discrepancy.

Another method of estimating the Thomas splitting consists in using the centripetal acceleration, v^2/r, in place of $(1/M)(\partial V/\partial r)$. This gives, instead of (1):

$$-\frac{\hbar^4}{4M^3c^2}\left(\frac{1}{r^4}\right)(\boldsymbol{\sigma}\cdot\mathbf{L})L^2. \quad (7)$$

The eigenvalue of L^2 is here $L(L+1)=2$. Proceeding as before, the splitting is

$$-\frac{3}{2}\frac{\hbar^4}{M^3c^2}\int r^2dr\left(\frac{1}{r^4}\right)P^2(r)=-\frac{\hbar^4}{M^3c^2\lambda^4}, \quad (8)$$

which, with the former assumptions, gives 4.1 kev.

Still another way to look at the Thomas splitting is the following. We can think of the effect as being due not to the uniform precession of the P neutron, but rather, somewhat more realistically, to the short-lived, violent precessions accompanying two particle collisions. If we neglect the instantaneous motion of the center of gravity of the two particles, and take the force between them to be central, we obtain an interaction:

$$-\frac{\hbar^2}{16M^2c^2}\sum_{\text{pairs}}\left(\frac{1}{r^{ij}}\right)\frac{\partial V(r^{ij})}{\partial r^{ij}}(\boldsymbol{\sigma}^i+\boldsymbol{\sigma}^j)\cdot\mathbf{L}^{ij}, \quad (9)$$

where $\mathbf{r}^{ij}=\mathbf{r}^i-\mathbf{r}^j$ and $\hbar\mathbf{L}^{ij}=M[\mathbf{r}^{ij}\times(\mathbf{v}^i-\mathbf{v}^j)]$. The evaluation of this term is similar to others that will be discussed in the next section. The result has the same functional dependence and is of the same order of magnitude as (3) for a square well potential and a range 2.2×10^{-13} cm.

The radial integral in which $V(\rho)$ occurs is of the form $\int\rho^3d\rho(\partial V/\partial\rho)$. It is clear that the magnitude of the effect will depend greatly on the form of V. A $1/\rho^3$ dependence for $V(\rho)$ would lead to a divergent result; using this dependence, a cut-off could be adjusted to make the splitting as large as desired. However, potentials with leading term $1/\rho^3$ occur in mesotron theory only through the "tensor force." As will be shown in the next section the tensor force itself leads to a considerable splitting; with the adjustment of shape required here, it can be shown that it would lead to a much larger splitting than the Thomas precession, and in the opposite direction. It is therefore probably not possible to explain the magnitude of the doublet on the basis of relativistic spin-orbit precession, as customarily formulated.

† *Note added in proof.*—Stephens gives for the nucleus He^5-Li^5 a "Coulomb radius" of 4.2×10^{-13} cm (W. E. Stephens, Phys. Rev. **57**, 938 (1940)).

A possible modification of the calculation of the Thomas splitting involves simply multiplying the expression (9) by a constant. Breit[5] has obtained the Thomas force as a necessary part of the nuclear Hamiltonian by demanding covariance, to order v^2/c^2, of the Dirac wave equation for a nuclear particle. If the force field is taken to be a fixed external field, the Thomas term is uniquely fixed as $(\hbar/4c^2)\boldsymbol{\sigma}\cdot[\dot{\mathbf{v}}\times\mathbf{v}]$. However, when the forces are due to interactions between particles, there is an arbitrariness essentially in the choice of a multiplicative constant.

If the inter-particle forces are derivable from an "ordinary" potential $J(r^{ij})$, then the additive term necessary to guarantee covariance is

$$Q_0=-\frac{\hbar}{4Mc^2}\sum_{\text{pairs}}\{[\nabla^i J(r^{ij})\times\boldsymbol{\sigma}^i]\cdot[\mathbf{v}^i b^{ij}+\mathbf{v}^j(1-b^{ij})]+[\nabla^j J(r^{ij})\times\boldsymbol{\sigma}^j]\cdot[\mathbf{v}^j b^{ij}+\mathbf{v}^i(1-b^{ij})]\},\quad(10)$$

where b^{ij} is, to this point, arbitrary. If we take $b^{ij}=1$ for all i, j, we get

$$Q_0=\frac{\hbar}{16Mc^2}\sum_{\text{pairs}}\{[\mathbf{v}_-^{ij}\times\nabla_-^{ij}J(r^{ij})]\cdot[\boldsymbol{\sigma}^i+\boldsymbol{\sigma}^j]+[\mathbf{v}_+^{ij}\times\nabla_-^{ij}J(r^{ij})]\cdot[\boldsymbol{\sigma}^i-\boldsymbol{\sigma}^j]\}.\quad(11)$$

Here $\mathbf{v}_+^{ij}=\mathbf{v}^i+\mathbf{v}^j$, $\nabla_+^{ij}=\nabla^i+\nabla^j$, etc. This result is the intuitive extension of $(\hbar/4c^2)\boldsymbol{\sigma}\cdot[\dot{\mathbf{v}}\times\mathbf{v}]$ and agrees with (9) if the center of gravity is at rest: $\mathbf{v}_+^{ij}=0$. Q_0 is not essentially different if Majorana or Heisenberg forces are considered.

On the other hand, if we take $b^{ij}=b\gg 1$, we get

$$Q_0=\frac{b\hbar}{8Mc^2}\sum_{\text{pairs}}[\mathbf{v}_-^{ij}\times\nabla_-^{ij}J(r^{ij})]\cdot[\boldsymbol{\sigma}^i+\boldsymbol{\sigma}^j].\quad(12)$$

The velocity of the center of gravity is not involved. This is just the expression (9) multiplied by $2b$. b could be adjusted to give correctly the experimental value of the splitting; a value of the order of 10–100 would be needed. The weakness of such a method of treatment is that such a large value of b renders meaningless the expansion in terms of powers of v^2/c^2 whereby (10) was obtained. One can conclude only that large spin-orbit forces may be present, but not that they will necessarily be of the form (12).

[5] G. Breit, Phys. Rev. **51**, 248 (1937).

III

One would expect a contribution to this splitting from the "tensor force" of mesotron theory

$$T=\sum T^{ij}=\sum_{\text{pairs}}[(\boldsymbol{\sigma}^i\cdot\hat{\mathbf{r}}^{ij})(\boldsymbol{\sigma}^j\cdot\hat{\mathbf{r}}^{ij})-\tfrac{1}{3}\boldsymbol{\sigma}^i\cdot\boldsymbol{\sigma}^j]V(r^{ij}),\quad(13)$$

where $\mathbf{r}^{ij}$ is the vector joining the positions of two nuclear particles, $\boldsymbol{\sigma}^i$ and $\boldsymbol{\sigma}^j$ the corresponding spin vectors.[4] The radial function $V(r^{ij})$ occurs in mesotron theory as a singular expression with leading term $\sim 1/r^3$; to avoid divergences, it must either be cut off at short distances or replaced by a less singular function of depth and range empirically determined. The procedure here followed will be to use for $V(r^{ij})$ a square well of radius $a=e^2/mc^2$ and of depth 25 Mev, which combination results in the observed quadrupole moment for the deuteron.

As has already been pointed out, T gives no shift to either the ${}^2P_{1/2}$ or the ${}^2P_{3/2}$ level in first order, and hence gives no splitting between them.[6] This results simply from the fact that T, in its dependence on rotations involving either the spin or space coordinates separately, behaves as D_2, a representation of second rank of the rotation group. The He^5 ground state has spin $=\frac{1}{2}$; it can have no diagonal matrix elements for T.

We have now to consider the second-order contribution to the splitting. The intermediate states will be taken to be solutions of the unperturbed Hamiltonian, that is, they have definite S and L. They must have components in their reduction with $J=\frac{1}{2}$ or $\frac{3}{2}$ and must also have odd parity. Because of the selection rule on the tensor force, they must have spin $\geqslant\frac{3}{2}$. This can only be obtained if the α-particle core is broken up; consequently the combining states must all possess large excitations, probably >15 Mev.

The question arises as to whether one can obtain the second-order splitting by calculating explicitly the contribution of a relatively small number of upper states possessing, presumably, the least excitation. To test this possibility, we made a calculation of the depression contributed

[6] S. M. Dancoff, Phys. Rev. **56**, 384 (1939).

by a given upper state, namely that formed by raising one of the protons out of the α-particle to an excited S orbit. The resulting configuration has components which have spin $\frac{3}{2}$ as well as total angular momentum $\frac{1}{2}$ and $\frac{3}{2}$, and odd parity. If one takes 15 Mev as the excitation of this level, one finds that it contributes the following amounts to the depressions of the two ground states:

$$\begin{aligned} &{}^2P_{1/2} - 23{,}000(a/\lambda)^{10} \text{ ev},\\ &{}^2P_{3/2} - \quad 1000(a/\lambda)^{10} \text{ ev}. \end{aligned}$$

In this calculation, the radial wave functions were taken of the form $e^{-r/\lambda}$ for the S orbits and $re^{-r/\lambda}$ for the P orbit. From the considerations of II, one sees that λ should be of the order of $4-5\times10^{-13}$ cm; a is about 2.8×10^{-13} cm. Consequently, setting $a/\lambda=1$ should provide a generous upper limit for the above depressions. Not only is the splitting extremely small, but it is in the opposite direction to that experimentally observed.

The following procedure should now serve as a check on whether the inclusion of a few excited states such as the above is sufficient for a good estimate of the second-order effects. The depression of either level may be written:

$$\sum_n \frac{(u_aTv_n)(v_nTu_a)}{E_0-E_n}, \tag{14}$$

where (u_aTv_n) is the matrix element of T between the ground state wave function u_a ($a=\frac{1}{2}$ or $\frac{3}{2}$) and the excited state wave function, v_n. An upper limit to this quantity is given by

$$\frac{1}{E_0-E_{\min}}\sum_n (u_aTv_n)(v_nTu_a),$$

where $E_{\min}$ is the energy of the lowest combining excited state. By closure, this expression becomes

$$(u_aT^2u_a)/(E_0-E_{\min}). \tag{15}$$

If it is true that only a few excited states are involved, this expression should also be quite small if a minimum energy denominator of $\sim$15 Mev is used; it should be not more than a few times the 23,000 ev found above.

The quantity T^2 should now be expressed in terms of its irreducible components. Such a reduction will yield: first, terms which transform like D_0 of space and spin separately; second, terms transforming like D_1 of space and spin separately; and last, terms representing higher rank transformation properties, which can be ignored because they have no diagonal matrix elements for the states u_a. The following expressions are obtained:

For terms in T^2 of the form $T^{ij}T^{ij}$:

$$\begin{aligned} &D_0\colon \tfrac{2}{3}(1+\tfrac{1}{3}\boldsymbol{\sigma}^i\cdot\boldsymbol{\sigma}^j)V^2(r^{ij}),\\ &D_1\colon \text{vanishes.} \end{aligned} \tag{16}$$

For terms in T^2 of the form $T^{ij}T^{jk}$ ($i\neq k$):

$$D_0\colon \tfrac{2}{3}\{\boldsymbol{\sigma}^i\cdot\boldsymbol{\sigma}^k\}\{(\hat{\mathbf{1}}^{ij}\cdot\hat{\mathbf{1}}^{jk})^2-\tfrac{1}{3}\}V(r^{ij})V(r^{jk}), \tag{17}$$

$$D_1\colon [\boldsymbol{\sigma}^i\times\boldsymbol{\sigma}^k]\cdot[\hat{\mathbf{1}}^{ij}\times\hat{\mathbf{1}}^{jk}](\hat{\mathbf{1}}^{ij}\cdot\hat{\mathbf{1}}^{jk})V(r^{ij})V(r^{jk}). \tag{18}$$

For terms in T^2 of the form $T^{ij}T^{kl}$ (no two indices alike): Terms of this sort can always be disregarded if one deals with a Hartree configuration in which one nuclear particle is in a P state, the other four in S states; one sees this by applying the selection rule for the tensor force to the integrals over the coordinates of the four particles.

An analysis of the integrals involved shows that the terms (17) and (18) have extremely small matrix elements for the ground states, giving a contribution to (15) of not over 10,000 ev for either state.[7] On the other hand, the terms

[7] The reason these terms are small is essentially the following. The transformation properties of $(\hat{\mathbf{1}}^{ij}\cdot\hat{\mathbf{1}}^{jk})(\hat{\mathbf{1}}^{ij}\times\hat{\mathbf{1}}^{jk})$ are $D_2(\hat{\mathbf{1}}^{ij})D_2(\hat{\mathbf{1}}^{jk})$. (The same is true of $(\hat{\mathbf{1}}^{ij}\cdot\hat{\mathbf{1}}^{jk})^2-\frac{1}{3}$.) This factor is to be integrated with a product of 3 initial and 3 final wave functions and with $V(r^{ij})V(r^{jk})$. We find that the product of 6 wave functions, if analyzed with regard to its transformation properties for rotations of $\hat{\mathbf{1}}^{ij}$ and $\hat{\mathbf{1}}^{jk}$, contains only a small amount of $D_2(\hat{\mathbf{1}}^{ij})D_2(\hat{\mathbf{1}}^{jk})$, resulting in approximate orthogonality.

Instead of the variables of integration $\mathbf{r}^i$, $\mathbf{r}^j$, $\mathbf{r}^k$, take $\mathbf{r}^{ij}$, $\mathbf{r}^{jk}$, $\mathbf{r}^j$. Functions of r^i (e.g.) that occur in the integral are expanded by means of the relation

$$\begin{aligned} r^i &= [(r^j)^2+(r^{ij})^2-2r^jr^{ij}\cos\gamma]^{\frac{1}{2}}\\ &\sim r^j-r^{ij}\cos\gamma+(r^{ij})^2/2r^j-(r^{ij})^2\cos^2\gamma/2r^j+\cdots. \end{aligned}$$

γ is the angle between $\mathbf{r}^j$ and $\mathbf{r}^{ij}$. Here we treat r^{ij} as smaller than r^j; this will be valid for most of the range of integration because of the fact that $V(r^{ij})$ vanishes for $r^{ij}>a=2.8\times10^{-13}$ cm, whereas the size of the wave functions is about $\lambda=4.5\times10^{-13}$ cm. If the wave functions are expanded for $r^{ij}<r^j$, the leading terms, namely those in $(\cos\gamma)^0$ and $\cos\gamma$ are orthogonal to $D_2(\hat{\mathbf{1}}^{ij})$; we must go to the terms in $\cos^2\gamma$, at the same time introducing a factor $(r^{ij})^2/(r^j)^2$ which results in an extra factor $(a/\lambda)^2$ in the result.

A similar expansion of the functions of r^k about $r^{jk}=0$ leads again to the elimination of two leading terms and to the introduction of a factor $(r^{jk})^2/(r^j)^2$.

Evaluating term (18) we find contributions on the order of $10^{-1}(a/\lambda)^{10}$ (Mev)^2. With an energy denominator of 15

(16) contribute in the neighborhood of 2 Mev to (15), this contribution being the same for both states. A comparison of this result with the upper limit of 23 kev found for the depression of the $\frac{1}{2}$ level due to a single excited state is indication that an extremely large number of states extending to high energies of excitation is important in the sum (14).

Equation (14) may always be rewritten, using an average energy denominator, in the form

$$(u_aT^2u_a)/(E_0-\bar{E}_a). \tag{19}$$

As was shown above, the factor $(u_aT^2u_a)$ can differ for the two states by not more than 10 kev, which is an upper limit for the contribution of the D_1 component of T^2. Therefore if the splitting of the two states by the tensor force is actually of the order of magnitude of the experimental result, it must be due almost entirely to the variation in $\bar{E}_a$ for the two states. In the following, to simplify the argument, we make the assumption that $(u_aT^2u_a)$ is the same for $a=\frac{1}{2}, \frac{3}{2}$.

By writing the energy denominator in (14) in terms of a partial expansion about its mean value, one obtains the identity:

$$\frac{1}{E_0-E_n}\equiv\frac{1}{E_0-\bar{E}_a}+\frac{E_n-\bar{E}_a}{(E_0-\bar{E}_a)^2}+\frac{(E_n-\bar{E}_a)^2}{(E_0-\bar{E}_a)^2(E_0-E_n)}. \tag{20}$$

The contribution of the second and third terms to (14) vanishes, since this was the criterion for the choice of $\bar{E}_a$; the splitting of the two states is due to the variation of $\bar{E}_a$.

But now let us make the expansion (20) not about $\bar{E}_a$, but about $\bar{E}$, chosen *the same* for the two states. The first term in the expansion (20) now gives an equal contribution to (14) for the states u_a. The splitting has become buried in the second and third terms. We will calculate the splitting due to the second term, making the assumption that the third term may be made to contribute a vanishing splitting by an appropriate choice of $\bar{E}$. The possibility of such a procedure rests on the fact that the splitting due to the second term does not depend critically on the choice of $\bar{E}$; in particular it cannot change its sign as a function of $\bar{E}$. On the other hand, the splitting due to the third term does depend critically on this choice. It may be written

$$\int_{E_0+15}^{\infty} dE_n\frac{(E_n-\bar{E})^2}{(E_0-E_n)}G(E_n), \tag{21}$$

where the function $G(E_n)$ has zero average value. The factor $(E_n-\bar{E})^2/(E_0-E_n)$ is always negative and has a zero point at $E_n=\bar{E}$ which may in general be adjusted so as to reduce the value of the integral as much as desired. Our ignorance of the precise form of $G(E_n)$, that is to say of the spectra of $|(u_aT^2v_n)|^2$, prevents the exact specification of this required value of $\bar{E}$. If the two spectra were very similar, but somewhat displaced, $\bar{E}$ would lie between $\bar{E}_{1/2}$ and $\bar{E}_{3/2}$. If one considers other forms which $G(E_n)$ might reasonably have, one finds values of $\bar{E}$ not much removed from the above. At any rate, the arguments that follow are essentially unaffected by the value of $\bar{E}$, within wide limits; it is only necessary that it can in principle be found.

One has now to calculate the splitting due to the expression

$$\sum_n\frac{(u_aTv_n)(v_nTu_a)}{(E_0-\bar{E})^2}(E_n-\bar{E}). \tag{22}$$

$\bar{E}$ may be dropped as not contributing to splitting. The remainder is

$$\sum_n\frac{(u_aTH_0v_n)(v_nTu_a)}{(E_0-\bar{E})^2}=\frac{(u_aTH_0Tu_a)}{(E_0-\bar{E})^2}$$

by closure, where H_0 is the unperturbed Hamiltonian. Proceeding as before, we look for the part of TH_0T which transforms like D_1. We permit H_0 to include the kinetic energies plus a sum of potential terms which may represent ordinary forces, spin exchange, space exchange, and complete exchange forces. If we examine these potential terms we find that they do indeed lead to D_1 components for TH_0T, but only by combining $D_2(\mathbf{\hat{1}}^{ij})$ with $D_2(\mathbf{\hat{1}}^{jk})$, in other words forming structures of the form $(\mathbf{\hat{1}}^{ij}\cdot\mathbf{\hat{1}}^{jk})(\mathbf{\hat{1}}^{ij}\times\mathbf{\hat{1}}^{jk})$; it is immediately clear that no other space vectors could result from such combinations. The contribution of these terms is again very

Mev, this results in a depression (15) of less than $10^{-2}(a/\lambda)^{10}$ Mev. With the estimated values of the constants $a/\lambda=0.62$, and the result is reduced to less than 10^{-4} Mev. The upper limit of 10^{-2} Mev should be quite safe in this case.

small, for the same reason that the contribution of (18) is small.[7]

On the other hand, the kinetic energy terms in H_0 lead to space vectors of lower rank symmetry properties. The D_1 component of

$$T^{ij}\sum_{\mu}[-(\hbar^2/2M)\nabla_\mu{}^2]T^{ij}$$

is found to be

$$-\frac{i\hbar^2 V^2(r^{ij})}{Mr^{ij}}(\boldsymbol{\sigma}^i+\boldsymbol{\sigma}^j)\cdot[\mathbf{r}^{ij}\times(\boldsymbol{\nabla}^i-\boldsymbol{\nabla}^j)], \quad (23)$$

which can be written

$$\frac{\hbar^2 V^2(r^{ij})}{M(r^{ij})^2}(\boldsymbol{\sigma}^i+\boldsymbol{\sigma}^j)\cdot\mathbf{L}^{ij}, \quad (24)$$

where $\hbar\mathbf{L}^{ij}$ is the relative angular momentum of the particles i and j. The above "diagonal" terms in $T\sum_\mu[-(\hbar^2/2M)\nabla_\mu{}^2]T$ prove to be more important than the "nondiagonal" terms—those that involve different T^{ij}'s on left and right; the latter have D_1 components of higher symmetry and give a contribution of the order of the terms (18).

Consequently the leading terms in the splitting will be given by the difference in the matrix elements of

$$\frac{\hbar^2}{M(E_0-\bar{E})^2}\sum_{\text{pairs}}\frac{V^2(r^{ij})}{(r^{ij})^2}(\boldsymbol{\sigma}^i+\boldsymbol{\sigma}^j)\cdot\mathbf{L}^{ij}, \quad (25)$$

where $E_0-\bar{E}$ has a value which is not less than 15 Mev and probably not greater than 40 or 50 Mev; we will take 30 Mev as an estimate which should give the right order of magnitude.

The interaction (25), in contrast to the Thomas interaction, (9), is a "normal" spin-orbit force; that is to say, one would expect it to give rise to a splitting in which the state of higher multiplicity lies higher, because for such a state the spin of a particular particle and its angular momentum relative to any other particle would have the maximum tendency to be aligned. The calculation verifies this conclusion and gives for the splitting $80(a/\lambda)^5$ kev, where as before a is the range of the tensor force and λ the scale of the nuclear wave functions; the state $J=\frac{1}{2}$ is found to lie lower.

It is clear from the form of (25) that the magnitude of the result will be extremely sensitive to the form of $V(r^{ij})$. The radial integral in which V occurs is of the form $\int\rho^2 d\rho V^2(\rho)$ so that the $1/\rho^3$ dependence of mesotron theory would give an extremely bad $1/\rho^4$ divergence. If one uses instead of the square well potential the cut off potential proposed by Bethe,[8] and fitted by him to the two levels of the deuteron, one obtains a much larger result. Using the neutral mesotron theory with zero cut-off inside the radius $x_0=0.318a$, one obtains $800(a/\lambda)^5$ kev for the splitting of the two levels, the order being of course as before. It is clear, then, that splitting of the order of magnitude of the experimental result could be caused by the tensor force.

The terms which were discarded in comparison with (25), and whose upper limit was said to be 10 kev, (e.g. (17), (18)) prove to be much less affected by the shape of V. The radial integrals of interest are of the form $\int\rho^4 d\rho V(\rho)$; they would not diverge if a $1/\rho^3$ potential was inserted. If Bethe's cut off potential is used, the upper limit previously assigned is practically unaffected.

As a result of this section it is indicated that the tensor force gives rise to a splitting in the opposite direction to that experimentally observed and of a magnitude which is extremely sensitive to the shape of the force but which is probably of the order of 100 kev. The direction of the splitting is of course independent of any ambiguity in the sign of the tensor force in the Hamiltonian, since in second order it occurs quadratically. The importance of higher order contributions is difficult to evaluate since it will depend to a great extent on the degree of "peakedness" assigned to the potential function. Because of this, as well as the considerable experimental uncertainty, the conclusion which may be drawn must be regarded as tentative. It is that the structure of the He^5 doublet is not determined by the tensor force, and can be assigned to Thomas relativistic coupling only with an adjustment of multiplicative constant such as described in II.*

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[8] H. A. Bethe, Phys. Rev. **57**, 390 (1940).
* See "Note Added in Proof" above.