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Fine Structure in the Directional Intensity of Cosmic Rays

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The effect relating to fine structure in the zenith-angle distribution of cosmic-ray intensity was investigated with a triple coincidence circuit of Geiger-Müller counters. Three counters, each of diameter 8.8 cm, active length 36 cm, were made according to the method of Shonka. The counters were used with a separation of 50 cm between the centers of adjacent counters in a modified circuit of the Neher-Harper type. Readings were taken in a cyclic fashion to minimize errors due to possible instrumental sensitivity drifts, changes in the barometric pressure, and changes in the magnetic field of the earth. Five-degree intervals were investigated in the zenith-angle range from 0° to 45°, inclusive, in all the directions studied. The survey of intensities in the east, west, north, south, southwest, southeast, northwest, and northeast azimuths exhibits intensity patterns with small oscillations. The prominences measured in terms of the largest positive deviations from the $\cos^2\theta$-curve are of the order of two or three percent of the intensity, and they occur at approximately 7°, 20°, and 37°. The directional intensity pattern shows an approximate symmetry about the zenith. These results tend to confirm the predictions of Schremp relative to the existence of a fine structure due to a line or banded nature for the energy spectrum for cosmic rays at infinity.

INTRODUCTION

SCHREMP[1,2] developed the theory of fine structure in the directional intensity of cosmic rays, i.e., prominences and depressions in the intensity curve plotted as a function of the zenith angle. He inferred that a fine structure might arise from: (1) magnetic effects associated with the edges of the "main" cones and the "penumbral" bands of the Lemaître-Vallarta[3,4] theory, which would give rise to patterns of asymmetrical loci of intensity prominences about the zenith; or, (2) absorption effects associated with the traversal of range limits of lines or bands in the energy spectrum for cosmic rays at infinity, which would provide a symmetrical pattern. The data from Johnson's[5] world-wide asymmetry measurements supplied evidence that a fine structure was present, despite the fact that the experimental conditions and methods would tend to mask the fine structure. In 1939, Ribner[6] and Cooper,[7] independently, reported results on fine-structure measurements that demonstrated the existence of the effect. Ribner's observations in the east and west azimuths showed positional symmetry of the fine structure. This experiment, which has been performed at Columbia, Missouri, supplies further information in the east and west azimuths and extends the observations to other azimuths.

APPARATUS

Three Geiger-Müller counter tubes were arranged for triple coincidence counting. Each counter had a copper cylindrical cathode 8.8 cm in diameter, and a tungsten central wire 0.005 in. in diameter. These parts were sealed in glass, and the counter was cleaned, baked, and filled with hydrogen according to the methods of Shonka.[8] The characteristics of the counters are shown in Fig. 1, where the counting rate for each counter is plotted as a function of the applied voltage. The counters were operated at 1520 volts, which is well within the plateau range for each counter. The counters exhibited "clean" pulses, and tests with gamma-rays showed them to be highly efficient.

A diagram of the circuit is shown in Fig. 2. The first stage is of the Neher-Harper[9] type. The second stage lengthens and changes the sign of the pulse. Coincidences operate the thyratron.

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[1] E. J. Schremp, Phys. Rev. **53**, 915A (1938); **54**, 157 (1938).

[2] E. J. Schremp and H. S. Ribner, Rev. Mod. Phys. **11**, 149 (1939).

[3] G. Lemaître and M. S. Vallarta, Phys. Rev. **49**, 719 (1936).

[4] R. A. Hutner, Phys. Rev. **55**, 15 (1939); **55**, 614 (1939).

[5] T. H. Johnson, Phys. Rev. **45**, 569 (1934); **48**, 287 (1935).

[6] H. S. Ribner, Phys. Rev. **55**, 127 (1939); **56**, 1069 (1939).

[7] D. Cooper, Phys. Rev. **55**, 1272 (1939).

[8] J. B. Hoag, *Electron and Nuclear Physics* (D. Van Nostrand Company, New York, 1938), p. 431.

[9] H. V. Neher and W. W. Harper, Phys. Rev. **49**, 940 (1936).

The final stage including the strobotron tube[10] was added as a Western Electric 5M message register will operate in the plate circuit of this tube. Small Edison type syd batteries supplied the high voltage, which was kept constant within 4 volts. The 250-volt plate voltages were supplied by two RCA type TMV-118-B power supply units. The various grid biases were obtained from battery units.

The counter train was mounted in a vertical plane on a large semicircular track, which was graduated in five-degree intervals on the track. The angular adjustments could be made to approximately three minutes of arc. The counters were separated 50 cm between adjacent counters. Under these conditions, the maximum angle of divergence of the cosmic-ray beam in the plane of rotation of the counter train was 10°, and the corresponding angle perpendicular to the plane of rotation of the counter train was 40°. However, the actual angular resolution is much better than the dimensions indicate. It can be shown from the geometry of the set-up that for a maximum angle of divergence of 10° in the plane of rotation of the counter train, more than one-half of the rays are incident through the angle within 1.4° on each side of the zenith angle. More than one-half of the rays are incident through an angle of 5.8° on each side of the plane of rotation. The counting rate was approximately 6 counts per minute with this separation of the counters. Eckart and Shonka's[11] formula gave a value of 0.00006 "accidental" coincidence per second.

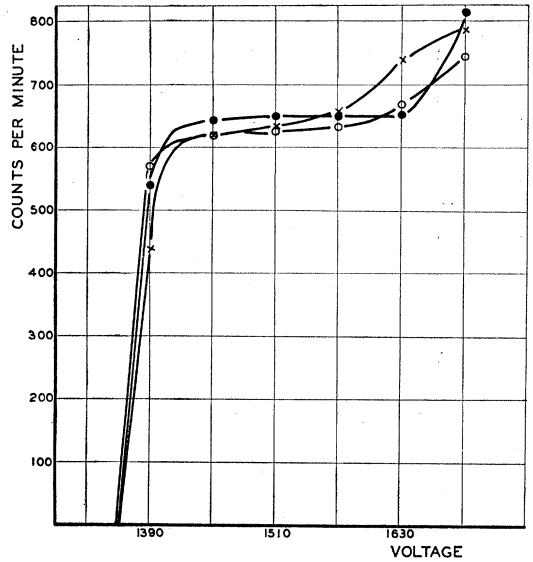
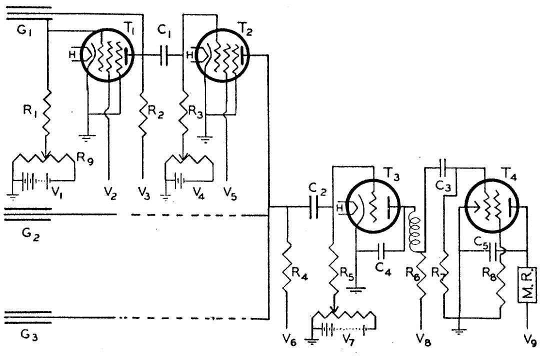


FIG. 1. Characteristic curves for the Geiger-Müller counters with the counting rate plotted as a function of the anode voltage.

OBSERVATIONS

The observations were made in the attic of the Physics Building at an altitude of 760 ft. above sea level and a geomagnetic latitude[12] of 49° N. Only a thin, uniform slate roof shielded the apparatus from the sky. Readings were taken at every five degrees of zenith angle from 0° to 45°, inclusive, in each azimuth that was investigated. It was necessary to use the "method of cycles" in collecting the data to minimize the effect of: (1) changes in the barometric conditions, especially changes in barometric pressure; (2) changes in the intensity of the magnetic field of the earth; and, possibly, (3) slight sensitivity drifts in the triple coincidence apparatus. The changes in the intensity arising from changes in the barometric pressure and changes in the magnetic field of the earth are not small



$G_1=G_2=G_3$—Geiger counters
$T_1=T_2$—Type 57
$T_3=885$ Thyratron
$T_4=$Strobotron
$V_1=10$ volts
$V_2=45$ volts
$V_3=1520$ volts
$V_4=2$ volts
$V_5=90$ volts
$V_7=-90$ volts
$V_6=V_8=V_9=250$ volts
$H=2.5$ volts—a.c.

$R_1=10^7$ ohms
$R_2=2\times10^6$ ohms
$R_3=5\times10^5$ ohms
$R_4=R_7=R_8=10^5$ ohms
$R_5=5\times10^4$ ohms
$R_6=10^3$ ohms
$R_9=2\times10^5$ ohms
$C_1=50\mu\mu f$
$C_2=0.02\mu f$
$C_3=0.1\mu f$
$C_4=0.5\mu f$
$C_5=6\mu f$

FIG. 2. Modified triple coincidence circuit with the Neher-Harper first stage.

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[10] N. S. Gingrich, Rev. Sci. Inst. **7**, 207 (1936).

[11] C. Eckart and F. R. Shonka, Phys. Rev. **53**, 752 (1938).

[12] A. H. Compton, Phys. Rev. **43**, 387 (1933).

TABLE I. *The analysis of the data. Cycle No. 48—North (15 min. of observation at each angle).*

Zenith Angle (θ)	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°
$N_{48}(\theta)$	6.20	6.40	5.93	6.07	6.20	4.33	4.80	3.93	2.60	2.80
$N_{48}(\theta)\sec^2\theta$	6.2	6.5	6.1	6.5	7.0	5.3	6.4	5.8	4.4	5.6
							· · ·		$N_{48}(0)=6.0$	
$I_{48}(\theta)$	1.03	1.07	0.99	1.01	1.03	0.72	0.80	0.66	0.43	0.47

$N_{48}(\theta)$ is the counts per minute at the angle θ.
$N_{48}(0)$ is the average value of $N_{48}(\theta)\sec^2\theta$.
$I_{48}(\theta)$ is $N_{48}(\theta)/N_{48}(0)$.

$I(\theta)=\frac{1}{n}\sum_{i=1}^{n} I_i(\theta)$, where $I(\theta)$ is the normalized intensity in a particular azimuth, and n is the number of cycles in this azimuth.

$\Delta(\theta)=I(\theta)-\cos^2\theta$, where $\Delta(\theta)$ is the deviation from the $\cos^2\theta$ curve.

(For convenience in comparison, $\Delta(\theta)$ has been shifted to zero in each azimuth.)

factors; in percentage change of the intensity, they compare in magnitude[12, 13] with the fine structure prominences. By completing in a relatively short time a cycle of the angles in the azimuth under study, these factors which are beyond experimental control remained essentially constant during the cycle. Theoretically, the error arising from these sources may be reduced to any desired value by increasing the number of cycles. By treating each cycle as a separate entity, only those sensitivity changes in the coincidence apparatus that occurred during the cycles affected the final result. The method of analyzing the data is illustrated in Table I. Since the intensity is approximately proportional to $\cos^2\theta$, where θ is the zenith angle, $N(\theta)\sec^2\theta$ has nearly a constant value throughout the cycle and the average value of this quantity for the cycle has been used to normalize the intensity curve for that cycle. It is more convenient to

TABLE II. *Directional intensity data. Zenith angle, θ; deviations from $\cos^2\theta$, $\Delta(\theta)$; probable error, ϵ.*

	EAST AZIMUTH[1]		WEST AZIMUTH[2]		NORTH AZIMUTH[3]		SOUTH AZIMUTH[4]					
θ	$\Delta(\theta)$	ϵ	$\Delta(\theta)$	ϵ	$\Delta(\theta)$	ϵ	$\Delta(\theta)$ FOR 68 CYCLES	ϵ	$\Delta(\theta)$ 60 REGULAR CYCLES	ϵ	$\Delta(\theta)$ 8 ANOMALOUS CYCLES	ϵ
0	0	0.008	0	0.010	0	0.009	0	0.009	0	0.009	0	0.025
5	+0.009	0.008	+0.020	0.011	+0.031	0.008	+0.015	0.008	+0.019	0.008	−0.017	0.024
10	+0.002	0.008	+0.034	0.011	+0.005	0.009	+0.007	0.009	+0.012	0.009	−0.034	0.024
15	−0.046	0.007	−0.008	0.009	−0.021	0.009	−0.007	0.008	−0.016	0.009	+0.056	0.024
20	−0.003	0.008	+0.006	0.012	−0.007	0.007	−0.003	0.009	+0.008	0.009	−0.088	0.024
25	−0.053	0.008	−0.006	0.010	−0.061	0.009	−0.015	0.008	−0.014	0.009	−0.016	0.023
30	−0.038	0.007	−0.036	0.009	−0.034	0.007	−0.015	0.008	−0.026	0.009	+0.017	0.022
35	+0.019	0.009	−0.025	0.014	−0.038	0.008	−0.014	0.007	−0.012	0.008	−0.038	0.021
40	−0.032	0.008	−0.016	0.010	−0.060	0.006	−0.028	0.006	−0.024	0.006	−0.008	0.020
45	−0.025	0.008	−0.023	0.011	−0.057	0.006	−0.029	0.006	−0.031	0.006	−0.011	0.018

	SOUTHWEST AZIMUTH[5]		SOUTHEAST AZIMUTH[6]		NORTHWEST AZIMUTH[7]		NORTHEAST AZIMUTH[8]	
θ	$\Delta(\theta)$	ϵ	$\Delta(\theta)$	ϵ	$\Delta(\theta)$	ϵ	$\Delta(\theta)$	ϵ
0	0	0.014	0	0.011	0	0.011	0	0.009
5	+0.041	0.012	+0.037	0.009	+0.001	0.011	+0.004	0.011
10	+0.040	0.015	+0.031	0.012	+0.034	0.011	+0.049	0.014
15	−0.037	0.012	−0.027	0.009	−0.002	0.010	−0.027	0.010
20	+0.015	0.011	−0.003	0.009	+0.009	0.011	−0.028	0.009
25	−0.034	0.012	−0.017	0.009	−0.044	0.010	+0.004	0.008
30	−0.023	0.013	−0.066	0.009	−0.025	0.009	−0.055	0.008
35	−0.022	0.011	−0.048	0.009	−0.036	0.009	−0.044	0.009
40	−0.021	0.011	−0.042	0.008	−0.039	0.011	−0.047	0.009
45	−0.033	0.011	−0.043	0.009	−0.049	0.007	−0.070	0.009

[1] 40 cycles from 0° to 45°, plus 20 cycles from 0° to 30°; 15-min. observations at each angle during each cycle.
[2] 20 cycles from 0° to 45°, plus 20 cycles from 0° to 30°; 15-min. observations at each angle during each cycle.
[3] 60 cycles from 0° to 45°; 15-min. observations.
[4] 68 cycles from 0° to 45°; 15-min. observations; 8 cycles run in 2 consecutive days in August showed a shift of the fine structure pattern. These 8 cycles are listed separately.
[5] 30 cycles from 0° to 45°; 10-min. observations.
[6] 30 cycles from 0° to 45° with 10-min. observations, plus 20 cycles from 0° to 45° with 15-min. observations; results weighted accordingly.
[7] 40 cycles from 0° to 45°; 15-min. observations.
[8] 40 cycles from 0° to 45°; 15-min. observations.

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[13] S. E. Forbush, Phys. Rev. **54**, 975 (1938).

plot the deviations $\Delta(\theta)$ from the empirical $\cos^2$-curve than the zenith-angle curve. This representation flattens out the distribution curve, and shows any prominences and depressions as oscillations about a perfect $\cos^2$-distribution.

The results of the survey of the directional intensities in eight azimuths are tabulated in Table II, and $\Delta(\theta)$ is plotted against θ for the various azimuths in Fig. 3. The azimuths were investigated in the order in which they are listed in the table. The observations were made during the time from May 15, 1939, to January 15, 1940.

The probable errors were computed by the "method of residuals" from the value of $I(\theta)$ by the relation

$$\epsilon=0.67\left[\sum_{i=1}^{n}\frac{r_i^2}{n(n-1)}\right]^{\frac{1}{2}},$$

where n is the number of cycles, r_i is the ith residual, and ϵ is the probable error of the average value. The probable errors for the results of eight anomalous cycles in the south (shown in Fig. 3) were computed from the total counts by the formula

$$\epsilon=0.67\sqrt{C/N(0)T},$$

where C is the total number of counts, and T is the time of observations.

Discussion of Results

The results of Ribner's[6] independent survey of intensities in the east and west azimuths are shown in Fig. 4. A comparison of the two surveys shows qualitative agreement and reasonably good quantitative agreement.

In the present work, 68 cycles were taken in the south. Sixty of these cycles exhibited a fine structure quite comparable in magnitude and position with the fine structure in the north (as is shown in Fig. 3). However, eight anomalous cycles, taken during two consecutive days in August, gave a complete inversion of the fine-structure pattern. Short period shifts of this type were observed by Ribner; and Schremp[14] has recently reported studies of the time variation of the directional intensity pattern. These shifts have been attributed to changes in the barometric or the magnetic conditions. It is interesting to note that the eight anomalous cycles were made during a period of activity of the aurora borealis. Similar shifts of shorter duration are believed to be present in the data. Because of the time shifts there is some question whether definite magnitudes can be ascribed to the prominences, even with a large number of counts.

The directional intensity curves of Fig. 3 show approximate symmetry of the fine structure

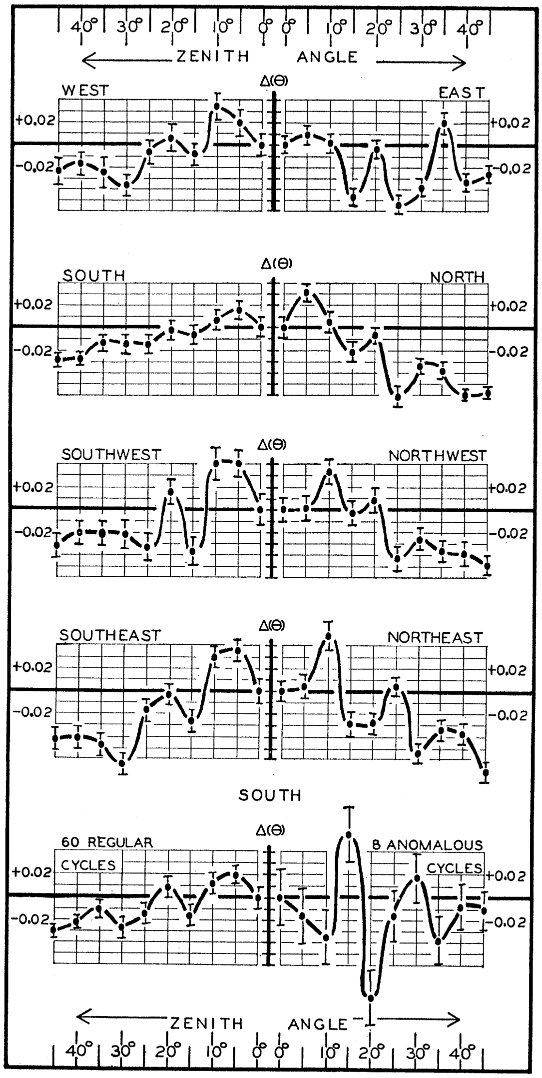

Fig. 3. Directional intensity patterns for eight azimuths. The deviations $\Delta(\theta)$ of the normalized directional intensity from the empirical $\cos^2$-curve are plotted as a function of the zenith angle θ, in degrees. The first eight graphs show the approximate positional symmetry of the fine structure in the various azimuths. The observed shift of the directional intensity pattern is shown in the last two graphs.

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[14] E. J. Schremp, Phys. Rev. 57, 1061A (1940).

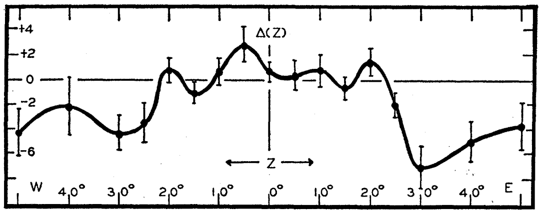

FIG. 4. Ribner's directional intensity results in the east and west azimuths. z is the zenith angle in degrees. $\Delta(z)$ is the absolute deviation, in percent of the zenith angle value, of the observed intensity from that given by the $\cos^2 z$ relation.

about the zenith with intensity prominences in the zenith-angle range from 5° to 10°, near 20°, and in the range from 35° to 40°. These results tend to confirm the predictions of Schremp relative to the existence of a fine structure due to a line or banded nature of the energy spectrum for cosmic rays at infinity. That the fine structure may not arise partly from magnetic effects is not entirely precluded, as the penumbral bands of the Lemaître-Vallarta theory form a sort of "C" and might give rise to a perturbation of the fine structure due to absorption without entirely destroying the approximate symmetry of the pattern. But as Columbia, Missouri is near the "knee" of the geomagnetic latitude intensity curve and the pattern shows an approximate symmetry about the zenith, the most plausible explanation in terms of primary cosmic rays seems to be that there are lines or bands in the energy spectrum at infinity. Then, it is necessary that these lines or bands be successively absorbed out by range absorption by increasing the atmospheric path traversed by the primary radiation. This explanation of the fine structure in terms of absorptive effects is not in discord with the present results, Ribner's results, and the deductions from Johnson's data. Absorption experiments are in progress to test this interpretation of the experimental results.

The author wishes to express his deepest appreciation to Professor N. S. Gingrich whose keen advice, earnest cooperation, and inspiring stimulation were invaluable throughout the course of this investigation. He also wishes to express his gratitude to Professor E. J. Schremp for many invaluable suggestions and constructive criticisms.

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Multiple Scattering of Fast Electrons and Alpha-Particles, and "Curvature" of Cloud Tracks Due to Scattering

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According to a number of observers the multiple scattering of fast electrons by thin foils is appreciably less than the scattering predicted by the theory of this effect given some time ago by the present author. The statistical part of the theory, which in the earlier work was developed with special regard to the scattering of cosmic-ray particles, has been reconsidered more closely for the conditions of the experiments on fast electrons. The results confirm within a few percent the scattering given by the earlier general formula. The discrepancies must accordingly be attributed to experimental error or to a failure of the basic collision theory. A new formula is given which represents the mean projected deflection within 1 percent over the whole range of experimental conditions. The *distribution* of the projected deflections is considered more closely than in the earlier paper, and a general expression for the most probable deflection in space is also given. The theory is extended to the multiple scattering of α-particles, the quantum-mechanical collision theory in this case taking the form of classical mechanics. The results are in satisfactory agreement with the early experiments of Geiger and of Mayer. A discussion is given of recent papers on the subject. Reasons are given for the nonoperation of the interference effect considered by Wheeler, and which according to him appreciably reduces the scattering. The numerical results of a new treatment of multiple scattering by Goudsmit and Saunderson are shown to be the same as those required by the general formula given by the writer's theory. Other points raised by these authors are also discussed. A summary of the theoretical results is given in Section (7).

IN a recent publication[1] (1939) the writer discussed the problem of the multiple scattering of cosmic-ray particles and fast electrons. The theoretical value of the mean deflection of cosmic-ray particles by metal plates was shown

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[1] E. J. Williams, Proc. Roy. Soc. **169**, 531 (1939).