The Scattering of 50-Kilovolt Electrons by Aluminum

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Apparatus is described for scattering electrons with thin foils and measuring the intensity of scattering in a narrow pencil making any angle from 30° to 120° with the direction of incidence. Electrons with energies below 95 percent of the energy on incidence are excluded by a retarding voltage. The theory of nearly single scattering is discussed, and a general expression is found for the error introduced by slight plural scattering. In the case of Rutherford scattering this expression gives for the fractional error the value $\epsilon^2\{\csc^2(\theta/2)-\frac{1}{2}\}$ where θ is the angle at which scattering is measured and ϵ^2 is the mean square deflection of particles traversing in the foil the same length of path as those scattered at the angle θ. The relation of this result to Wentzel's criterion is shown. Electrons accelerated by 50 kv were scattered by thin composite foils of Al on collodion. A difference was found, though not explained, between the intensities of scattering at 90° on the two sides of the foil, which was inclined at 45° to the direction of incidence. With allowance made for this asymmetry in comparing the scattering at angles above and below 90°, relative intensities at angles from 30° to 120° were within 5 percent of those predicted by Mott. Comparison of the observed and predicted absolute intensities of the scattering from Al was subject to an uncertainty of about 20 percent, principally in the determination of the thickness of the Al film. Within this uncertainty, the observation agreed with the prediction. If the actual scattering differs from the predicted, it appears from these observations to be more probably less than greater.

INTRODUCTION

If a beam of electrons of speed v is incident normally on a thin foil of thickness t, having, per unit volume, n atoms of atomic number Z, then the fraction scattered into a small solid angle Ω around a direction making an angle θ with the direction of incidence may conveniently be written

$$i=nt\Omega(Ze^2/2mv^2)^2(1-\beta^2)\csc^4(\theta/2)R. \quad (1)$$

For the factor R, Mott[1] has calculated from the Dirac equations, assuming a Coulombian field, the expression

$$R=1-\beta^2\sin^2(\theta/2)+(Z\beta\pi/137)\cos^2(\theta/2)\sin(\theta/2)$$

+terms of the second and higher powers in $Z/137$. (2)

The results of several experiments have indicated exceptions to this equation and to the closely related prediction of an asymmetry in double scattering, and it has been suggested that they show an actual invalidity of the Dirac equations in a range of electron speeds where it is hardly to be expected on other evidence.[2] On the other hand, the experiments offer difficulties in technique, and not all the experimental results are in mutual agreement. It seems desirable therefore to have more observation on the subject.

In the present experiment tests were made of Eq. (2) with 50-kv electrons scattered at angles between 30° and 120° by thin composite foils of Al evaporated on collodion.

APPARATUS AND EXPERIMENTAL PROCEDURE

The chamber in which the electrons were scattered is shown in Fig. 1(a) in vertical section and in Fig. 1(b) in horizontal section. Its vertical wall is a short section of 1-ft. steel pipe, supported for insulation on a glass cylinder of the same diameter and 2 feet high. The beam of electrons enters the chamber, as shown by the arrow, through a sylphon bellows and ground joint from the accelerating tube (not shown). This beam, already rendered parallel by the distribution of voltage among the electrodes of the accelerating tube, is defined by the diaphragm D_1. This diaphragm is at the end of a steel tube set in a bracket fastened to the top of the scattering chamber. The bracket is flanked by blocks of lead to absorb x-rays. The electrons passing the diaphragm strike the foil, which is mounted on one of two diaphragms carried by a cylindrical rod projecting through the top of the scattering

[1] N. F. Mott, Proc. Roy. Soc. **A124**, 425 (1929).
[2] A general discussion of theory and experiment is given in an article by M. E. Rose, Phys. Rev. **57**, 280 (1940).

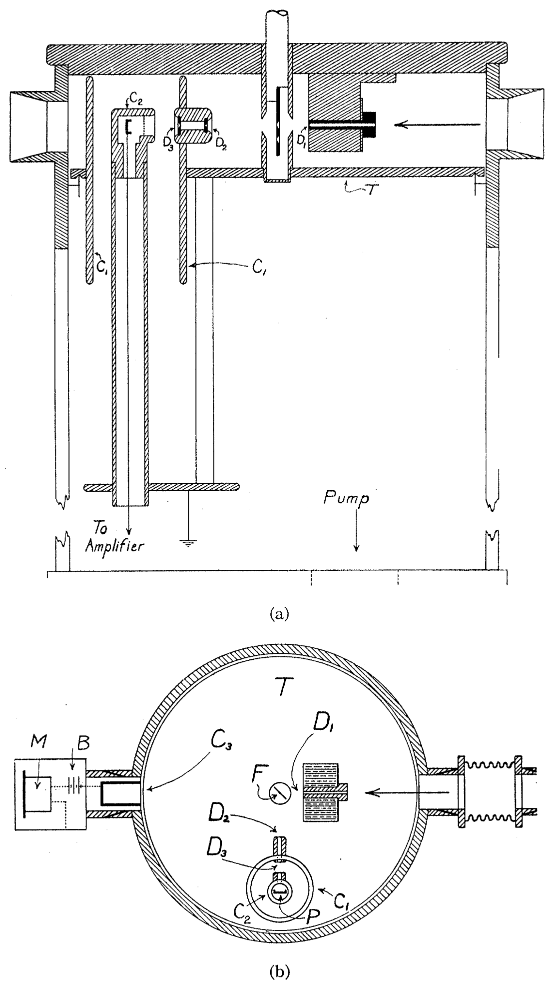

FIG. 1 (a) vertical section and (b) horizontal section of scattering chamber. The wall of the chamber is 1-ft. steel pipe. The support is a glass cylinder. Arrow shows direction of incident beam. T, turning floor of chamber. D_1, diaphragm defining incident beam. D_2, diaphragm defining scattered beam. D_3, larger diaphragm behind D_2. C_1, brass cylinder enclosing region of retarding voltage. C_2, collecting chamber for scattered beam. C_3, collecting chamber for incident beam. B, collecting voltage for primary beam. M, microammeter for measuring incident current. F, scattering foil. P, collecting electrode for scattered beam.

chamber. The rod is attached to a mechanism (not shown) housed in sylphon bellows above the top of the scattering chamber and evacuated with it. This mechanism permits placing either of the two diaphragms in the electron beam and also setting their plane for incidence at 45° to either side of normal, all by manipulation outside the vacuum.

The floor of the scattering chamber is a steel disk with a circular groove on the under side near the rim. This groove rides on three small wheels set on axles 120° apart on the inner wall of the scattering chamber. Two of the wheels are idlers; the third can be driven from outside the vacuum by a crank housed in a flexible metal tube. In this way the floor can be rotated about a vertical axis through the center of the scattering foil.

A brass pipe C_1 projects through the floor from below almost to the top of the chamber. In the wall of this pipe is a copper cylinder carrying two steel diaphragms D_2 and D_3. Inside C_1 but insulated from it by three supporting Pyrex glass rods is the copper chamber C_2 for the collection of the beam of scattered electrons.

The potentials of the parts of the apparatus are shown schematically in Fig. 2. The chamber C_2 was at ground potential. The pipe C_1 and all the parts of the scattering chamber outside it were at a potential V which was the highest positive potential of the apparatus. It was maintained by a transformer and electron-tube rectifier with a large condenser to eliminate ripple. The voltage was found by measuring with a microammeter the current through a calibrated high resistance.

The filament was maintained below ground potential by means of a power pack producing a voltage V_R which was a small fraction of the total voltage used to accelerate the electrons. The electrons were thus incident on the foil with an energy corresponding to the total voltage $V+V_R$. The scattered beam was defined in field-free space at high potential by the diaphragm D_2 (which was considerably smaller than D_3). Passing through D_3 the electrons were retarded by the voltage V, applied between D_3 and a grid of fine tungsten wire in the opening of C_2. Hence only the energy corresponding to the voltage V_R could be retained by any electron passing through the grid. For this reason V_R may be called the residual voltage. Any electron which lost in being scattered an energy as great as that corresponding to this residual voltage was repelled before reaching the grid.

Not only these electrons were excluded by the retarding voltage V but also secondary electrons ejected from the foil by the primary beam and any that may have been ejected by x-rays from any part of the apparatus except the interior of C_2.

Electrons passing the grid of C_2 were collected on the plate P. This was maintained at a positive potential of 270 volts with respect to the grid to prevent the escape of electrons by recoil or the escape of secondary electrons ejected from P. The current to P passed thence to an amplifier of the Barth type, described by Penick[3] and the amplified current was measured by a shunted Leeds and Northrup Type R galvanometer.

The current incident on the foil was commonly about 10 μa. With the precautions taken against slow electrons and x-rays it was possible to measure scattered currents as small as 10^{-6} μa. This allowed the diaphragm D_2 to be made small enough to give a rather high angular resolution. It subtended a plane angle of about 3° at the foil, its solid parallax, Ω in Eq. (1), being 0.00223 rad.[2]

In measuring the fraction of the incident electrons scattered at any angle through the diaphragm D_2, readings of the incident and scattered currents were alternated. To measure the incident current, the foil was removed from the incident beam by the mechanism above the scattering chamber, which was operated from a distance by gas pressure, without impairing the insulation of the scattering chamber or jarring the apparatus. The incident current then passed directly to the chamber C_3 and was measured by the microammeter M, which could be read from a distance. When the diaphragm carrying the foil was removed from the incident beam it was replaced by an open diaphragm carefully made to be as nearly as possible identical with it. This diaphragm was used for "blank" readings to detect any current to C_2 other than that of electrons scattered by the foil. Whatever such current there was, it was too small to measure.

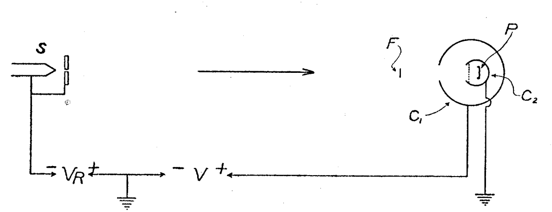

FIG. 2. Schematic diagram of apparatus showing potentials of different parts. S, source of electrons, filament of accelerating tube. F, scattering foil. C_1, chamber enclosing region of retarding voltage. C_2, collecting chamber. P, collecting electrode. V_R+V, total accelerating voltage. V, retarding voltage. V_R, residual voltage.

[3] D. B. Penick, Rev. Sci. Inst. **6**, 115 (1935).

The zero of the galvanometer was subject to some drift. To eliminate the effect of this each reading made with the beam of electrons incident on the foil was bracketed between two readings with the beam cut off.

Retardation of Electrons in the Foil

Any electron traversing the foil must lose some energy in remote collisions with other electrons. It would obviously be a mistake to set the residual voltage so low as to exclude from the measured scattered current any electrons which had lost only such energy as this. To find a proper value for the residual voltage, the current scattered to the collecting electrode was measured with various values of the residual voltage. Figure 3 shows the result of observations made with one foil (foil II of the later discussion). The scattered current was measured at 90°, with the electrons emerging from the same face of the foil as that on which they were incident. Incidence was at 45°, so that an electron traversing almost the whole oblique thickness of the foil and being reflected through 90° at the far side would have a length of path in the foil $2\sqrt{2}$ times the thickness of the foil. No other conditions of observation would provide so great a possible length of path and hence so great a possible retardation.

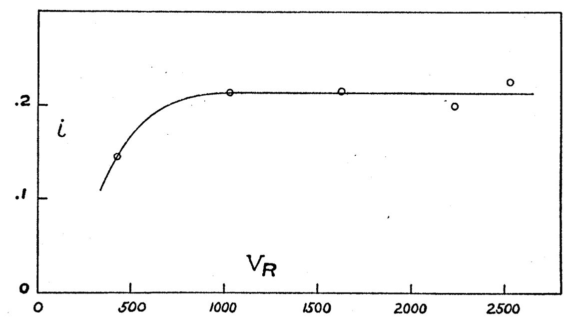

FIG. 3. Variation with residual voltage of current to collecting electrode (in units of 1 cm galvanometer deflection per microampere incident on foil). Scattering at 90° on side of incidence. Foil II.

It is evident from the figure that the energy lost by ordinary retardation was less than 1 kev, since the current was not increased by the increase of residual voltage from 1 to 2.5 kev. The thickest foil used in the observations on scattering was about twice as thick as the one used here.

The residual voltage employed was 2.5 kv, which is judged more than enough to compensate for the ordinary retardation of the electrons.

Tests of Single Scattering

Wentzel[4] has given as a criterion for single scattering of the Rutherford type that the angle θ at which scattering is observed shall be several times greater than 4ω, where

$$\omega = 2 \cot^{-1} \frac{2V}{Ze}\left(\frac{2}{\pi n t}\right)^{\frac{1}{2}},$$

eV being the kinetic energy of the electron. It has been customary to consider scattering as single at angles above 12ω. In the present experiment ω, as reckoned from the oblique thickness of the foil, was about 2° for the thinner foils used and something less than 3° for the thickest. Consequently the criterion would be satisfied at angles around 30° and higher. Instead of depending wholly on Wentzel's criterion, however, we have applied a more empirical test of single scattering derived from the following considerations.

When the foil is thin enough for most of the electrons to pass through with only small deflections, it is very improbable that one electron should undergo two large deflections. The deviations from strictly single scattering at any large angle will rather be due almost entirely to the combination of a single deflection through an angle nearly equal to this one with one or more much smaller deflections.

In Fig. 4 let OP be the direction of the beam of electrons incident on the foil. Let OQ be the direction of the observed scattered ray making an angle θ with the direction of incidence, OP. Let it be assumed that an electron observed in the ray OQ entered the ray not by a single deflection from OP to OQ through the angle θ but by the combination of two deflections, one from OP to OP' through a small angle ϵ and another from OP' to OQ through the angle θ' nearly equal to θ. The angle Φ between the planes of ϵ and θ may be taken at random. Also the sequence of the two deflections has no significance in the result; ϵ may even be considered as the resultant of several small deflections of which some may precede and some follow the large deflection.

[4] G. Wentzel, Ann. d. Physik 69, 333 (1922).

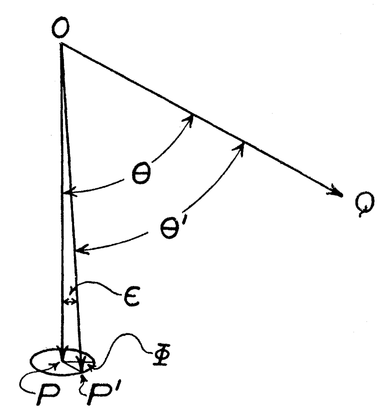

FIG. 4. Combination of small deflection ϵ with large deflection θ' to produce deflection at angle of observation θ. OP, direction of incidence. OP', direction after deflection ϵ. OQ, direction after deflections ϵ and θ'. Φ, angle, assumed random, between planes of ϵ and θ.

If powers of ϵ above the second are neglected,

$$\theta' = \theta - \epsilon \cos \Phi + \tfrac{1}{2}\epsilon^2 \cot \theta \sin^2 \Phi.$$

Let $p(\theta)$ be the probability of single deflection through the angle θ from the ray OP to the ray OQ, and let $p(\theta')$ be the probability of single deflection through the angle θ' from the ray OP' to the ray OQ. Then to the same degree of approximation as before

$$p(\theta') = p(\theta) - \frac{\partial p}{\partial \theta}(\epsilon \cos \Phi - \tfrac{1}{2}\epsilon^2 \cot \theta \sin^2 \Phi) + \frac{1}{2}\frac{\partial^2 p}{\partial \theta^2}\epsilon^2 \cos^2 \Phi.$$

Let $p'(\theta)$ denote the average probability of deflection into the ray OQ of an electron initially incident in the direction OP but deflected through the angle ϵ, the average to be taken with all values of Φ equiprobable. Then

$$p'(\theta) = \frac{1}{\pi}\int_0^{\pi} p(\theta')d\Phi.$$

With the expression given above for $p(\theta')$, we obtain

$$p'(\theta) - p(\theta) = \tfrac{1}{4}\epsilon^2\left(\cot \theta \frac{\partial p}{\partial \theta} + \frac{\partial^2 p}{\partial \theta^2}\right). \qquad (3)$$

To find the error made when the actual, approximately single scattering is taken as strictly single, it is necessary to average this expression over all values of ϵ. The error is thus the right-hand member of the equation with ϵ^2 denoting the mean square deflection of electrons having a mean length of path in the foil equal to that of the electrons deflected at the angle θ from the direction of incidence. In our experiment, incidence being at 45°, the mean length of path was not very different among the electrons observed and was not greater at any angle than that of an electron passing undeflected through the foil. Consequently ϵ^2 was not greater than the mean square deflection of all the electrons incident on the foil, which could be estimated experimentally. Equation (3) is valid for any law of scattering provided the observed large deflections may be ascribed to the combination of a single large deflection only with much smaller deflections. The derivatives of p may in this case be replaced without serious error by those of the experimentally determined function p'. Thus the experimental data furnish all the means of the test.

The observations in this experiment show p' varying very nearly as $\csc^4(\theta/2)$. In this case Eq. (3) gives for the fractional error caused by plural scattering:

$$\{p'(\theta)-p(\theta)\}/p(\theta)=\epsilon^2\{\csc^2(\theta/2)-\tfrac{1}{2}\}. \quad (4)$$

To estimate ϵ^2, the scattering at small angles was observed with the movable collecting chamber. The observations made with one foil (foil II of the later discussion) are shown in Fig. 5, the current collected in the chamber being plotted against the angle. Since the diaphragm in the front of the chamber subtended approximately 3°, the resolution was low at these small angles, and ϵ^2 could not be determined accurately. But since the collected current at 0° was 0.3 of the incident current, it is clear that 0.3 of the incident electrons were not scattered through more than 1.5°, half the angle subtended by the diaphragm. From this and the shape of the curve, it seems reasonable to take 2° or 0.035 rad. as the value of ϵ. With this value, the error caused by plural scattering would be 1.8 percent at 30° and 0.18 percent at 90°. With the thickest foil used, the errors would be not quite twice as large as with this one.

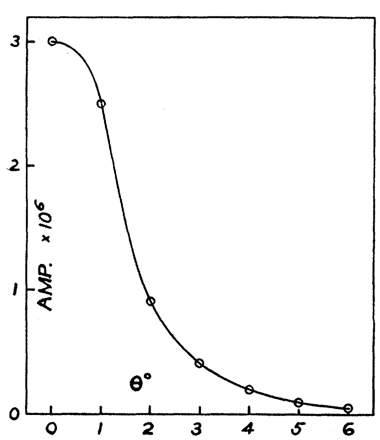

FIG. 5. Scattered current at small angles, in microamperes. Foil II. The incident current was 10 μa.

From the preceding calculation it may be seen why the simple form of Wentzel's criterion, which states that θ shall be at least equal to about 12ω, serves as well as it does. From Eq. (4) it follows that, if the fractional error is not to exceed a certain value δ, then it is necessary that

$$\frac{\theta}{\epsilon} \geqq \frac{1}{\delta^{\frac{1}{2}}}\theta[\csc^2(\theta/2)-\tfrac{1}{2}]^{\frac{1}{2}}.$$

Now $\theta[\csc^2(\theta/2)-\frac{1}{2}]^{\frac{1}{2}}$ varies only slowly with θ. From the value 2 when θ is zero it goes to a minimum of 1.9 when θ is about 0.6π and rises to 2.2 when θ is π. To set a maximum for the fractional error is thus approximately the same as to set a minimum for θ/ϵ (about 14 for a maximum error of 2 percent).

If the coherence of the scattering is not important, ϵ is proportional to $(nt)^{\frac{1}{2}}$. So also is ω if it is small. Hence setting a maximum value for the fractional error is also approximately setting a minimum value for θ/ω, as is done in applying Wentzel's criterion.

If Wentzel's criterion is to be valid, however, ω must be reckoned with t taken as the mean length of path in the foil of the electrons scattered at the angle θ, not as that of an undeflected electron. A similar point has already been made in regard to ϵ. The exceptions to Wentzel's criterion reported by Neher[5] may probably be ascribed to the fact that the average path of the scattered electrons in his experiment was much greater than the foil thickness, from which ω was computed.

[5] H. V. Neher, Phys. Rev. **38**, 1321 (1931).

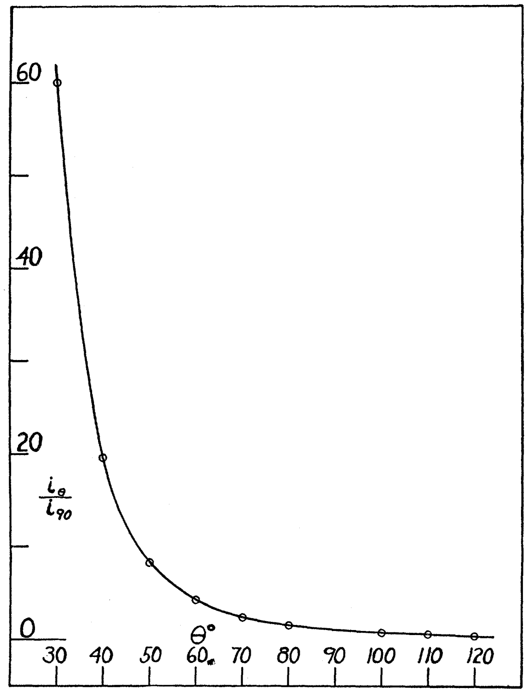

FIG. 6. Variation with angle of relative scattered current. Plotted points show averages of measurements with three foils. Curve shows Mott's theory.

Relative Scattering at Various Angles

Three foils were used in the final observations, all made in the same way. Al wire was wrapped around a tungsten filament and evaporated in vacuum by heating the filament. Some of the Al was allowed to condense on the very thin collodion film used as the support for the scattering foil.

The collodion film was too thin to allow a measurement of its thickness, but it was not so thin that its contribution to the scattering could be ignored. Consequently in order to compare the absolute intensity of the scattered current with that predicted it was necessary to determine the difference in the scattering by a film before and after a known layer of Al was deposited on it. Moreover the scattering by the collodion alone could not be measured directly, for uncoated films of collodion were quickly destroyed by the beam of electrons. Probably they became so highly charged that they were torn apart by electrostatic forces.

Foils I and II were collodion films thinly coated with Al. Foil III was the same foil as II but with an additional coating of Al of about the same thickness as the first. The difference in the scattering by foils II and III was ascribed to the second layer of Al. The determination of this difference was thus subject to the inaccuracies of the observations with both foils. The measurement of all absolute intensities is also subject of course to inaccuracies of calibration and to the considerable uncertainty in the determination of the thickness of the Al layer.

The comparison of the relative scattering at different angles with the prediction of the theory was free from several of the errors of calibration and from the error in measuring the thickness of the foil. Also it did not require taking the difference between the intensities observed with foils II and III. For elements of atomic number as low as those of Al and the constituents of collodion, the third term in the expression for R, Eq. (2), is small, and the difference in its extreme values between 30° and 120° does not amount to more than 2 percent of R. Since the probable errors of even the relative intensities at different angles were greater than this, the third term in R was ignored in the comparison. The other two terms do not involve Z and are the same for collodion as for Al. Hence the data obtained with the three foils could be averaged together.

With these three foils and others used in preliminary observations, an effect was observed for which we have as yet no explanation. It appeared in a comparison of the intensities of the beams scattered at 90° on the two sides of the foil, the side on which the beam was incident and the opposite side. Single nuclear scattering should be equally intense on both sides. But the observed scattering was consistently more intense on the side on which the beam was incident. The ratio of the two intensities was 1.1 for the thinner foils I and II and 1.2 for the thicker foil III.

It is hard to see what sort of scattering could combine with the nuclear scattering to cause such a difference. Some kind of boundary reflection, if it could be imagined great enough to be

TABLE I.

θ	30°	40°	50°	60°	70°	80°	100°	110°	120°
i_θ/i_{90} obs.	60	19.5	8.2	4.1	2.31	1.50	0.69	0.53	0.44
i_θ/i_{90} pred.	60	19.6	8.3	4.2	2.38	1.49	0.71	0.54	0.42

detectable, might produce a difference in this sense, but the absolute value of this difference should remain constant once the boundary was established, and the relative value should diminish with increasing thickness of the foil. The observed difference varied in the opposite way. Any effect of the coherence of the scattering at different scattering centers might increase in its relative value with increasing thickness of the foil, but it is hard to imagine either that coherence would be important at 90° with 50-kv electrons or that coherent scattering would be asymmetrical.

On the other hand we have not been able to ascribe the asymmetry to any defect of the experiment. Tests already described seem to make it unlikely that it was caused by scattering from parts of the apparatus other than the foil, or by a selective action of the retarding voltage on electrons traversing different lengths of path in the foil, or by plural scattering. To test for any accidental asymmetry in the apparatus, the foil was turned through 90°, so that its sides of incidence and emergence were reversed with respect to the two sides of the scattering chamber, but the asymmetry persisted.

Measurement of the scattering on both sides of the foil could only be made around 90°. Over the higher range of angles, scattering could be measured only on the side of incidence, and over the lower range only on the other side. Consequently in comparing the scattering above 90° with that below allowance for the observed asymmetry could only be made in some rather arbitrary way.

We have followed the simplest procedure possible by comparing the scattering measured at every angle with that observed at 90° and using the scattering measured at 90° on the side of incidence for the comparison at angles above 90° and the scattering measured at 90° on the side of emergence for the comparison at angles below 90°.

Table I shows the observed values of i_θ/i_{90} with those predicted by Eqs. (1) and (2). The observed values are averages for all the readings with the three foils in which every reading was given the same weight except that 7 out of 260 were discarded, either because they were very far from the mean or because of some suspicious circumstance in the behavior of the apparatus noted when they were made.

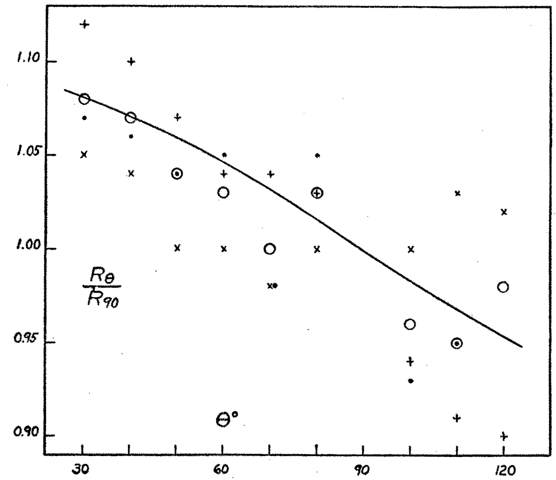

FIG. 7. Variation with angle of R_θ/R_{90}, Eq. (1). Dots, foil I. xxx, foil II. Crosses, foil III. Circles, average. Curve, Mott's theory.

The same data are shown in Fig. 6, in which the points show the observations and the curve shows the prediction. To the scale of this figure only the average values obtained with the three foils can be shown, as the points representing the separate observations fall too near together to be plotted. Although the deviations of the plotted points from the curve are scarcely discernible, it should be noted that the graph of i_θ/i_{90} against θ does not give a very critical comparison of observation with prediction, since the factor $\csc^4(\theta/2)$ must provide the dominant variation in any theory of scattering in a Coulombian field. A stricter comparison is made in Fig. 7, in which R_θ/R_{90} is plotted against θ. The curve is plotted from Eq. (2) with neglect of terms after the second. The plotted points show the observations made with the three foils and their average. The curve passes about as near to the open circles, which show the average values of the measurements, as any smooth curve could. The agreement is, indeed, perhaps closer than can be considered significant in view of the spread of the measurements with the foils separately.

Absolute Intensity of Scattering

To obtain the scattered intensity as a fraction of the incident current, it was necessary to find what part of the current passing through the diaphragms D_2 and D_3 was intercepted by the grid in front of the collecting plate P. This was done by swinging the diaphragm D_2 into the incident beam with the scattering foil removed and measuring the current to P with a microammeter, and alternately swinging the chamber C_1 out of the beam and measuring the current to the fixed collecting chamber C_3. The current to P was found to be 90 percent of that to C_3.

To compare the absolute values of i_θ observed and predicted, it was necessary to determine the thickness or, more directly, the mass per unit area, of the second Al layer deposited on foil II to produce foil III. Two estimates were made. The first was by assuming that all the Al wire wrapped around the tungsten filament was evaporated, as it appeared to be, and that the fraction deposited per unit area on the foil was $1/(4\pi r^2)$, where r is the distance from the filament to the foil. The surface density of the foil so estimated was 16.2 micrograms per cm^2, which would make the thickness 6.0×10^{-6} cm. Two factors may have made this estimate too high. The Al may not have been entirely evaporated from the filament and some of the atoms striking the foil may not have adhered. Of course some atoms may have reached the foil after rebounding from other surfaces, but this possibility was limited by baffle-plates above the foil. The values of i_θ predicted from this estimate of the surface density are for these reasons more likely to be high than low, and the ratio of the observed i_θ to that predicted more likely to be low than high.

The other estimate was kindly made for us by Mr. Hermann Yagoda by a colorimetric microchemical method applied to a layer of Al deposited on a blank collodion film by the evaporation of the same quantity of Al wire and under the same conditions as in the preparation of foil III.

A disk of this foil 4.8 mm in diameter was immersed in 0.5 cm^3 of 10-percent hydrochloric acid, which was then warmed over steam until the film lost its metallic luster. The solution, together with water used in two subsequent washings of the collodion film, was transferred to a micro-Nessler tube. The contents of the tube were treated with 0.4 cm^3 of 50-percent ammonium acetate and 0.1 cm^3 of freshly prepared 0.5-percent solution of the ammonium salt of aurin tricarboxylic acid. The solution was diluted to 2 cm^3 and, 15 minutes later, when the pink of the Al lake became fully developed, the sample was compared in color with a set of standards made by treating standardized Al-ion solutions in the same way as the solution of the Al of the foil had been treated.

The surface density of the foil found by this method was 13.6 micrograms per cm^2, which would make the thickness 5.0×10^{-6} cm.

Other samples of the Al layer meant to be measured by another chemical method were unfortunately destroyed in the process without yielding a result.

In reckoning the difference in the currents scattered by foil III and foil II, which is to be ascribed to the additional layer of Al, only the readings at angles from 30° to 60° have been used, since the random errors of the measurements were relatively larger at the higher angles. The ratio of the observed scattering to the predicted is 0.80 if the surface density of the foil is reckoned by the inverse-square calculation and 0.96 if it is taken as given by the colorimetric method. Both values would be lower if the readings at larger angles were included. The first and smaller of these values might be expected to be low, as was explained before. Either one is uncertain by as much as the difference between them. It would appear, however, from these observations that if the actual scattering is different from that predicted by Mott it is more likely lower than higher.

Comparison with Neher's Observations

Among the many experiments on the scattering of electrons, a part of the work of Neher[5] seems most nearly comparable to ours in respect to the experimental conditions. He measured the scattering by Al of electrons at voltages around that of our experiment and at higher angles above 90°. He found, as we do, that the relative scattering at different angles is in agreement with Mott's theory, though because of the small difference between Mott's distribution and Rutherford's he could not discriminate between them. He found the ratio of the observed to the predicted scattering to be 1.32, an apparent difference from our result given above. Since all his observations were made above 90°, where ours are least accurate, the two results may not be strictly comparable. If they may be compared, then our observation of the asymmetry between the scattering on the side of incidence and on the other side may perhaps be relevant. All of his observations were on the side of incidence, and if the excess scattering to this side is a general characteristic of scattering by foils and not a peculiarity of our apparatus, the difference between his observations and ours might be explained.

We wish to thank Mr. A. C. Weid for help in the construction of the apparatus and in making preliminary observations, Mr. Hermann Yagoda for measurements on the foil thickness, our colleagues Professors F. E. Myers and O. Halpern and Dr. R. D. Huntoon for helpful discussions, and Mr. William Werker for technical assistance.

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Partially Inverted Multiplets in MgI

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(Received December 11, 1939)

With the use of Fermi-Thomas statistical eigenfunctions the separations of the $3s3d\ ^3D$ term of MgI are evaluated, taking into account all the magnetic interactions in the atom and the electrostatic interactions with other configurations. The smallness of the observed separations is found to be due to the large value of the exchange integrals of the magnetic actions.

1. Introduction

THIS note discusses some remarkable fine structures found by Meissner[1] in the magnesium spectrum (MgI), namely partially inverted multiplets showing very small separations. It will be shown that they arise from several causes whose importance can be evaluated; the exact calculation is, however, impossible because of the approximations which one is compelled to introduce and because of the unavailability of accurate atomic eigenfunctions. In any case the study of these phenomena in a particular example may be interesting in view of conclusions that are generally valid in similar cases.

We shall discuss the $3s3d\ ^3D$ term, placed at 13,712 cm^{-1}, whose levels, relative to the center of gravity of the term, are, respectively:

$$^3D_3 \text{ at } +0.0032 \text{ cm}^{-1};\quad ^3D_2 \text{ at } -0.0140 \text{ cm}^{-1};$$
$$^3D_1 \text{ at } +0.0157 \text{ cm}^{-1}.$$

These positions are those which would be expected according to the theory if only the spin-spin interaction of the two outer electrons, and no other magnetic energy, existed. In fact, according to this hypothesis, the positions of the levels would be:

$$^3D_3 \text{ at } +\tfrac{1}{7}L\alpha^2;\quad ^3D_2 \text{ at } -\tfrac{1}{2}L\alpha^2;\quad ^3D_1 \text{ at } +\tfrac{1}{2}L\alpha^2,$$

(α = fine structure constant;

$$L=\int_0^\infty R_{3s}^2(r_1)r_1^2dr_1\int_{r_1}^\infty R_{3d}^2(r_2)\frac{dr_2}{r_2};$$

R_{3s}, R_{3d} = radial eigenfunctions of the $3s$ and $3d$ electron, respectively). Thus the ratios between the experimental values are very near to the theoretical ones, and also the value of the integral L, evaluated with Fermi statistical eigenfunctions, is

$$L\alpha^2=0.055 \text{ cm}^{-1},$$

leading to the correct order of magnitude of the separations.

Now the interaction considered is only one, and not the most important, of the causes which contribute to the splitting of the term. We have now to discuss all these causes.

[1] K. W. Meissner, Ann. d. Physik **31**, 518 (1938).