Radiative K Capture

P. MORRISON AND L. I. SCHIFF
Department of Physics, University of California, Berkeley, California

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Accompanying K capture, as in β-activity, there is a weak continuous high energy gamma-ray spectrum. The major contribution to the effect is from magnetic radiative capture of a K electron. For an allowed transition using Fermi or Gamow-Teller coupling, the number of gamma-quanta per K electron captured is $(\alpha/12\pi)(W/mc^2)^2$, where W is the available energy. For other couplings and forbidden transitions, the effect is of the same order of magnitude.

ACCOMPANYING the emission of electrons and positrons from a radioactive nucleus, there is a low intensity continuous spectrum of gamma-radiation. This radiation has been measured by several observers,[1] and the effect has been discussed theoretically by Bloch[2] and by Knipp and Uhlenbeck.[3] The origin of the radiation is the sudden change in the atomic electric moment when the charge of the heavy particle, confined to the nucleus, is released as a free electron or positron. An entirely analogous effect will take place when a nucleus falls to a stable state by the capture of an atomic electron. Here the available energy will most frequently go off with the neutrino, but may be shared between a neutrino and a gamma-quantum. If the energy difference between the isobaric nuclei is $E_Z - E_{Z-1} = \Delta E$, the available energy for the capture process includes the rest energy of the atomic electron and is $W = \Delta E + mc^2$. When $0 < W < 2mc^2$ only electron capture will be energetically allowed. Then only the unobservable neutrino and the low energy x-ray quanta from the final nucleus will be emitted in the ordinary capture process, and the radiative capture gamma-rays will be observable without a background of other ionizing radiation; we shall see that most of the gamma-quanta have a good fraction of the energy W. The most favorable cases for observation will be those where: (1) there is only electron capture, with no positrons; (2) Z is not too large, so that the x-rays are of low energy; and (3) the available energy W is of the order of hundreds of kilovolts. Under these conditions the capture radiation will be observable without the necessity for the elaborate corrections for ordinary bremsstrahlung made in the case of the radiation accompanying electron or positron activity. The elements V^{47} and Y^{86} satisfy conditions (1) and (2), and may be suitable substances for a study of this effect.[4]

We have calculated the ratio of the number of gamma-quanta emitted with a given energy to the number of electrons captured. The radiationless capture is a first-order process, and its probability per unit time is:

$$w_C = (2\pi/\hbar)\rho_\nu(W)\,|(f|H|0)|^2,$$

where ρ_ν is the density of final neutrino states per unit energy range, and H is the coupling energy between heavy and light particles. For all interesting cases the available energy will be greater than the K-shell binding energy, and the K electrons, which spend the most time in the region of the nucleus, will give the great bulk of the capture processes whether they are allowed or forbidden. The radiative capture is a second order process, and its probability per unit time when the gamma-quantum has an energy between k and $k+dk$ is:

$$w_k dk = \frac{2\pi}{\hbar}\rho_\nu(W-k)\rho_\gamma(k)dk \times \left|\sum_i \frac{(f|H|i)(i|e\boldsymbol{\alpha}\cdot\mathbf{A}_k|0)}{E_0 - E_i}\right|^2,$$

where ρ_γ is the density of final gamma-ray states per unit energy range. For allowed transitions,

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[2] F. Bloch, Phys. Rev. **50**, 472 (1936).

[3] J. K. Knipp and G. E. Uhlenbeck, Physica **3**, 425 (1936).

[4] J. J. Livingood and G. T. Seaborg, Rev. Mod. Phys. **12**, 30 (1940).

the intermediate state of the electron must be an s state. From an initial K-state magnetic dipole radiation will be produced, while from an initial L_{II}-state electric dipole radiation can occur. The electric radiation depends on the transverse component of the current which is proportional to the momentum of the initial state; that is, to αZ, where α is the fine structure constant. As is well known from the theory of the photoelectric effect, the electric radiation cannot be calculated correctly without taking into account the effect of the Coulomb field on the intermediate state wave functions. However, an estimate shows that the electric radiation probability is of order $(\alpha Z)^2/10$ compared to the magnetic radiation probability for $W\sim mc^2$, and only becomes comparable with it for the uninteresting case of available energies of the order of the K-shell binding energy. For forbidden transitions, the intermediate electron state need not be an s state, since either the electron (and thence the gamma-quantum) or the neutrino can take up the angular momentum of the nucleus. There are then many more possible transitions; however, those transitions which start from a K state will be most likely, since their probability contains the fewest powers of αZ.

Since we are dealing with an initial K state whose binding energy is small compared to the gamma-ray recoil energy, we can neglect Z consistently throughout and use plane waves for the electrons, taking the initial state to have zero momentum. For allowed transitions, the calculation can then be performed in the usual way, assuming a point nucleus, and summing over spins of the initial and final states and over spins and both signs of the energy of the intermediate state by the usual method of projection operators. With either Fermi or Gamow-Teller couplings, we obtain:

$$\frac{w_k dk}{w_C}=\frac{\alpha}{\pi}\left(\frac{W}{mc^2}\right)^2(1-\epsilon)^2\epsilon d\epsilon, \quad \epsilon\equiv\frac{k}{W},$$

giving for the ratio of the total number of gamma-rays to the total number of K captures:

$$\int_0^W \frac{w_k dk}{w_C}\equiv\frac{w_R}{w_C}=\frac{\alpha}{12\pi}\left(\frac{W}{mc^2}\right)^2.$$

The Konopinski-Uhlenbeck coupling gives a slightly different result:

$$\frac{w_k dk}{w_C}=\frac{\alpha}{\pi}\left(\frac{W}{mc^2}\right)^2(1-\epsilon)^4\epsilon d\epsilon, \quad \frac{w_R}{w_C}=\frac{\alpha}{30\pi}\left(\frac{W}{mc^2}\right)^2.$$

Because of the magnetic character of the radiation, most of the quanta have a considerable fraction of the available energy.

As is usually the case with forbidden transitions, the calculations here are essentially more complicated, since they are more dependent on the type of coupling and on the nuclear matrix element. With the Fermi coupling, for example, the contributions of the nuclear charge and current terms become comparable, and it is no longer possible to cancel out the nuclear matrix elements between w_k and w_C. However, it is easy to see that w_R/w_C will not be changed in order of magnitude, although the spectrum of the emitted gamma-rays will be altered considerably in shape. An estimate indicates that w_R/w_C is about half as large for a singly forbidden as for an allowed transition.

Knipp and Uhlenbeck[3] have suggested an alternative method of calculation for such problems as radiative emission and capture. It consists in treating the effect not as a second-order process involving the form of both the beta-coupling and the electromagnetic coupling, but as a process of the first order. They calculate the probability of a radiative transition from an electron state which represents a source of electrons at the nucleus to a plane wave. (For the capture effect, a state representing a sink at the nucleus would be used.) They indicate then that it is sufficient to use the observed momentum distribution of outgoing electrons to predict the total effect. But it is easy to see that the "first-order" and the "second-order" calculations will give the same result only for a restricted form of beta-coupling. For if we take the beta-coupling energy to be a function of the electron and neutrino momenta $f(\mathbf{k}_e, \mathbf{k}_\nu)$, we find that the "first-order" probability is proportional to $|f(L, \mathbf{k}_\nu)|^2$ where L is the momentum of the outgoing electron before it has radiated, while the "second-order" probability is proportional to $|f(\mathbf{K}, \mathbf{k}_\nu)|^2$, where $\mathbf{K}$ is the difference of the momenta of the gamma-quantum and the final electron. In general, L and

K are not the same, so that the two calculations will give the same result only if f is independent of $\mathbf{k}_e$. This condition is satisfied for allowed transitions with the three couplings commonly employed, although not for forbidden transitions. A detailed study of the radiative beta-activity and radiative capture might thus give further information on the actual form of the beta-coupling.

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The Scattering of Thermal Neutrons by Deuterons*

LLOYD MOTZ, *Columbia University, New York, New York*

AND

JULIAN SCHWINGER,† *The University of California, Berkeley, California*

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The cross section for the scattering of very slow neutrons by deuterons is calculated by numerical methods. Polarization is completely neglected and the wave equation for the process is set up in such a form as to take correctly into account exchange effects between the incident neutron and the neutron initially in the deuteron. This wave equation is then replaced by an integral equation the solution of which is correctly symmetrized and has the right asymptotic value to describe the scattering process. The numerical integration is performed by replacing the integral equation by a finite set of simultaneous linear algebraic equations. The work is greatly simplified by the use of a sum of two Gauss functions to approximate the ground state deuteron wave function. It is assumed throughout this paper that the interactions between like and unlike particles are equal and are of the general form

$$V_{ij}=-[(1-g-g_1-g_2)P_{ij}+gP_{ij}Q_{ij}+g_1+g_2Q_{ij}]J(r_{ij}),$$

where the symbols have their usual meanings and where $J(r_{ij})$ is a Gauss function. The calculation is carried out for two sets of g's. For the first set, $g_1=g_2=0$, $g=0.2$, the cross section is found to be equal to 4.57×10^{-24} cm^2, and for the second set of g's, $g_2=2$, $g=0.22-g_2$, $g_1=0.25-0.8g_2$, the value of the cross section is found to be equal to 6.91×10^{-24} cm^2. The experimental value is at least 20 percent smaller than the first of these values.

INTRODUCTION

SINCE the scattering of neutrons by deuterons involves a fundamental process in which only three particles take part, the solution of this problem can be expected to throw a great deal more light on the nature of the forces between elementary particles than is now available. This, coupled with the fact that an exact theoretical treatment is impossible because of the complexity of the equations, must justify a step-by-step attack on the problem in which various simplified models are considered in some detail. A complete treatment must take into account polarization as well as exchange effects. If polarization is entirely neglected, as has been done in the present calculation, it is possible to set the problem up in such a form that one can obtain a numerical solution without an undue amount of work. Calculations in which polarization has been neglected have already been carried out by Schiff[1] and by Yukawa and Sakata[2] who proceeded somewhat indirectly by introducing auxiliary potentials which enabled them to simplify their equations considerably. The present paper differs from the aforementioned ones in two respects: the calculations are carried through using the most general type of interaction between the particles (we assume only that the forces between like and unlike particles are equal); exchange effects are accurately taken account of to the order of approximation employed.

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† National Research Fellow.

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