range is due to protons from $Ne^{22}(dp)Ne^{23}$ and corresponds to the transition to the ground state. The Q value calculated is 3.6 Mev which gives a mass of 23.0012 for the mass of Ne^{23}. The agreement is reasonable but such data should not be pushed too far.

The proton yield for the $Ne^{22}(dp)Ne^{23}$ reaction can be calculated from the radioactivity observed. With our solid angle for counting we should obtain between ten and thirty per minute. Since when set for great resolution the counter only records about one-twentieth of the total protons passing through it, the size of group to be expected is about that of the indicated group at 41 cm so that it is not surprising that the heavy yields due to Ne^{20} mask the Ne^{22} group. It might be pointed out that without separated isotopes the temptation to ascribe the 33-cm group to Ne^{22} would have been irresistible and an incorrect level scheme for Ne^{21} thus deduced.

In conclusion we wish to thank Mr. W. L. Davidson, Jr., for help in general operations, Messrs. Harry Schultz and A. R. Tobey for keeping the beam in tune during many long hours, and Professor E. O. Lawrence for the gift of the vacuum chamber. The cyclotron construction has been greatly helped by a grant from the George Sheffield Fund.

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On the Resonance Levels of Rhodium and Indium

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Self-absorption curves as well as mutual absorption curves of the In and the Rh neutron capture levels have been measured. From the experiments, the following values were obtained for the absorption coefficients for self-indication K_s, the resonance cross sections σ_0, the natural widths Γ, the neutron width Γ_N and the spacing $|E_{0\text{In}}-E_{0\text{Rh}}|$ of the two levels: $K_s=11.5$ cm²/g, $\sigma_0=4100\times10^{-24}$ cm², $\Gamma=0.13$ ev, $\Gamma_N=3.5\times10^{-4}$ ev for Rh; $K_s=52$ cm²/g, $\sigma_0=23{,}000\times10^{-24}$ cm², $\Gamma=0.07$ ev, $\Gamma_N=1\times10^{-3}$ ev for In; $|E_{0\text{In}}-E_{0\text{Rh}}|=0.15$ ev.

THE total and the neutron width of a resonance level for slow neutron capture are quantities important in nuclear theory, since from them the probabilities of γ-ray emission and neutron re-emission of compound nuclei can be calculated. On the assumption that only one resonance level is responsible for the neutron capture, the total width Γ of the level may be computed from

$$\Gamma=2E_0^{\frac{3}{2}}E_{th}^{\frac{1}{2}}(K_{th}/K_0)^{\frac{1}{2}}. \tag{1}$$

Here, E_{th} is the thermal and E_0 the resonance energy, K_{th} the thermal absorption coefficient and K_0 the absorption coefficient at exact resonance, and it is assumed that $E_0\gg E_{th}$ and $E_0\gg\Gamma$. The neutron width Γ_N may be found from the relation[1]

$$\sigma_0=\frac{1.30\times10^6}{E_0}\left(1\pm\frac{1}{2j+1}\right)\frac{\Gamma_N}{\Gamma}, \tag{2}$$

if the cross section at exact resonance σ_0 is measured in units of 10^{-24} cm² and E_0 in ev. The factor $(1\pm1/(2j+1))$, with j designating the spin of the capturing nucleus, is unknown and shall, therefore, be included in Γ_N.

In an earlier work,[2] self-absorption measurements to determine σ_0 for the one-volt level of rhodium were performed with thick detectors.

[1] H. A. Bethe, Rev. Mod. Phys. **9**, 69 (1937) (Henceforth referred to as B.), Eq. (548).
[2] Manley, Goldsmith and Schwinger, Phys. Rev. **55**, 39 (1939).

The mathematical evaluation was complicated by the well-known phenomenon of self-reversal of the absorption line which causes a strong dependence of the observed absorption on the thickness of the detector. This effect is so large that for great absorber and detector thicknesses the absorption becomes independent of σ_0. As a consequence, the evaluation of the Rh data was somewhat unreliable and in the case of In only a lower limit for the resonance cross section was obtained.[3]

In view of these difficulties it was found desirable to measure the self-absorption coefficients of In and Rh with thin detectors.

Supplementary information was gained from the mutual absorption method which can be applied because of the proximity of the Rh and In levels. From data on the mutual absorption, the ratio of the widths of the two levels Γ_1/Γ_2 and their spacing $|E_{01}-E_{02}|$ can be computed from[4]

$$\Gamma_1/\Gamma_2=K_{12}K_{02}/K_{21}K_{01} \tag{3}$$

and

$$|E_{01}-E_{02}|=\tfrac{1}{2}(\Gamma_1+\Gamma_2)\times\left(\frac{K_{01}K_{02}}{K_{12}K_{02}+K_{21}K_{01}}-1\right)^{\frac{1}{2}}. \tag{4}$$

Here, K_{01} and K_{02} are the resonance cross sections of the two substances, K_{12} and K_{21} the absorption coefficient of substance 1 or 2 for the activity of substance 2 or 1, respectively. The equations are valid only if both detectors and absorbers are thin and if Doppler broadening can be neglected.

Experimental Procedure

The neutron source was a mixture of 200 mC of Ra with Be, embedded 3 cm below the surface of a paraffin cylinder 15 cm high and 20 cm in diameter. The Rh detectors were made from foil weighing 16.5 mg/cm² cemented to a nickel backing. As In detectors nickel slabs were used on which 7.3 mg/cm² of In was electrolytically deposited. (In the earlier work, the "thin" Rh detector weighed 110 mg/cm² and the In detector 80 mg/cm².) The thinnest absorbers consisted of sets of slabs identical with the detectors; the thicker ones were made either from Rh (123 mg/cm²) and In (10, 26, 89 and 185 mg/cm²) foil, or from powdered Rh metal (60 and 100 mg/cm²). In each case, sets of four Cd shielded detector slabs, each measuring 2.2 by 4 cm, were irradiated at a standard elevation of 35 mm above the paraffin surface in an arrangement in which they covered an area of 4.5×8 cm. For counting, the four detectors were placed on a frame which fitted around a thin-walled Geiger-Mueller counter. The counter was connected to a scale-of-eight circuit. The 44″ period of Rh and the 54′ period of In were used. The routine of counting and irradiation was, of course, strictly standardized, both with regard to geometrical arrangement and to timing. The statistical error in the determination of each point was less than 2 percent. The results are represented in Figs. 1 and 2, where the transmissions are plotted against the absorber thickness in mg/cm².

Geometrical Considerations

In the earliest papers on neutron absorption[5] it was already pointed out that the absorption shown by a given absorber will strongly depend on the angular distribution of the emerging neutrons. For the thermal region, Fermi[6] has shown that the number of neutrons emerging between the angles ϑ and $\vartheta+d\vartheta$ is proportional to $(\cos\vartheta+\sqrt{3}\cos^2\vartheta)$. Frisch[7] has computed expressions for the absorption of a monokinetic beam of neutrons which obeys Fermi's formula for the angular distribution up to a maximum angle θ, but which contains no neutrons emerging at angles larger than θ. His results showed that for θ near 90° accurate knowledge of θ is essential for correct evaluation of absorption experiments. For $\theta\leqslant 65°$, the absorption varies more slowly with θ and is, both for thin and thick detectors, very nearly exponential, down to transmissions of the order of 10 percent. The slope of the exponential (plotted semi-logarithmically) is larger than in the case of a parallel beam by a

[3] Manley, Goldsmith and Schwinger, Phys. Rev. **55**, 107 (1939).
[4] B., Eqs. (540), (541).
[5] E. Amaldi and E. Fermi, Phys. Rev. **50**, 899 (1936).
[6] E. Fermi, Ricerca Scient. **72**, 13 (1936).
[7] O. R. Frisch, Kgl. Danske Vidensk. Selskab. Math. Fys. Medd. **14**, No. 12 (1937).

factor f which is function of θ. In other words, experiments carried out with a partly collimated beam ($\theta \leqslant 65°$) of neutrons show the same absorption in a layer of the thickness η as a parallel beam in a layer of the thickness $f\eta$.

Frisch's formulae cannot, however, be applied directly to our experimental conditions, since the value of θ is unknown. θ cannot be computed from the size of the neutron source (paraffin cylinder) and the distance between the paraffin and the detector, because the paraffin surface does not act as a homogeneous source of neutrons. On the contrary, the neutron intensity as measured by cadmium-shielded Rh detectors, drops almost linearly from a maximum at the center of the cylinder to practically zero at the edges. The proper reduction factor f was, therefore, determined semi-empirically instead by comparing with Cd shielded *thick* Rh and In detectors the boron absorption curve obtained in the above-described standard arrangement with a curve obtained with a practically parallel beam.[8] These investigations confirmed Frisch's calculations in that the absorption was exponential within the experimental error and yield $f=1.32$, corresponding to $\theta=65°$. For $\theta=65°$, the f factor for *thin* detectors is found from Frisch's results to change from 1.41 to 1.37 for transmissions varying from near 100 percent down to 30 percent. We have here assumed that the ratio of the f factors for thin and thick detectors is not materially affected by the deviation of the actual angular distribution of resonance neutrons from Fermi's expression for thermal neutrons. Calculations for a cosine distribution show, for instance, that in this case the ratio differs by less than 2 percent from the former value.

The preceding discussion is not generally applicable to self-absorption experiments where the neutron beam is not monokinetic. However, as long as the factor f is the same for all parts of the neutron spectrum, that is as long as θ is not larger than 65° and the transmission at exact resonance is not smaller than 10 percent, the only effect of the angular distribution is to increase the apparent thickness of the absorber and the above considerations still apply. Inspection of a theoretical absorption spectrum shows that even for still thicker absorbers quite accurate results may be expected from this representation. If, therefore, the transmission is plotted against $fK_s\eta$, K_s being the self-absorption coefficient and η the thickness of the absorber, the resulting curve should follow closely the self-absorption curve[9] for a parallel beam.

[8] These experiments were performed and will be described by Dr. W. J. Horvath to whom we are obliged for his kind permission to quote his results before publication.

Detector Thickness

Our detectors are so thin that the electron absorption in the detector material may be neglected. They may not, however, be considered ideally thin neutron absorbers. If the Doppler width $\Delta \ll \Gamma \ll E_0$, correct evaluation of the experimental results, taking into account the finite detector thickness, is obtained in the following way: Let $G(y)=e^{-y}J_0(iy)$ be the self-absorption function for ideally thin detectors (J_0 is the zero-order Bessel function). $y=fK_s\eta$ and will be called the absorptive power. If the absorptive power of the detector, having a thickness ζ, is z, the transmission T_y of the absorber is given by

$$T_y=\int_y^{y+z} G(\xi)d\xi \Big/ \int_0^z G(\xi)d\xi. \qquad (5)$$

Since the ratio $z/y=\zeta/\eta$ is known, y can be obtained from this equation. By numerically carrying out the integration in the special case $y=z$ for various values of z, the z corresponding to the observed T_z was found. It was then used to evaluate the transmissions of the other absorbers.

If the detectors can be considered to be ideally thin in first approximation, the calculation is greatly simplified by first obtaining an approximate absorptive power z_1 from the equation

$$T_z=G(z_1), \qquad (6)$$

Table I. *Indium self-absorption.*

η mg/cm²	T_y%	z_1	T_y'%	$y+\frac{1}{2}z$	f	K_s cm²/g
7.3	66.6±0.7	0.45	54.0	0.76	1.41	52±1.8
17.3	45.5±0.7	—	36.8	1.49	1.40	52±1.5
26.0	37.4±0.6	—	30.3	2.02	1.38	50±1
33.3	34.0±0.6	—	27.6	2.42	1.37	48±1

[9] B., Fig. 16, curve z.

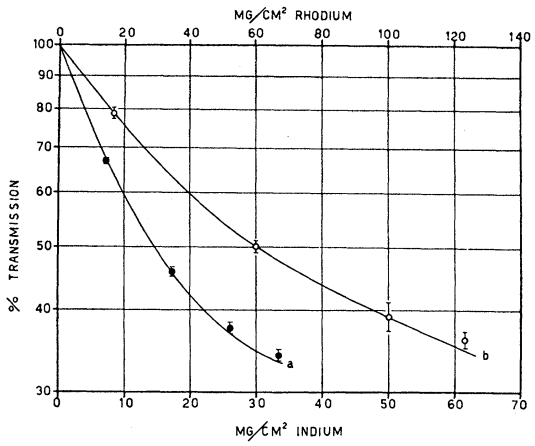

FIG. 1. Self-absorption, percent transmission *vs.* absorber thickness. *a*, In-In (lower abscissa scale); *b*, Rh-Rh (upper abscissa scale).

by then substituting $z_1G(\frac{1}{2}z_1)$ and $z_1G(y+\frac{1}{2}z_1)$ for the integrals

$$\int_0^z G(\xi)d\xi \quad \text{and} \quad \int_y^{y+z} G(\xi)d\xi,$$

respectively, and solving for y. In the special case $y=z$, z was larger than z_1 by 11 percent for the Rh detectors, and by 20 percent for the In detectors. Both the direct and the approximate evaluation gave the same result, showing that the approximations are adequate.

It follows from the computations that a detector may, in first approximation, be considered thin, if its activity is reduced by an absorber of the same thickness by less than 40 percent. For this limiting value, the approximate method of evaluation yields an absorption coefficient too low by only 3 percent.

Table I shows the procedure of evaluation for the case of indium. Column 1 gives the thickness of the absorbers in mg/cm²; column 2 the experimental transmissions; column 3 the preliminary absorptive power z_1 of the thinnest absorber, as found from Eq. (6); column 4 the quantities $T_y'=G(\frac{1}{2}z_1)T_y$ and column 5 the absorptive powers $y+\frac{1}{2}z_1$ determined from the relation $T_y'=G(y+\frac{1}{2}z_1)$; column 6 the f factors. From columns 1, 5, and 6, the absorption coefficients K_s in column 7 are computed. The first two values are deemed to be most reliable and $K_s=52\pm2$ cm²/g was, therefore, adopted as a basis for the subsequent calculations. In Fig. 1, curve (*a*) represents the absorption curve as computed from Eq. (6) for $K_s=52$ cm²/g. Similarly, for Rh $K_s=11.5\pm0.5$ cm²/g and curve (*b*), Fig. 1, were obtained.

The limits of error include the experimental fluctuations as well as the uncertainties in the f factors. The corrections for Doppler broadening which must be made before the cross section at exact resonance can be derived from the absorption coefficient for self-indication will be discussed below.

MUTUAL ABSORPTION

The evaluation of experiments on the mutual absorption of the levels of two different substances is complicated by the fact that the shape of the absorption curve depends on the ratio of the width Γ_1/Γ_2 of the two levels and their spacing $|E_{01}-E_{02}|$. Rough values for these quantities can be derived from the initial slope of the absorption curves by means of Eqs. (3) and (4). However, for accurate values these equations must be replaced by an expression which gives the transmission as a function of the parameters Γ_1/Γ_2 and $|E_{01}-E_{02}|$ not only for thin but also for thick absorbers. The finite thickness of the detector (substance 2, absorptive power z) may approximately be taken into account by replacing in the computation the actual fairly thin detector by a very thin detector in combination with a filter of the absorptive power $\frac{1}{2}z=\frac{1}{2}fK_{s2}\zeta$. Neglecting the effects of Doppler broadening,[10] assuming Γ_1, Γ_2 and $|E_{01}-E_{02}|$ all $\ll E_{01}$, and using the abbreviations

$$x=\frac{E-E_{02}}{\frac{1}{2}\Gamma_2}, \quad a=(\Gamma_2/\Gamma_1)^2, \quad b=\frac{E_{01}-E_{02}}{\frac{1}{2}\Gamma_2},$$

we derive from the Breit-Wigner formula[11] for the transmission T of an absorber of the absorptive power $y=fK_{s1}\eta$:

$$T=\frac{\int_{-\infty}^{+\infty}\frac{dx}{1+x^2}\exp\left(\frac{-2y}{1+a(x-b)^2}-\frac{z}{1+x^2}\right)}{\int_{-\infty}^{+\infty}\frac{dx}{1+x^2}\exp\left(\frac{-z}{1+x^2}\right)}. \quad (7)$$

[10] Note that in this case $K_0=2K_s$.
[11] B., Eq. (530).

By substituting $x=tn\frac{1}{2}\psi$ and performing the integration in the denominator, we may bring this in a form convenient for numerical evaluation:

$$T=\frac{1}{2\pi G(\frac{1}{2}z)}\int_{-\pi}^{+\pi} \dot{e}\exp\left[(1+\cos\psi)\times\left(\frac{-2y/c}{1-\{1-(4a/c^2)\}^{\frac{1}{2}}\sin(\psi-\varphi)}-\frac{z}{2}\right)\right]d\psi, \quad (8)$$

with $c=1+a+ab^2$ and $tn\varphi=(1-a+ab^2)/2ab$.

Width and Doppler Broadening

It is, of course, impracticable to compute T from Eq. (8) for all possible values of the parameters y, a, and b in order to obtain, by comparison with the experiments, the widths and spacing. Eq. (8) was, therefore, evaluated only for a few slightly differing tentative sets of the parameters. These were obtained by inserting into Eqs. (3) and (4) approximate values for K_{InRh} and K_{RhIn}, determined from the initial slope of the mutual absorption curves. The other parameters in these equations, $K_{0\text{In}}$ and $K_{0\text{Rh}}$, were derived from the experimental $K_{s\text{In}}$ and $K_{s\text{Rh}}$ by applying Doppler correction in the following way: First, an estimate for the width of the Rh level was deduced from Eq. (1) by inserting $K_s=11.5$ cm²/g, $K_{th}=0.74$ cm²/g,[12] $E_{th}=0.024$ ev, and $E_0=0.85$ ev.[8] Here, E_{th} is a function of the transmission of the absorber used in determining K_{th} and was taken from a plot given by Bethe.[13] The implicit assumption that only one level contributes to the thermal neutron absorption in Rh is well justified, since it has been shown[14] that the 44″ and 4.2′ periods are due to the same level and since no other periods induced by slow neutrons are known. (The hypothetical levels of higher energy considered below should not greatly affect the thermal absorption.)

It is not safe to apply the same procedure to the In level because of the unknown contribution of the 40^d period to the thermal absorption. An approximate Γ_{In} was found, therefore, from Eq. (3) by inserting the approximate K_{InRh} and K_{RhIn}, and $K_{s\text{In}}$ and $K_{s\text{Rh}}$ instead of $K_{0\text{In}}$ and $K_{0\text{Rh}}$. For the Doppler correction, K_s/K_0 as a function of Δ/Γ, with Δ the Doppler width, was taken from a graph given by Bethe and Placzek.[15] The equation

$$\Delta=(2/A)(E_0E_{th})^{\frac{1}{2}} \quad (9)$$

(A is the atomic weight) yields $\Delta=0.031$ ev both for In and Rh. Because of the effects of lattice binding,[16] the effective thermal energy $E_{th}\backsimeq 1.2kT$ of the crystal is used in (9). Combining Δ with the tentative values for Γ, $K_{0\text{Rh}}=24\pm1$ cm²/g and $K_{0\text{In}}=116\pm6$ cm²/g are obtained as final Doppler-corrected absorption coefficients at exact resonance. The corresponding cross sections are $\sigma_0=4100\times10^{-24}$ cm² for Rh and $\sigma_0=23{,}000\times10^{-24}$ cm² for In[115].

The In cross section is larger than the lower limit of $20{,}000\times10^{-24}$ cm² deduced from the thick detector work (l.c.). The Rh cross section agrees with Jaeckel's[14] result, but is lower by 30 percent than the value given by Manley, Goldsmith and Schwinger.[2,17]

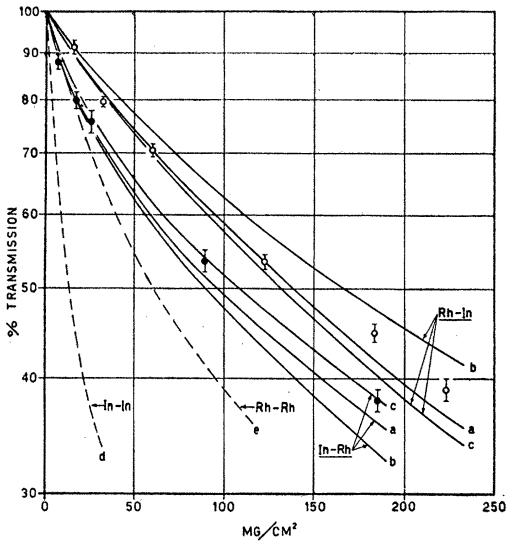

FIG. 2. Mutual and self-absorption of Rh and In, percent transmission *vs.* absorber thickness. Full curves calculated for (*a*), $K_{\text{InRh}}=12.5$ cm²/g, $K_{\text{RhIn}}=4.8$ cm²/g; (*b*), $K_{\text{InRh}}=12.5$ cm²/g, $K_{\text{RhIn}}=4.2$ cm²/g; (*c*) $K_{\text{InRh}}=11.5$ cm²/g, $K_{\text{RhIn}}=4.8$ cm²/g.

[12] Powers, Fink and Pegram, Phys. Rev. **49**, 650 (1936).
[13] B. Fig. 15.
[14] R. Jaeckel, Zeits. f. Physik **107**, 669 (1937).
[15] H. A. Bethe and G. Placzek, Phys. Rev. **51**, 450 (1937).
[16] W. E. Lamb, Phys. Rev. **55**, 190 (1939).
[17] In the case of the data these authors got with their "thin" detector (see their Fig. 2, lower curve), better agreement could be obtained by fitting a calculated curve more closely to the three experimental points at more than

The calculated mutual absorption curves are shown in Fig. 2. The best simultaneous fit (curves *a*) with the experimental points both of the Rh-In and the In-Rh absorption experiments was obtained with $K_{\mathrm{InRh}}=12.5$ cm^2/g, $K_{\mathrm{RhIn}}=4.8$ cm^2/g, $\Gamma_{\mathrm{In}}/\Gamma_{\mathrm{Rh}}=0.5_3$, $|E_{0\mathrm{In}}-E_{0\mathrm{Rh}}|/\Gamma_{\mathrm{Rh}}=1.1_4$.

To show the sensitivity of the shape of the curves to changes in the parameters, the curves (*b*) Fig. 2, were computed with $K_{\mathrm{InRh}}=12.5$ cm^2/g, $K_{\mathrm{RhIn}}=4.2$ cm^2/g, ($\Gamma_{\mathrm{In}}/\Gamma_{\mathrm{Rh}}=0.60$, $|E_{0\mathrm{In}}-E_{0\mathrm{Rh}}|/\Gamma_{\mathrm{Rh}}=1.26$) and the curves (*c*) with $K_{\mathrm{InRh}}=11.5$ cm^2/g and $K_{\mathrm{RhIn}}=4.8$ cm^2/g, ($\Gamma_{\mathrm{In}}/\Gamma_{\mathrm{Rh}}=0.49$, $|E_{0\mathrm{In}}-E_{0\mathrm{Rh}}|/\Gamma_{\mathrm{Rh}}=1.12$). Curves (*b*) and (*c*) show that the ratios of widths and spacing have an error of less than 10 percent. For comparison, the self-absorption curves of Fig. 1 are also reproduced in Fig. 2.

The final absolute values of the natural width and the spacings are $\Gamma_{\mathrm{Rh}}=0.13$ ev, $\Gamma_{\mathrm{In}}=0.07$ ev and $|E_{0\mathrm{In}}-E_{0\mathrm{Rh}}|=0.15$ ev. They are less accurate than the ratios because of their dependence on K_{th} which is not too accurately known. The spacing is in agreement with the difference in resonance energy as determined by the cadmium[18] and the less sensitive boron[19] absorption method.

From Eq. (1), we obtain $\Gamma_{\mathrm{In}}=0.063$ ev by inserting $K_{th}=0.74$ cm^2/g [5] and $E_0=1.0$ ev (the In resonance energy is known to be higher than the Rh resonance energy[19]). Since the above value of K_{th} is rather uncertain, the agreement can be considered as satisfactory and shows that the major portion of the thermal absorption of In is contributed by the one-volt level.

By inserting the data given in the preceding paragraphs into Eq. (2), we obtain for the neutron width $[1\pm1/(2j+1)]\Gamma_{N\mathrm{Rh}}=3.5\times10^{-4}$ ev and $[1\pm1/(2j+1)]\Gamma_{N\mathrm{In}}=1\times10^{-3}$ ev.

The deviation of the experimental points from the tail of the calculated curves which is also apparent in the self-absorption curves of Fig. 1 shows that some of our assumptions become invalid at smaller transmissions. Neglect of Doppler broadening in the derivation of Eqs. (5) and (8) should introduce only a very small error and would cause the observed transmissions to be *smaller* than the calculated ones at larger absorber thicknesses. Inadequacy of the *f* factors to account for the angular distribution gives a deviation from the computed data by less than 0.3 percent. The assumption, however, that only one level is responsible for the activity is open to doubt. In view of the fact that the average spacing between levels is theoretically expected to be of the order of 10 ev, and that experiments in the case of silver[19] and iodine[20] have confirmed that expectation, it is reasonable to assume that a small portion of the activity is contributed by levels of higher energies which should have a smaller K_s and would not contribute to the mutual absorption. A contribution of only 3 percent by these higher levels to the In activity would be sufficient to explain the discrepancy at the end of curve *a*, Fig. 1, and curve *a*, Rh-In, Fig. 2.

We wish to thank Mr. S. Hedelman for his help with the experiments. One of us (J. H.) gratefully acknowledges a grant from the Diamond Jubilee Fund.

40 percent transmission. However, the discrepancy with the absorption coefficient derived from the initial portion, particularly the first point, of their "thick" detector curve (their Fig. 2) is difficult to explain. The tail of this curve is already insensitive to changes in the absorption coefficient.

[18] H. H. Goldsmith and J. H. Manley, Phys. Rev. **51**, 382 (1937).

[19] H. H. Goldsmith and F. Rasetti, Phys. Rev. **50**, 328 (1936).

[20] J. Hornbostel and F. A. Valente, Phys. Rev. **55**, 108 (1939).