The Theory of the Diffuse Scattering of X-Rays from Crystals

We wish to discuss a recent paper by Zachariasen[1] on the theory of the diffuse scattering of x-rays by crystals. In this discussion we shall use the symbols and terms of Zachariasen's paper and we shall refer to an equation in his paper by placing a Z in front of the equation number, thus (Z5).

Following Zachariasen, we shall consider the unmodified (non-Compton) part of the scattered radiation from a small crystal of linear dimensions of the order 10^{-4} cm. Absorption effects in such a crystal may be neglected. We prefer to distinguish between the cases $l=l'$ and $l\neq l'$ in (Z5), (Z6) and (Z7) and we write the result in the form

$$\begin{aligned} J&=Sf^2\{N+e^{-2M}\Sigma'_l\Sigma'_{l'}\exp[2\pi i(\mathbf{k}-\mathbf{k}_0)\cdot\mathbf{r}_{ll'}]\exp P_{ll'} \\ &\backsimeq Sf^2\{N+e^{-2M}\Sigma'_l\Sigma'_{l'}\exp[2\pi i(\mathbf{k}-\mathbf{k}_0)\cdot\mathbf{r}_{ll'}] \\ &\quad+e^{-2M}\Sigma'_l\Sigma'_{l'}P_{ll'}\exp[2\pi i(\mathbf{k}-\mathbf{k}_0)\cdot\mathbf{r}_{ll'}]\}, \end{aligned} \quad (A)$$

where $\Sigma'\Sigma'$ denotes omission of those terms for which $l=l'$. Equation (A) holds both at and away from a Laue spot. It may be simplified to

$$\begin{aligned} J=J_0e^{-2M}+Sf^2\{N(1-e^{-2M}) \\ +e^{-2M}\Sigma'_l\Sigma'_{l'}P_{ll'}\exp[2\pi i(\mathbf{k}-\mathbf{k}_0)\cdot\mathbf{r}_{ll'}]\} \end{aligned} \quad (B)$$

where

$$J_0=Sf^2\Pi_i\frac{\sin^2[N^{\frac{1}{3}}\pi(\mathbf{k}-\mathbf{k}_0)\cdot\mathbf{a}_i]}{\sin^2[\pi(\mathbf{k}-\mathbf{k}_0)\cdot\mathbf{a}_i]}. \quad (C)$$

It is convenient to write (B) as

$$J=J_1+J_2 \quad (D)$$

as in (Z8). Away from the maxima of J_1, J_2 is the only term of importance, and we shall write it in the form

$$J_2=J_D+J_{\mathrm{FWZ}} \quad (E)$$

with

$$J_D=Sf^2N(1-e^{-2M}), \quad (F)$$

the Debye part, and

$$J_{\mathrm{FWZ}}=Sf^2e^{-2M}\Sigma'_l\Sigma'_{l'}P_{ll'}\exp[2\pi i(\mathbf{k}-\mathbf{k}_0)\cdot r_{ll'}], \quad (G)$$

the Faxen-Waller-Zachariasen part. For an isotropic crystal at a given temperature the Debye part is a function of $(\sin\phi/2)/\lambda$ alone, while the FWZ part is a function not only of $(\sin\phi/2)/\lambda$ but also of the orientation of the crystal relative to the primary rays. The FWZ part may be either positive or negative. Now it follows from (Z26) and (Z27) that

$$\langle J_{\mathrm{FWZ}}\rangle_{\mathrm{Av}}=0 \quad (H)$$

upon averaging over a sufficient range of orientations of the crystal. Thus unless precaution be taken to use a rather perfect crystal, *what one actually measures is the Debye diffuse scattering*. This is suggested by Zachariasen[1] but not, we believe, with the proper emphasis.

Further, in all of the work of Jauncey and his collaborators the technique of Jauncey and May[2] or the Jauncey-Bruce[3] modification of this technique has been used. The purpose of this technique has been to avoid any scattered radiation which is *orientation-sensitive*. As we now know, this technique would not only avoid Laue spots but in addition the peaks of the FWZ radiation which is also *orientation-sensitive*. In view of this, together with the fact that the crystals used were imperfect* we believe that Jauncey and his collaborators have come very close to determining the Debye part of the diffuse scattering.

In the theory as developed by Jauncey and Harvey[4–7] the simplifying assumption was made that the probability of a set of displacements of the atoms of a crystal was equal to the product of the probabilities of the separate displacements. This is only true if the displacements are independent of each other. In this case $P_{ll'}=0$ and the Faxen-Waller-Zachariasen part of the diffuse scattering becomes zero. However, as Zachariasen points out, this simplifying assumption is not justified for any real crystal and so we have the Faxen-Waller-Zachariasen part as well as the Debye part of the diffuse scattering.

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[1] W. H. Zachariasen, Phys. Rev. **57**, 597 (1940).
[2] G. E. M. Jauncey and H. L. May, Phys. Rev. **23**, 128 (1924); G. E. M. Jauncey and W. D. Claus, Phys. Rev. **46**, 941 (1934).
[3] G. E. M. Jauncey and W. A. Bruce, Phys. Rev. **50**, 413 (1936).
* Had imperfect crystals not been used this technique would give values *less* than the Debye formula.
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[5] G. E. M. Jauncey and G. G. Harvey, Phys. Rev. **37**, 1203 (1931).
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The Latitude Effect in Cosmic Rays at Far Southern Latitudes

Two meters for measuring the total cosmic-ray intensity from all directions, namely a single Geiger counter[1] and an electroscope of the type described by Millikan and Neher,[2] were operated on board the *U.S.S. North Star* during a recent trip through the Antarctic regions. The electroscope was operated on the Philadelphia-Panama-Dunedin, New Zealand-Little America run, while the counter set was operated on that from Dunedin to Little America-Valparaiso-Palmer Land-Valparaiso-Panama. The electroscope was inside a 10-cm thick lead shield, and the single Geiger counter was inside 7.5 cm of lead. The counter counts, approximately 190,000 per day, were scaled down with a conventional thyratron scale-of-four circuit[3] and then recorded on a dial recorder. The daily mean counting rate and the daily mean electroscope reading were corrected for changes in atmospheric pressure by applying the same barometer coefficient to each. The daily mean counting rate was then multiplied by an arbitrary constant to enable it to be shown on the same scale as the electroscope ionization. The corrected readings were then plotted against geomagnetic latitude and are presented in Fig. 1.

It will be observed that the cosmic-ray intensity shows the familiar "knee" at approximately geomagnetic latitude 38° south. Between the knee and 77° south the intensity rises by 3 ± 0.3 percent, this rise being shown by both the counter and the electroscope. At latitudes nearer the equator the curves obtained by the two meters are not