a uniform field equal to H. A change in the uniform field H must make an *equal* change in any average field acting upon the electrons. However, when I is kept constant and H is changed, no change is observed[2–4] in the transverse effects. This seems to indicate that the average field "seen" by the conduction electrons must be so large that changes due to H are negligible by comparison.

The second explanation would assume that the difficulty lies in the treatment of the interaction between electrons and the nuclei entirely in terms of the electron mean free path. Jones and Zener[8] have had considerable success in clearing up a similar discrepancy found in the change of resistance with magnetic field by more exact treatment of the interaction between electrons and nuclei. It is probable that both effects are responsible for the discrepancy.

The authors are indebted to L. A. Young for helpful advice and criticism.

[8] Jones and Zener, Proc. Roy. Soc. **A145**, 268 (1934).

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Fresnel Formulae Applied to the Phenomena of Nonreflecting Films

Katharine B. Blodgett
Research Laboratory, General Electric Company, Schenectady, New York

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The Fresnel formulae are applied to the calculations of the amplitudes of rays reflected and transmitted by nonreflecting films of transparent isotropic substances. Two cases are considered. In the first, the film of refractive index n_0 is bounded on both sides by a medium of refractive index n_0. In the second, the film is bounded by media of refractive indexes n_0 and n_2. It is shown how in the first case these formula lead to zero reflection when the thickness t of the film is given by $n_1t \cos r_1=(2n+2)\lambda/4$. In the second case for zero reflection the thickness must be given by $n_1t \cos r_1=(2n+1)\lambda/4$.

SEVERAL workers[1–3] have coated glass with films for the purpose of diminishing or extinguishing the reflection of light from the glass. Their published papers have given the following analysis of the requirements which must be met in order that a film shall reflect no light of wave-length λ. A film which meets these requirements is called a nonreflecting film.

There are two types of nonreflecting films made of transparent substances: *Type* (*a*). A film having a refractive index n_1 bounded on both sides by a medium of refractive index n_0. (Example: wall of a soap-bubble.) *Type* (*b*). A film having a refractive index n_1, bounded by media of refractive indices n_0 and n_2, where $n_0<n_1<n_2$. (Example: film of fluorite on glass.) The requirements for the extinction of reflected light are given by the following equations.

[1] J. Strong, J. Opt. Soc. Am. **26**, 73 (1936).
[2] K. B. Blodgett, Phys. Rev. **55**, 391 (1939).
[3] C. H. Cartwright and A. F. Turner, Phys. Rev. **55**, 595(A) (1939).

Thickness

The film must have a thickness t which is given by the equations

$$n_1t \cos r_1=(2n+2)\lambda/4 \text{ for Type (a)}, \quad (1)$$

$$n_1t \cos r_1=(2n+1)\lambda/4 \text{ for Type (b)}, \quad (2)$$

where r_1 is the angle of refraction of light in the film, and n has the values 0, 1, 2, $\cdots$.

Refractive Index of Film

Type (a). The film may have any value of refractive index.

Type (b). Case (1). Film viewed by perpendicular light, $i=0$. The film must have a refractive index which satisfies the equation

$$n_1^2=n_0n_2. \quad (3)$$

Case (2). Film viewed at an angle of incidence $i\neq 0$. The refractive index must satisfy one of

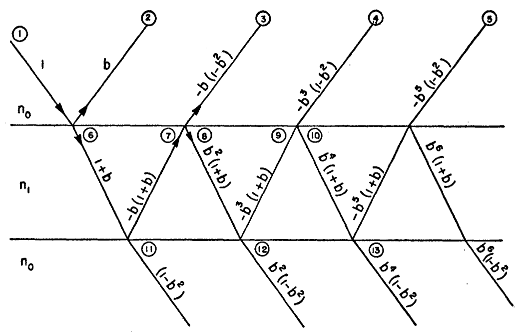

FIG. 1. Paths of a ray of light polarized perpendicular to the plane of incidence. The film of refractive index n_1 is bounded on both sides by a medium of refractive index n_0.

the following equations:

For the R_s ray $\quad \tan i \tan r_2 = \tan^2 r_1,$ (4)

For the R_p ray $\quad \sin^2 r_1 \cos^2 r_1 = \sin i \cos i \sin r_2 \cos r_2,$ (5)

where R_s and R_p are the polarized rays having the plane of polarization perpendicular and parallel, respectively, to the incident plane; and r_1 and r_2 are the angles of refraction of light in the media of refractive indices n_1 and n_2.

The present paper will apply the Fresnel formulae to calculations of the amplitudes of the rays reflected and transmitted by nonreflecting films of transparent isotropic substances. The Fresnel formulae are[4]

$$R_s = -E_s \frac{\sin(i-r)}{\sin(i+r)}, \qquad (6)$$

$$D_s = E_s \frac{2 \cos i \sin r}{\sin(i+r)}, \qquad (7)$$

$$R_p = -E_p \frac{\tan(i-r)}{\tan(i+r)}, \qquad (8)$$

$$D_p = E_p \frac{2 \cos i \sin r}{\sin(i+r)\cos(i-r)}, \qquad (9)$$

where E_s, R_s, D_s represent the amplitudes of the incident ray, reflected ray, and transmitted ray, respectively, for R_s-polarized light (as defined above), and E_p, R_p, D_p have corresponding values for R_p-polarized light.

Equations (6), (7), (8) and (9) are in agreement with the requirement imposed by energy relationships between the three rays. The energy of a light wave per unit volume is $na^2/8\pi$ where a is the amplitude of the wave. Therefore the energy per unit time of a wave having a wave front of cross section A and traveling with the velocity c is $Acna^2/8\pi$. From this we obtain the following value for the energy of the ray D,

$$D^2 = (E^2 - R^2) A_0 n_0 / A_1 n_1, \qquad (10)$$

where A_0 and A_1 represent the cross sections of the waves in the media in which the light is reflected and refracted, respectively. This equation may be written

$$D^2 = (E^2 - R^2) \cos i \sin r / (\cos r \sin i). \qquad (11)$$

The values of D_s, E_s and R_s from Eqs. (6) and (7) satisfy Eq. (11); also the values of D_p, E_p and R_p from Eqs. (8) and (9).

From Eqs. (6) and (7) we obtain the result

$$E_s + R_s = D_s. \qquad (12)$$

That is, the sum of the amplitudes of the incident and reflected rays on one side of a boundary is equal to the amplitude of the refracted ray on the opposite side of the boundary.

From Eqs. (8) and (9) we obtain

$$E_p - R_p = D_p \sin i / \sin r, \qquad (13)$$

which can be written $E_p - R_p = n_1 D_p / n_0$ where $n_1/n_0 = \sin i / \sin r$.

Figure 1 represents the paths which a ray of R_s-polarized light takes when it strikes a film of Type (a). The amplitude of the incident ray is taken as unity. The diagram shows the amplitude

TABLE I. *Amplitudes of reflected and transmitted rays for incident light polarized both perpendicular (R_s) and parallel (R_p) to the plane of incidence. The film of refractive index n_1 is bounded on both sides by a medium of refractive index n_0.*

RAY	R_s-POLARIZED	R_p-POLARIZED
1	1	1
2	b	d
3	$-b(1-b^2)$	$-d(1-d^2)$
4	$-b^3(1-b^2)$	$-d^3(1-d^2)$
5	$-b^5(1-b^2)$	$-d^5(1-d^2)$
6	$1+b$	$(1-d)n_0/n_1$
7	$-b(1+b)$	$-d(1-d)r'_0/n_1$
8	$b^2(1+b)$	$d^2(1-d)n_0/n_1$
9	$-b^3(1+b)$	$-d^3(1-d)n_0/n_1$
10	$b^4(1+b)$	$d^4(1-d)n_0/n_1$
11	$1-b^2$	$1-d^2$
12	$b^2(1-b^2)$	$d^2(1-d^2)$
13	$b^4(1-b^2)$	$d^4(1-d^2)$

[4] *Handbuch der Physik*, Vol. 20, p. 211.

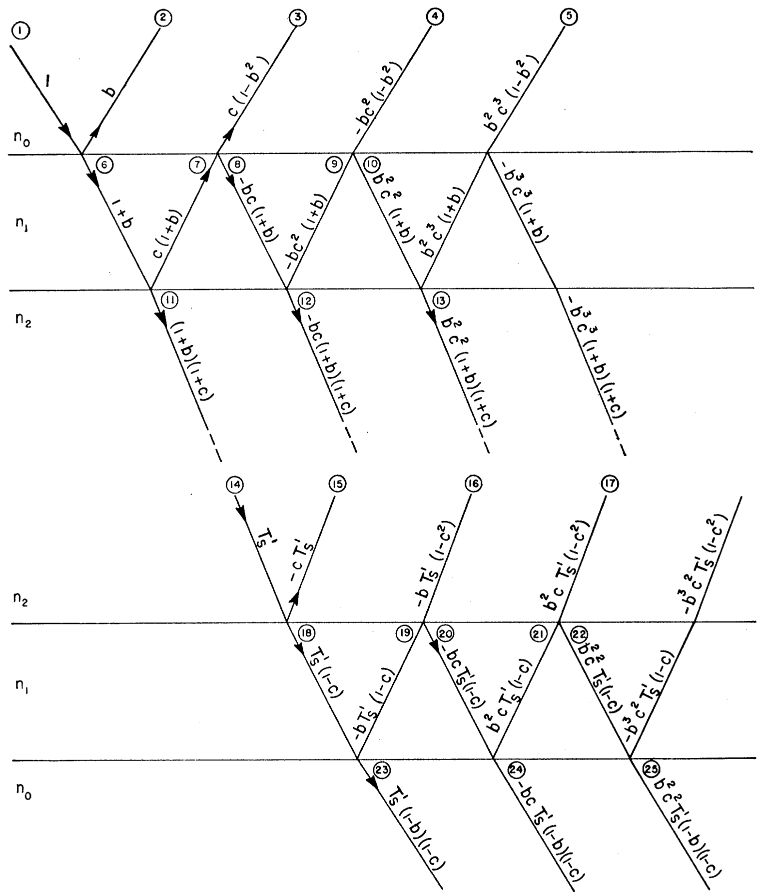

FIG. 2. Paths of a ray of light polarized perpendicular to the plane of incidence. The film of refractive index n_1 is bounded by media of refractive indexes n_0 and n_2.

of the fraction of the light which follows each path. Ray (1) is split into the following components.

(A) Ray (2) reflected from the upper surface of the film. The amplitude of (2) is $b=\sin(i-r)/\sin(i+r)$ from Eq. (6).

(B) Ray (6) refracted in the film. From Eq. (12) ray (6) has the value $1+b$.

(C) Ray (7) reflected from the second surface of the film, with the value $-b(1+b)$, since the values of R_s given by Eq. (6) are equal in value and opposite in sign for rays striking the inner and outer surfaces of a film of Type (a).

(D) Ray (8), and other successively reflected rays (9), (10) $\cdots$ having the values given on the diagram.

(E) Ray (3) refracted with the value $-b(1-b^2)$ calculated from rays (7) and (8) by Eq. (12). Other refracted rays are (4), (5) $\cdots$.

(F) Transmitted ray (11) calculated from rays (6) and (7) by Eq. (12). Other transmitted rays are (12), (13) $\cdots$.

When the thickness of a film of Type (a) is given by Eq. (1), the path difference between the successive reflected rays (2), (3), (4) $\cdots$ is λ, and also between the successive transmitted rays (11), (12), (13) $\cdots$. Therefore the amplitude of the sum of the rays is

$$\begin{aligned} R &= b-b(1-b^2)(1+b^2+n^4\cdots) \\ &= b-b(1-b^2)/(1-b^2) \\ &= 0. \end{aligned} \tag{15}$$

The amplitude of the sum of the transmitted rays is

$$\begin{aligned} T &= (1-b^2)(1+b^2+b^4\cdots) \\ &= (1-b^2)/(1-b^2) \\ &= 1. \end{aligned} \tag{16}$$

TABLE II. *Amplitudes of reflected and transmitted rays for incident light polarized both perpendicular (R_s) and parallel (R_p) to the plane of incidence. The film of refractive index n_1 is bounded by media of refractive indexes n_0 and n_2.*

RAY	R_s-POLARIZED	R_p-POLARIZED
1	1	1
2	b	d
3	$c(1-b^2)$	$e(1-d^2)$
4	$-bc^2(1-b^2)$	$-de^2(1-d^2)$
5	$b^2c^3(1-b^2)$	$d^2e^3(1-d^2)$
6	$1+b$	$(1-d)n_0/n_1$
7	$c(1+b)$	$e(1-d)n_0/n_1$
8	$-bc(1+b)$	$-de(1-d)n_0/n_1$
9	$-bc^2(1+b)$	$-de^2(1-d)n_0/n_1$
10	$b^2c^2(1+b)$	$d^2e^2(1-d)n_0/n_1$
11	$(1+b)(1+c)$	$(1-d)(1-e)n_0/n_2$
12	$-bc(1+b)(1+c)$	$-de(1-d)(1-e)n_0/n_2$
13	$b^2c^2(1+b)(1+c)$	$d^2e^2(1-d)(1-e)n_0/n_2$
14	T_s'	T_p'
15	$-cT_s'$	$-eT_p'$
16	$-bT_s'(1-c^2)$	$-dT_p'(1-e^2)$
17	$b^2cT_s'(1-c^2)$	$d^2eT_p'(1-e^2)$
18	$T_s'(1-c)$	$T_p'(1+e)n_0/n_1$
19	$-bT_s'(1-c)$	$-dT_p'(1+e)n_0/n_1$
20	$-bcT_s'(1-c)$	$-deT_p'(1+e)n_0/n_1$
21	$b^2cT_s'(1-c)$	$d^2eT_p'(1+e)n_0/n_1$
22	$b^2c^2T_s'(1-c)$	$d^2e^2T_p'(1+e)n_0/n_1$
23	$T_s'(1-b)(1-c)$	$T_p'(1+d)(1+e)$
24	$-bcT_s'(1-b)(1-c)$	$-deT_p'(1+d)(1+e)$
25	$b^2c^2T_s'(1-b)(1-c)$	$d^2e^2T_p'(1+d)(1+e)$

This means that when parallel rays of light of unit amplitude strike the surface of the film, the rays which leave the film also have unit amplitude since each ray is the sum of a series of rays, the sum being unity. The successive members of the series usually diminish very rapidly in value. For example, when light strikes a film of refractive index $n=1.50$ at an angle $i=30°$, the value of b calculated by Eq. (6) is 0.240. Therefore the rays (11), (12), and (13) $\cdots$ have the values 0.9422, 0.0545, 0.0031, 0.0002, $\cdots$. Table I gives the corresponding values of the amplitudes of the rays for R_p-polarized light. The value of d can be calculated from Eq. (8). In this case also the sum of the reflected rays is zero, and of the transmitted rays is unity.

Figure 2 gives the paths of the rays and their formulae for a film of Type (b). The formulae are listed in Table II. The values of b and d in the formulae can be calculated by means of Eqs. (6) and (8) for the upper boundary of the film which separates the media of refractive indices n_0 and n_1, and c and e for the lower boundary between n_1 and n_2.

When a film satisfies Eq. (2) the path difference between the successive reflected rays is $\lambda/2$. Therefore in writing the sum of the reflected rays (2), (3), (4), (5) $\cdots$, alternate members of the series must be written with the sign opposite to that which appears in Fig. 2 and in Table II. The sum R is

$$\begin{aligned} R &= b-c(1-b^2)(1+bc+b^2c^2+\cdots) \\ &= b-c(1-b^2)/(1-bc). \end{aligned} \tag{18}$$

Eqs. (4) and (5) were derived from Eqs. (6), (7), (8), (9) for the cases $b=c$ and $d=e$. When Eq. (4) is satisfied, Eq. (18) becomes

$$R=0. \tag{19}$$

The path difference between the rays (11), (12), (13) transmitted in the medium n_2 is also $\lambda/2$. Therefore the sum T_s' of these rays is

$$\begin{aligned} T_s' &= (1+b)(1+c)(1+bc+b^2c^2\cdots) \\ &= (1+b)(1+c)/(1-bc) \\ &= (1+b)/(1-b) \text{ when } b=c. \end{aligned} \tag{20}$$

For R_p-polarized light, the sum is

$$T_p' = n_0(1-b)/n_2(1+b). \tag{21}$$

Figure 2 shows the paths of the rays into which a ray T_s' [ray (14)] is split when it strikes the second boundary of the medium n_2. The sum of the rays (15), (16), (17) $\cdots$ is zero. The sum T of the rays (23), (24), (25) $\cdots$ is

$$T=T_s'(1-b)(1-c)/(1-bc).$$

When $b=c$ and $T_s'=(1+b)/(1-b)$ from Eq. (20), we have the result

$$T=1.$$

Similarly the equations for the case of R_p-polarized light give the result $T=1$.

The writer is indebted to Dr. F. Seitz for assistance in deriving some of the formulae in this paper.