ences. A direct comparison of the γ-rays from $Li^6(d,p)Li^7$ and $Be^9(d,\alpha)Li^7$ in the low energy region would be of interest.

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Internal Scattering of Gamma-Rays

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If an element is irradiated with gamma-rays of sufficiently high energy, the upper limit of the Compton recoils is less than the minimum energy of photo- or conversion electrons from any shell. But electrons of energy between these two limits may be ejected in processes in which the momentum condition can be relaxed. Such processes are (1) the scattering of an external gamma-ray by a bound electron, where momentum can be taken up by the nucleus, (2) the internal scattering by the electrons of the radioactive atom itself, where the radiation field of the near-by nucleus can fulfill momentum conditions impossible for a plane wave. We consider the second case, for scattering of an electric dipole gamma-ray by s electrons. We use Dirac electron theory with Born approximation. The process is of order α compared to the internal conversion, as expected. Our small result indicates that most of the electrons observed in such a region—for instance from the 2.62-Mev gamma-ray of Th C″—are of instrumental origin. This is in agreement with the results of the latest experiments.

I. Introduction

In the study of the electron energy spectrum of radioactive elements with the magnetic spectrograph, certain observers[1] found an unexpectedly large number of electrons in energy regions just below strong K-conversion lines. A particularly clear-cut case in point was that of the region 2.39–2.52 Mev in the spectrum of the thorium elements, in which really no electrons were expected. This is below the 2.62-Mev gamma-ray of ThC″, but near no other strong gamma-ray. Any electrons near 2.62 Mev and above the upper limit of the beta-ray spectrum at 2.25 Mev, must be indirectly produced by this gamma-ray.

The ordinary processes by which a gamma-ray can produce electrons are internal conversion, and photo-effect and Compton scattering in the material of the source. There is an upper limit to the energy that can be given a recoil electron in Compton scattering. In general this is $E_c = h\nu/(1+mc^2/2h\nu)$. The sharp K-conversion line sets a lower limit for both of the absorption processes: $E_K = h\nu - mc^2[1-(\alpha Z)^2]^{\frac{1}{2}}$. For any element E_K will be greater than E_c if only the gamma-ray energy, $h\nu$, is large enough. In the case mentioned this condition is fulfilled. In fact $E_K = 2.52$ Mev and $E_c = 2.39$ Mev. Thus, between 2.39 and 2.52 Mev, no electrons from these processes can appear. The problem was whether it was necessary to attribute the many electrons observed in the excluded region to unknown instrumental difficulties, or whether some other process for their production was actually involved.

Now, internal conversion and photo-effect are the only processes of first order in the interaction of matter and radiation; i.e., with a probability proportional to α, the fine structure constant. Any other process will be proportional to a higher power of α. Only one process seemed important here. This is internal scattering, which we shall

[1] Reported by Professor C. D. Ellis and F. Oppenheimer in discussion. Also W. J. Henderson, Proc. Roy. Soc. **A147**, 572 (1934).

define. It is of second order, proportional to α^2. It did not seem likely that this effect could account for all the observed electrons because α is only 1/137. We have calculated the probability of this effect in order definitely to decide about the instrumental origin of the unexpected electrons.

The two absorption processes internal conversion and photoelectric effect are closely related. Internal conversion is an analog to the photo-effect. However, the radiation is not a beam from an external source, but from the nucleus of the very atom whose electrons carry off the excitation energy which would otherwise go off as a gamma-quantum.[2] The chief difference between photo-effect and internal conversion is in the radiation field, which is a plane wave beam for photo-effect, but for internal conversion is the spherical wave of the nearby nuclear multipole. There is, of course, a process which differs from Compton scattering in the same way as internal conversion differs from photo-effect. This we call internal scattering. Here the excitation energy is not carried off by an electron alone, but is shared between an electron and a quantum. Just as the probability of Compton scattering is proportional to one higher power of the fine structure constant than the photo-effect, so internal scattering is related to conversion. Internal scattering has not previously been considered both because it is small compared to conversion, and because it does not yield any such direct information as do the sharp conversion lines, which give gamma-ray energies.

Internal scattering, unlike Compton effect, can give electrons in the energy range in question. The scattered quantum in the Compton effect cannot transfer an arbitrary fraction of its energy to the recoil electron because of the conservation of momentum. The incident plane wave has a well-defined momentum, and the electron is free. The total momentum must then be conserved between electron and quantum. The presence of a heavy body which can take up momentum without taking up energy will permit the transfer of any fraction of the gamma-ray energy to the recoil electron, and allow the emission of electrons in the excluded energy range. What we call internal scattering depends upon the absence of a unique momentum vector in the spherical field close to the nuclear multipole. Then the nucleus can take up momentum through the radiation field. We have not considered the effect of the Coulomb binding, which also allows the nucleus to take up momentum. That this omission of the binding is justified for a qualitative treatment of the internal scattering is shown by the experience with internal conversion, where calculations neglecting binding give a good estimate of the effect.

We have calculated the number of internally-scattered electrons as a function of electron energy for any gamma-ray energy, neglecting binding. We have considered scattering from s electrons only, since they have much the highest density at the nucleus. For gamma-ray energies safely above the threshold, the effect will be simply proportional to the electron density, and scattering from the s electrons in the K shell will give the bulk of the effect. We have found the formula only for electric dipole radiation. This is no essential restriction, since the dependence on multipole field is similar for scattering and conversion, this ratio, which is what is observed, will change little with multipole order. In the limit of high gamma-ray energy, $\nu\rightarrow\infty$, we get for the number of internally-scattered electrons in the range dE per electron internally converted in the K shell, for an electric dipole, neglecting binding:

$$\frac{N_E dE}{N_K}=\frac{\alpha}{\pi\nu^2}\left\{2\nu^2\left(\frac{\nu}{Q}-\frac{1}{\kappa}\right)+\frac{4}{\kappa}(2EQ-\nu^2)\ln\frac{\nu}{Q}-\frac{8E\nu}{\kappa}\ln\left[E-(E^2-1)^{\frac{1}{2}}\right]\right\}.$$

$Q=\nu-2E\kappa$. Where ν, κ and E are the energies of the initial and final quantum and the electron, respectively, in units of mc^2. We apply the above asymptotic formula to the example of ThC″. The gamma-ray is believed from conversion measurements to be an electric quadripole. The number of

[2] This should not be taken as an exact description of internal conversion. Actually each conversion electron represents, not the absorption of a gamma-ray, but an additional nuclear transition induced by the atomic electrons. The nucleus radiates essentially as though it were bare. Cf. H. M. Taylor and N. F. Mott, Proc. Roy. Soc. **A142**, 215–236 (1933).

internally scattered electrons turns out to be inappreciable. This is indeed in agreement with the absence of such electrons in the latest experiments.[3] The asymptotic expression gives about one electron scattered in the range in question for 30 in the K conversion. The exact formula gives half as many.

II. Calculations

We will find the rate of radiation of the emitted gamma-ray from the electron current in the transition from initial to final state. The energy radiated per second per unit solid angle with frequency $c\kappa/2\pi$ from a current distribution $\mathbf{S}(x, y, z, t)$ is:

$$R_\kappa d\Omega_\kappa = \frac{c\kappa^2}{2\pi}\left|\int d\mathbf{r}\mathbf{S}_\perp \exp\left[i\boldsymbol{\kappa}\cdot\mathbf{r}\right]\right|^2 d\Omega_\kappa \tag{1}$$

where $\mathbf{S}_\perp = (\mathbf{S}-\mathbf{n}(\mathbf{S}\cdot\mathbf{n}))$; $\mathbf{n}$ is a unit vector in the direction of radiation, and $\mathbf{r}$ is the position vector (x, y, z). We take the relativistic form of the current, appropriate to the energy range considered:

$$\mathbf{S} = e\psi^+\boldsymbol{\alpha}\varphi. \tag{2}$$

Here α is the vector whose components are the Dirac velocity matrices. ψ and φ are Dirac wave functions depending on r, t and spin. φ represents the electron in the final state, and ψ represents the initial state. We shall take the states normalized in a unit volume. We are interested in the electron transitions to a group of states in the energy range dE, the momentum direction being in a solid angle $d\Omega_E$. The number of these states is $(pE/h^3)dEd\Omega_E$. Therefore the energy radiated in $d\Omega_\kappa$ for electron transitions to $dEd\Omega_E$ is:

$$R_\kappa' d\Omega_\kappa d\Omega_E dE = \frac{c\kappa^2}{2\pi}\left(\frac{pE}{h^3}\right)\left|\int d\mathbf{r}\mathbf{S}_\perp \exp\left[i\boldsymbol{\kappa}\cdot\mathbf{r}\right]\right|^2 d\Omega_\kappa d\Omega_E dE. \tag{3}$$

We consider transitions through intermediate states which are plane waves. In the intermediate states the electron has energy E_1 and momentum $\hbar k_1$; in the final state, E and $\hbar k$. The wave functions are then:

$$\begin{aligned}\psi &= u_0 e^{-imc^2t} + \sum \frac{H_{01}u_1 \exp\left[-i\tau_1 E_1 t + i\boldsymbol{\kappa}_1\cdot\mathbf{x}\right]}{(\hbar\nu + mc^2) - E_1},\\ \varphi &= u_f \exp\left[iEt + i\boldsymbol{\kappa}\cdot\mathbf{x}\right] + \sum \frac{H_{1f}u_1 \exp\left[-i\tau_1 E_1 t + i\boldsymbol{\kappa}_1\cdot\mathbf{x}\right]}{E - (\hbar\nu + E_1)},\end{aligned} \tag{4}$$

where τ_1 is the sign of the energy of the intermediate state, and the u's are wave amplitudes for a state defined by a given value of spin and τ. The normalization to unit volume means that $\Sigma|u_s|^2 = 1$ where the subscript s refers to the components of the Dirac wave functions. We use the Born approximation. We take u_0 as $e^{-r/a}/(\pi a^3)^{\frac{1}{2}}$; $a = \hbar/mc\alpha Z$, and we shall treat a as large. H_{01} and H_{1f} are matrix elements of the perturbing Hamiltonian: $H = e(\varphi - \boldsymbol{\alpha}\cdot\mathbf{A})$, where φ and $\mathbf{A}$ are the scalar and vector potentials of the multipole field. We take the radiation field as that of an electric dipole oscillator of angular frequency ν, and moment D. (D will not have to be determined):

$$\mathbf{A} = \left(0, 0, D\frac{\nu}{r}\cos\left[\frac{\nu r}{c} - \nu t\right]\right), \quad \varphi = -\int dtc \operatorname{div}\mathbf{A} = -c\int \frac{\partial A_z}{\partial z}dt. \tag{5}$$

The matrix elements will contain integrals which are not simple δ-functions in momentum space as

[3] A. Alichanian and S. Nikitin, Comptes rendus de l'Acad. des Sciences U. S. S. R. **19**, 337 (1938).

they are for a plane wave radiation field. There occur the integrals:

$$\lim_{a\to\infty}\int d\mathbf{r}A_z \exp\left[i\boldsymbol{\kappa}_1\cdot\mathbf{x}-r/a\right]=a_1(e^{i\nu t}+e^{-i\nu t}),\quad \lim_{a\to\infty}\int d\mathbf{r}\varphi \exp\left[i\boldsymbol{\kappa}_1\cdot\mathbf{x}-r/a\right]=b_1(e^{-i\nu t}-e^{i\nu t}),\tag{6}$$

where

$$a_1=\frac{4\pi D\nu}{(\nu^2-k_1{}^2)};\quad b_1=\frac{(k_1)_z}{\nu}a_1.$$

When we put these into the expression (4) for the wave functions, and form the current we find for

$$\left|\int d\mathbf{r}\mathbf{S}_\perp \exp\left[i\boldsymbol{\kappa}\cdot\mathbf{r}\right]\right|$$

two time-dependent terms:

$$\mathbf{q}_1e^{-i\nu t}+\mathbf{q}_2e^{i\nu t}.$$

In squaring to get the rate of radiation we will get the proper result only if we make the replacement[4]

$$\left|\int d\mathbf{r}\mathbf{S}_\perp \exp\left[i\boldsymbol{\kappa}\cdot\mathbf{r}\right]\right|^2\rightarrow 2\mathbf{q}_1\cdot\mathbf{q}_2.$$

Doing this we find

$$R_\kappa' d\Omega_\kappa d\Omega_E dE=\frac{c\kappa^2}{2\pi}\left(\frac{pE}{h^3}\right)\mathbf{I}^+\cdot\mathbf{I}^- d\Omega_\kappa d\Omega_E dE,\tag{7}$$

where

$$\mathbf{I}^+=e^2\sum\int d\mathbf{r}\Bigg\{\exp\left[i(\mathbf{k}_1+\boldsymbol{\kappa})\cdot\mathbf{r}\right]\frac{\left[-a_1(u_0,\,\boldsymbol{\alpha}_\perp u_1)(u_1,\,\alpha_z u_f)-b_1(u_0,\,\boldsymbol{\alpha}_\perp u_1)(u_1,\,u_f)\right]}{E-(\hbar\nu+E_1)}$$
$$+\exp\left[-i(\mathbf{k}_1-\mathbf{k}-\boldsymbol{\kappa})\cdot\mathbf{r}\right]\frac{\left[-a_1(u_0,\,\alpha_z u_1)(u_1,\,\boldsymbol{\alpha}_\perp u_f)+b_1(u_0,\,u_1)(u_1,\,\boldsymbol{\alpha}_\perp u_f)\right]}{(\hbar\nu+mc^2)-E_1}\Bigg\},\tag{8}$$

$$\mathbf{I}^-(a_1,\,b_1)=\mathbf{I}^+(a_1,\,-b_1).$$

From the expressions (6) we saw how the presence of the dipole field allows the relaxation of the momentum conditions in inducing transitions to the intermediate states. Here the space integrals are δ-functions and give for the first term in (8) $\mathbf{k}_1=-\boldsymbol{\kappa}$ and for the second $\mathbf{k}_1=\mathbf{k}+\boldsymbol{\kappa}$. This expresses the conservation of momentum in that step of the double transition in which the final quantum is emitted. The conservation of momentum then reduces the sum over intermediate states to a sum over spins and signs of energy for a given momentum. This sum is easily done using the completeness theorems for the u's.[5] We are then left with the rate of radiation for transitions to states with a given final spin component. We must sum over the final spins and over the spins of the two K electrons which can make the transitions. This we do by introducing the projection operators for initial and final states and summing over both spin components and signs of the energy, thus taking the trace of the resultant matrix products.[5] We integrate over the angles for both the electron and the final quantum. We obtain

$$\left(\int\int R_\kappa' d\Omega_\kappa d\Omega_E\right)dE=(2/\pi)\alpha\kappa D^2Z^3W_E dE$$

where W_E is an expression too long to be reproduced here, but whose asymptotic value, $\nu\rightarrow\infty$ is:

$$\lim_{\nu\to\infty}W_E=\frac{2}{3}\left\{2\nu^2\left(\frac{\nu}{Q}-\frac{1}{\kappa}\right)+\frac{4}{\kappa}(2EQ-\nu^2)\ln\frac{\nu}{Q}-\frac{8E\nu}{\kappa}\ln\left[E-(E^2-1)^{\frac{1}{2}}\right]\right\};\quad Q=\nu-2E\kappa$$

[4] W. Heitler, *The Quantum Theory of Radiation* (Oxford, The Clarendon Press, 1936), p. 106.
[5] W. Heitler, reference 4, pp. 149–154.

ν, κ and E are energies of the initial quantum, final quantum, and electron in units of mc^2 and α is the fine structure constant 1/137. We divide this by κ to get the number of scattered quanta per second. This is also, of course, the number of recoil electrons per second. It is given in terms of D which is unknown. But the total number of electrons internally converted in the K shell for a dipole transition is also given in terms of D. It is[6]

$$N_K = \frac{4}{3}\left(\frac{\nu+2}{\nu}\right)^{\frac{1}{2}}(\nu^2+2)D^2Z^3\alpha^4.$$

Dividing the above expression for the number of electrons scattered per second in the energy range dE by this gives:

$$\frac{N_E dE}{N_K} = \frac{3\alpha}{2\pi}\left(\frac{\nu}{\nu+2}\right)^{\frac{1}{2}}\frac{W_E}{\nu^2+2}dE,$$

i.e., the number of electrons scattered in dE per electron internally converted. The above asymptotic expression for W_E gives a number about twice as large as that calculated from the more exact formula in the energy range 2.39–2.52 Mev for the 2.62-Mev gamma-ray. The average magnitude of the exact form of W_E in this range is about 800. This gives about 1/60 for the number of electrons scattered in this range per electron internally converted.

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[6] S. Dancoff and P. Morrison, Phys. Rev. 55, 122 (1939).

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The Oppenheimer-Phillips Process

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A critical study of the previous theoretical treatments of the process of neutron capture by heavy nuclei bombarded with deuterons shows that while the dependence of the transmutation function on the incident deuteron energy, W, has been given correctly, the energy distribution of the outgoing protons has not been satisfactorily estimated. In I, the application to the Oppenheimer-Phillips process of the usual formula for the cross section is shown to be justified for restricted values of the atomic number Z and of the deuteron energy. Bethe's method is used to express the partial cross section as a product of three factors: the penetrability of the potential barrier, the sticking probability of the neutron, and the energy transfer factor. In II, methods of obtaining the deuteron wave-function are discussed. Calculations of the transmutation function are extended to higher values of Z and W, and results obtained by using the O-P-Bethe and the Kapur methods are compared. In III, the proton energy distribution is discussed. A re-evaluation of the dependence of the transfer factor on the proton energy, K, leads to a result differing from Bethe's. The transfer factor is found to have a fairly sharp maximum, and to determine essentially the proton energy distribution. For high Z and low W the position and half-breadth of this maximum is given roughly by $K_0 \sim W$ and $\Delta K_0 \sim 3.3\ WZ^{-\frac{1}{2}}$. Lifshitz' and Kapur's treatments of proton energies are examined, and found to be unsatisfactory.

Introduction

A GENERAL theoretical treatment of the nuclear reactions induced by deuteron bombardment is made difficult by the fact that they are many-body processes to which ordinary perturbation methods are not applicable because of the strong short-range forces involved. An attempt to simplify the problem may be made by considering first the probabilities of the proton

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