Preliminary Experimental Study of New Diffraction Maxima in X-Ray Photographs*

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A revised theoretical treatment of the diffuse scattering of x-rays by crystals predicts a rapid variation of intensity with scattering angle, thus leading to intensity maxima in specific directions. The new diffraction maxima were observed with a rocksalt crystal and Cu $K\alpha$ radiation. The peak intensities and the half-widths were studied as functions of the direction of incidence. The observations were found to be in general agreement with theory.

IN a recent[1] paper a new intensity expression for the diffuse scattering of coherent x-rays by crystals has been found. According to this theory intensity maxima are to be expected in certain specific directions. The present article describes the results of experiments designed to test the theoretical predictions. These investigations are preliminary to a detailed study of the diffuse scattering which has been begun by one of us (S. S.).

The theory shows that a given intensity maximum becomes sharper and more intense as the directions of incidence approaches a Bragg direction. However, the maximum cannot be directly observed when the Bragg equation is satisfied because of coincidence with the much more intense Bragg maximum. The best condition for observing the new diffraction maximum is accordingly attained when the direction of incidence is made to deviate from the Bragg direction by an amount just sufficient to completely suppress the Bragg scattering.

* The results given in this article were presented at the Chicago Meeting of the American Physical Society, December 1, 1939.

[1] W. H. Zachariasen, Phys. Rev. **57**, 597 (1940).

EXPERIMENTAL PROCEDURE

An x-ray beam from a copper target tube was filtered by nickel foil, passed through a narrow slit (0.04 mm wide, 1 mm high and 30 mm deep) and allowed to fall on the cleavage plane of a stationary rocksalt crystal under a glancing angle of $\theta_B+\Delta$, where θ_B is the Bragg angle for the (200) reflection and Cu $K\alpha$ radiation. The scattered radiation was registered on a photographic plate placed normal to the incident beam. The slit was mounted normal to the plane of incidence and the intensity distribution in this plane near the Bragg scattering angle $2\theta_B$ was obtained from photometer recordings of the photographic plates. The angle was varied in the range from -80 to $+80$ minutes of arc.

In order to obtain a better definition of the orientation of the lattice planes the more heavily warped surface layers of the crystal were removed by etching. With this precaution the distribution function representing the orientation of the

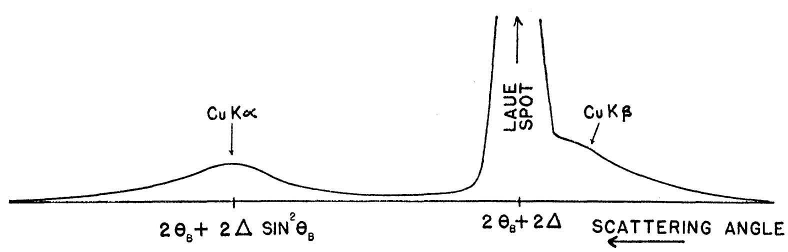

FIG. 1. Photometer tracing showing the intensity maximum due to Cu $K\alpha$ radiation diffusely scattered by a rocksalt crystal. Δ is $-68'$ and the half-width is $29'$. When the crystal is set at the Bragg angle the half-width of the Cu $K\alpha$ line (due to horizontal divergence) is only $1'$. The enhanced intensity maximum due to Cu $K\beta$ is seen close to the Laue spot. The exposure time for this photograph was 10 hours; for the sake of comparison it may be mentioned that the Cu $K\alpha$ line is easily observed after a few seconds exposure when Δ is zero so that Bragg scattering takes place.

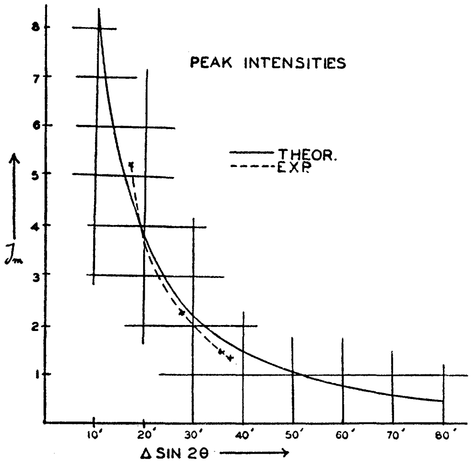

FIG. 2. Variation of peak intensity with $|\Delta|$.

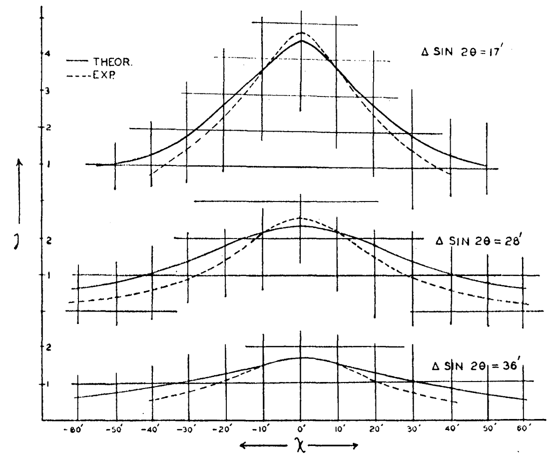

FIG. 3. Shape of the diffraction maximum for different values of $|\Delta|$.

various sections of the lattice planes about the mean normal showed a half-width of about two minutes of arc. But in order to completely get rid of the Bragg reflection of Cu $K\alpha$ in long exposed photographs, it was necessary to make $|\Delta|$ greater than 30′.

The filtered incident radiation naturally was not monochromatic; but the intensity of the continuous radiation was very small in comparison with that of the Cu $K\alpha$ line. In long exposed photographs the continuous radiation did give rise to intense Laue spots, while the diffuse scattering of the continuous radiation could be neglected.

Results

The direction of scattering corresponding to an intensity maximum of the diffuse radiation lies, according to theory, in the plane of incidence with a scattering angle of $2\theta_m = 2\theta_B + 2\Delta \sin^2 \theta_B$. If the incident beam is a narrow pencil the intensity near a maximum is given by the expression

$$J_2 = \frac{K}{(\Delta \sin 2\theta_B)^2 + \chi^2 + \varphi^2}, \qquad (1)$$

where χ and φ measure the angular deviations from the maximum in the plane of incidence and normal to it. As long as Δ, χ and φ remain small quantities, K may be treated as a constant. In our experiments a slit rather than a pinhole was used, so that Eq. (1) cannot be directly applied. Since the intensity measurements were made in the median plane of incidence, Eq. (1) must be integrated with respect to φ, and one finds accordingly

$$J_2 \propto \tan^{-1}(\varphi_m/y)/y\,; \quad y \equiv [(\Delta \sin 2\theta_B)^2 + \chi^2]^{\frac{1}{2}} \qquad (2)$$

with $\varphi_m = l/2R$. The height of the slit is l and R is the distance from crystal to point of observation. It is seen from Eq. (2) that the half-width of the intensity peak varies from $\Delta \sin 2\theta_B$ for a pinhole to $3^{\frac{1}{2}}\Delta \sin 2\theta_B$ for an infinitely tall slit. The actual value of φ_m was 65′ in our experiments.

Figure 1 shows the photometer tracing of a typical photograph. The observed variation of the peak intensity with $|\Delta|$ agrees well with

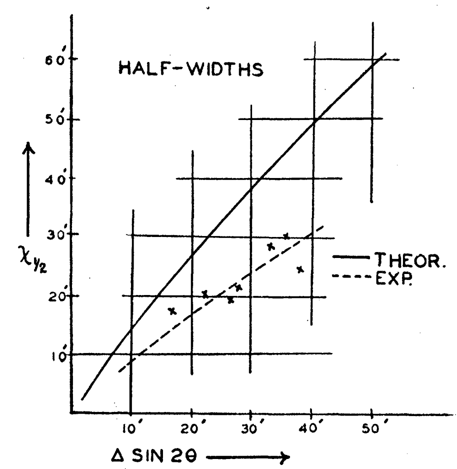

FIG. 4. Variation of half-width with $|\Delta|$.

theory as shown in Fig. 2. Fig. 3 gives the shape of the diffraction maxima. The increasing diffuseness with $|\Delta|$ is well illustrated; but the numerical values for the half-widths are—as Fig. 4 shows—smaller than the theory demands. This discrepancy may be real, but more likely is due to the limited accuracy of our photographic measurements.

More accurate and more extensive measurements, including observations at different temperatures and with different crystals, are of course desirable and are already under way. It may be mentioned that we have observed the new diffraction maxima with calcite crystals and that the maxima show the same general behavior as Δ is varied. Indeed, the theory of diffuse scattering should apply also to perfect crystals since the coupling interaction between incident and scattered radiation is weak when no Bragg scattering takes place.

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The Equations of Motion in Electrodynamics

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A formulation of the equations of motion of singularities in classical electromagnetic theory is obtained. The general method introduced by Einstein, Infeld and Hoffmann leads in a simple way, without any difficulty with "infinities," to the equations of motion obtained before by Dirac. It is shown further that Dirac's introduction of the inertial term into the equations of motion correspond in this method to the assumption of an energy-momentum tensor of matter. The attempt to remove this arbitrary assumption leads in a simple and natural way to the general theory of relativity, in which the equations of motion are obtained from the Einstein-Maxwell field equations.

1. Introduction

In Maxwell's theory the motion of charged particles represented as singularities of the field is not determined by the field equations. Dirac[1] has shown that the equations of motion are suggested by the conservation law for the electromagnetic energy-momentum tensor. By virtue of this law, it is demonstrated that the integral representing the flux of energy and momentum over a thin tube surrounding the world-line of a singularity (electron) depends only on the conditions at the ends of the tube. Then the equations of motion are obtained by assuming a simple expression for this flux. However, the integral takes an infinite value as the tube shrinks to the world-line, and Dirac is compelled to remove this difficulty artificially by equating the flux to an expression containing an infinite term and a finite term representing the product of mass and acceleration. This, and the formal complications of the paper, constitute its weak points.

The above procedure leads to familiar equations for the motion of electrons, but whereas these equations were formerly considered to be approximate, Dirac concludes that there is good reason to assume them exact.

The general method of obtaining equations of motion in the theory of relativity introduced by Einstein, Infeld and Hoffmann[2] appears at first sight fundamentally different from that of Dirac. It is shown here, however, that the former method can be adapted to the problem of motion in an electromagnetic field. The method involves only two-dimensional and not three-dimensional surface integrals, avoids the difficulties of "infinities," and leads in a simple manner, without the use of δ-functions, to the results obtained by Dirac. But we believe that the advantage of the

[1] P. A. M. Dirac, Proc. Roy. Soc. **A167**, 148 (1938).

[2] Einstein, Infeld and Hoffmann, Ann. Math. **39**, 65 (1938).