these moments are $\mu_{71}=2.5$ and $\mu_{69}=2.0$. The agreement in the ratios is excellent and the values of the moments agree as well as this method of calculation allows. The values of the moments one gets from the ${}^{2}P_{3/2} \Delta \nu$'s are too small by a factor 1.36. This number is obtained by taking the ratio of the expected $\Delta \nu$'s from the Goudsmit formula and comparing it with the experimental results. However, from the considerations given above, the diminished $\Delta \nu$ may be due to the effect of a perturbation by a higher configuration.

The experiment on gallium depended, as was seen, very critically on the ratios of the zeromoment peaks. We had occasion on this same apparatus to measure the ratio of the sodium and lithium hyperfine $\Delta \nu$'s given directly by the ratio of the zero-moment peaks. We obtained for this ratio $\Delta \nu_{\rm Na}/\Delta \nu_{\rm Li} = 2.2098 \pm 0.0035$. Millman, Kusch and Rabi,¹⁶ using the new and veryaccurate molecular beam resonance method, obtained the value $\Delta \nu_{Na}/\Delta \nu_{Li} = 2.2052 \pm 0.0002$. The discrepancy of $\frac{1}{5}$ of 1 percent implies a systematic error of this magnitude in the zeromoment experiments. We believe, therefore, that this gives a good indication of the precision of the apparatus in locating zero-moment peaks.

The author wishes to thank Professor I. I. Rabi for suggesting this application of atomic beams to the measurement of quadrupole moments. He also wishes to express his great indebtedness to Mr. D. R. Hamilton for his continuously invaluable assistance during the course of the research. The other members of the molecular beam laboratory were generous in making available their valuable experience.

¹⁶ The author is indebted to these authors for the communication of this result before its publication.

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The Radiofrequency Spectra of Atoms

Hyperfine Structure and Zeeman Effect in the Ground State of Li⁶, Li⁷, K³⁹ and K⁴¹*

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The molecular beam magnetic resonance method previously used for the study of molecules has been extended to the study of atoms. Transitions between the members of hyperfine structure multiplets of the ground state of atoms have been observed directly. In this way the hyperfine structure intervals of the normal states of Li⁶, Li⁷, K³⁹ and K⁴¹ have been measured. Since the measurement of frequency alone is involved, the results are of very high precision. These spectra have been observed in external magnetic fields varying from 0.05 to 4000 gauss, i.e., from the ordinary Zeeman to the complete Paschen-Back region. The lines in the pattern are completely resolved even at the low fields. The h.f.s. separations derived from measure-

INTRODUCTION

 $\mathbf{B}^{\mathrm{ECAUSE}}$ of the existence of nuclear spin the ground states of many atoms consist of a set of closely spaced energy levels. Each level of this

ments at different fields are in excellent agreement. Comparison is made between the ratio of the nuclear moments of Li^{7} and Li^{6} as derived from the h.f.s. measurements with the directly measured ratio. The two ratios are 3.9610 and 3.9601, respectively. They agree within the experimental error of 0.04 percent. The h.f.s. separations are given both in absolute frequency units and in wave numbers.

	sec. ⁻¹ ×10 ⁻⁶	cm ⁻¹
Li ⁶	228.22	0.007613
Li ⁷	803.54	0.026805
K ³⁹	461.75	0.015403
K^{41}	254.02	0.008474

hyperfine structure multiplet corresponds to a value of the total angular momentum of the atom. The spacings are caused chiefly by the feeble interactions of the magnetic and electric fields of the electrons with the nuclear magnetic moment and the electrical quadrupole moments,

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respectively. Since the magnitudes of these interactions depend on the angle between the angular momentum of the nucleus and that of the electronic configuration, the states of different total angular momentum, F, will differ in energy.

Left to themselves, the atoms would radiate this energy in the form of electromagnetic radiation of frequency given by the Bohr formula,

$$W_F - W_{F'} = h\nu \tag{1}$$

and settle down to their lowest energy state. Since the different F states correspond to different orientations of the atomic magnetic moment, this radiation has the character of magnetic dipole radiation.

The region of frequency in which these radiations are emitted lies in the rapidly developing upper end of the technical radiofrequency range, as most hyperfine structures of ground states which have been observed are included between the limits of 0.005 cm^{-1} and 0.4 cm^{-1} , i.e., between frequencies of 1.5×10^8 and 1.2×10^{10} sec.⁻¹ or between wave-lengths of 200 and 2.5 cm. Because of these low frequencies the lifetime of a hyperfine structure level is very long and the intensity of spontaneous emission very feeble. Direct observation of this radiation would be very difficult.

However, it is possible to illuminate the atom with electromagnetic radiation of the correct frequency and of such intensities as to cause it to absorb, or, by the Einstein process of stimulated emission, to emit a quantum of this frequency in the reasonably short time of about 10^{-4} second. If such a process is detected it offers a direct method of measuring hyperfine structure. Such a method has many important advantages over existing optical methods. Firstly, the results are simple to interpret since we are concerned with only one atomic energy level; secondly, the accuracy is very high because only the measurement of the frequency of a radio wave is involved: thirdly, it is possible to measure extremely small energy separations. It is, of course, clear that similar considerations apply to all metastable states which have sufficiently long lifetimes, as well as to the ground states of atoms.

The problem of the detection of the induced transitions between the hyperfine structure levels

of the atom has been solved by the utilization of atomic beams in a manner similar to that of the molecular-beam magnetic-resonance method.¹

In practice these methods have proved superior to the older optical methods as well as to the atomic beam method of zero moments. Measurements of hyperfine structure with an accuracy of 0.005 percent are made without difficulty. The high resolution which can be obtained is exemplified by the fact that it was possible to observe a clearly resolved Zeeman pattern of a hyperfine structure line in a magnetic field of 0.05 gauss, and to obtain half-widths of observed lines of 10^{-6} cm⁻¹.

The principal physical question to which we have applied these methods is the accurate measurement of the hyperfine structures of Li7 and Li⁶ and the comparison of the ratio of the nuclear moments of Li7 and Li6 deduced from such measurements with that obtained from the moment values directly measured by the molecular-beam magnetic-resonance method.¹ Present theory accounts for the h.f.s. of an atomic energy level by the assumption that h.f.s. is due solely to the magnetic interaction of the nuclear moment with the external electrons. On the basis of this assumption, the ratio, $(\Delta \nu)_1/(\Delta \nu)_2$, of the h.f.s. separations of a given atomic energy state of two isotopes should yield the ratio, μ_1/μ_2 , of the nuclear moments, in a manner which does not involve the electronic wave functions since they are the same for the two isotopes. If there are nonelectromagnetic interactions between the electrons and the nucleus, the directly measured ratio of moments may be expected to differ from that obtained from h.f.s. They would be the same only if the additional interactions were proportional to the magnetic moment, which is hardly likely. Since the two nuclei (Li⁶ and Li⁷) differ widely in structure it seems that this case is a particularly favorable one for searching for nonelectromagnetic interactions, particularly since the ratio of the moments of the lithium isotopes is accurately known from direct measurements.

This question has already been discussed in an earlier paper¹ where a comparison was made between the directly measured moment ratio and

¹ I. I. Rabi, S. Millman, P. Kusch and J. R. Zacharias, Phys. Rev. 55, 526 (1939).

the ratio obtained by Manley and Millman² from the h.f.s. ratio measured by the atomic-beam zero-moment method. The ratio of moments as determined from h.f.s. was subject to considerable uncertainty, and the discrepancy of about 2 percent between the two ratios, although outside the experimental error, was not considered sufficiently great to give conclusive indication of a real physical effect.



FIG. 1. The magnetic levels, at low field. of the h.f.s. doublet of a ${}^{2}S_{4}$ state for a nuclear spin of 3 and the transiallowed between tions these levels. The lower part of the figure indicates the spectrum resulting from these transitions.

THEORY OF THE EXPERIMENT

The levels which compose an h.f.s. multiplet are, by the rules of quantum mechanics, characterized by total angular momenta, F=i+J, $i+J-1, i+J-2, \cdots$, where *i* is the nuclear and J the total electronic angular momentum. The lowest F is J-i if J>i and i-J if i>J. Transitions between the levels are governed by the selection rules for magnetic dipole radiation,

$$\Delta F = 0, \pm 1; \quad \Delta m = 0, \pm 1,$$

where *m* is the magnetic quantum number.

Only in the absence of an applied magnetic field will the transition |F-F'|=1 result in a single line. In a magnetic field the Zeeman effect of this line will be observed. For sufficiently weak fields the Zeeman pattern can be calculated in the usual way by the Landé g formula:

$$g_{F} = g_{J} \frac{F(F+1) + J(J+1) - i(i+1)}{2F(F+1)} + g_{i} \frac{F(F+1) - J(J+1) + i(i+1)}{2F(F+1)}.$$
 (2)

Neglecting the second term and restricting ourselves to atoms for which $J = \frac{1}{2}$ we have for the two F states, $i + \frac{1}{2}$ and $i - \frac{1}{2}$,

$$g_{i+1} = \frac{g_J}{2i+1}$$
 and $g_{i-1} = -\frac{g_J}{2i+1}$, (3)

respectively. The weak field Zeeman lines should therefore be found at frequencies:

$$\nu = \frac{W_F - W_{F-1}}{h} + (m_F + m_{F-1}) \frac{g_J}{2i+1} \frac{\mu_0 H}{h}.$$
 (4)

For ${}^{2}S_{1}$ states $(g_{J}=2)$ we have two sets of lines corresponding to the π and σ lines of spectroscopy. The π lines, for which $\Delta m = \pm 1$, have frequencies,

$$\nu = \Delta \nu + \frac{2m \pm 1}{i + \frac{1}{2}} \frac{\mu_0 H}{h} \tag{5}$$

and the σ lines, for which $\Delta m = 0$, have frequencies,

$$\nu = \Delta \nu + \frac{2m}{i+\frac{1}{2}} \frac{\mu_0 II}{h},\tag{6}$$

where in (5) and (6), $\Delta \nu = (W_F - W_{F-1})/h$ and is the frequency corresponding to the energy difference between the two F states, $i+\frac{1}{2}$ and $i-\frac{1}{2}$, (expressed in sec.⁻¹) at zero magnetic field.

The π lines represent transitions caused by the component of the oscillating magnetic field perpendicular to the constant field, H, and the σ lines arise from the component parallel to H. This is opposite to the usual assignment because we are dealing with magnetic dipole radiation.

As an example we shall consider the magnetic levels of the h.f.s. multiplet of a ${}^{2}S_{4}$ state for a nuclear spin of $\frac{3}{2}$. These are indicated in Fig. 1 by means of the customary energy level diagram. The transitions which give rise to the π lines are represented by solid lines and those which give rise to the σ lines by means of dotted lines. The spectrum is indicated in the lower portion of the

² J. H. Manley and S. Millman, Phys. Rev. 51, 19 (1937).

diagram. The energy separation between adjacent magnetic levels is $\mu_0 H/(i+\frac{1}{2})$ and the frequency difference between two adjacent π lines as well as between two adjacent σ lines is $2\mu_0 H/[(i+\frac{1}{2})h]$.

It may be seen by use of Eqs. (5) and (6) or by reference to Fig. 1 that the frequency separation of the two *F* levels in an h.f.s. doublet at zero field, $\Delta \nu$, is given by the mean of the frequencies of all the lines in the Zeeman pattern at a low magnetic field, by the mean of all the π lines, by the mean of all the σ lines, or by the mean frequency of any symmetrical pair of lines.

When $\Delta F = 0$, the frequencies of the lines are given by,

$$\nu = \frac{1}{i + \frac{1}{2}} \frac{\mu_0 II}{h}$$
(7)

since $\Delta m = \pm 1$. However we shall present no experiments to fit this case in the weak field region.

In the region of high magnetic field $(\mu_0 H/h \gg \Delta \nu)$ the total angular momentum, F, is no longer constant and a more suitable description is given in the manner customarily employed for the Paschen-Back effect, i.e., in terms of m_i and m_J , the components in the direction of H of the nuclear and electronic angular momenta, respectively. The energy of the states is given³ by the formula,

$$Wm_{i}m_{J} = m_{i}g_{i}\mu_{0}H + m_{J}g_{J}\mu_{0}H + [\Delta W/(i+\frac{1}{2})]m_{i}m_{J}.$$
 (8)

The selection rules for the transitions are $\Delta m_i = \pm 1$, $\Delta m_J = 0$ and $\Delta m_i = 0$, $\Delta m_J = \pm 1$. From this equation it is clear that at large values of Hthe transitions for which $\Delta m_i = \pm 1$, $\Delta m_J = 0$ give rise to lines whose frequencies become independent of the field and approach the limiting value $\Delta \nu/(2i+1)$, since the term $m_i g_i \mu_0 H$ is still relatively small in comparison with the term $\left[\Delta W/(i+\frac{1}{2})\right]m_im_J$ for the highest fields used in our experiments (about 6000 gauss). This happy circumstance is very important for these experiments because it is then possible to measure $\Delta \nu$ with the use of frequencies which lie in the region of $\Delta \nu/(2i+1)$. One can thus avoid some of the technical difficulties involved in the use of extremely high frequencies.

The preceding considerations are only approximate. To utilize the full power of our experimental method one must employ the exact theory of the energy levels of an h.f.s. multiplet in a magnetic field. We shall follow the customary usage by defining the Landé g factor as the negative ratio of the magnetic moment, expressed in units of μ_0 , the Bohr magneton, to the angular momentum expressed in units of $h/2\pi$. The g_J values of the ground states of all the alkali atoms are, therefore, positive, while the g_i values of the alkali atoms are negative, since the moments are positive. The energies of the magnetic levels as a function of the field are given⁴ by the formulae:

$$W_{i+\frac{1}{2},m} = -\frac{\Delta W}{2(2i+1)} + g_{i}\mu_{0}Hm + \frac{\Delta W}{2} \left(1 + \frac{4m}{2i+1}x + x^{2}\right)^{\frac{1}{2}}$$
(9)

for $F = i + \frac{1}{2}$; $m = i - \frac{1}{2}$, $i - \frac{3}{2}$, $\cdots - (i - \frac{1}{2})$,

$$W_{i+1,\pm(i+1)} = -\frac{\Delta W}{2(2i+1)} \pm g_i \mu_0 H(i+\frac{1}{2}) + \frac{\Delta W}{2}(1\pm x) \quad (10)$$

for $F = i + \frac{1}{2}$; $m = \pm (i + \frac{1}{2})$,

.

$$W_{i-\frac{1}{2},m} = -\frac{\Delta W}{2(2i+1)} + g_{i}\mu_{0}Hm -\frac{\Delta W}{2} \left(1 + \frac{4m}{2i+1}x + x^{2}\right)^{\frac{1}{2}}$$
(11)

for $F = i - \frac{1}{2}$; $m = i - \frac{1}{2}$, $i - \frac{3}{2}$, $\dots - (i - \frac{1}{2})$, where the parameter *x* in Eqs. (9), (10) and (11) is defined by

$$x = (g_J - g_i)\mu_0 H / \Delta W. \tag{12}$$

Figs. 2a and 2b exhibit graphically the behavior of the ratio $W_m/\Delta W$ as a function of the parameter x for nuclear spins of $\frac{3}{2}$ and 1, respectively. For weak fields where x^2 and g_i may be neglected, Eqs. (9)–(12) yield the same result as Eqs. (5) and (6).

By taking the differences of the energies of the

³ E. Back and S. Goudsmit, Zeits. f. Physik 47, 174 (1928).

⁴S. Millman, I. I. Rabi and J. R. Zacharias, Phys. Rev. 53, 384 (1938).



FIG. 2. The variation with magnetic field of the energy of magnetic levels of a normal h.f.s. doublet of a ${}^{2}S_{1}$ state. (a) describes the case when $i=\frac{3}{2}$ and (b) the case when i=1. The solid lines correspond to the levels arising from the state $F=i+\frac{1}{2}$ and the dotted lines to the levels arising from the state $F=i-\frac{1}{2}$.

states between which transitions are allowed and dividing by h, the frequencies of the lines as a function of the field are obtained. These frequencies are shown in Figs. 3 and 4 in units of $\Delta \nu$, for nuclear spins of $\frac{3}{2}$ and 1, respectively, in the region of the weak field and intermediate field Zeeman effect.

The values of the field, H, corresponding to any value of x may be found from Eq. (12), if

 $\Delta \nu$ is known. The values of H in Fig. 3 are appropriate to the case of K³⁹. However, the curves apply equally well to any atom in a ${}^{2}S_{4}$ state whose nuclear spin is $\frac{3}{2}$ if a suitable conversion is made from the field parameter x to the field H. Similarly the values of the field in Fig. 4 correspond to the case of Li⁶. The curves apply to the case of any atom in a ${}^{2}S_{i}$ state whose nuclear spin is 1 for an appropriate conversion of the parameter x to the field. The full lines represent the $\pi(\Delta m = \pm 1)$ components and the dotted lines the $\sigma(\Delta m = 0)$ components. The approximate equations (5) and (6) which describe the Zeeman pattern at weak fields give the frequencies of the lines only in that range of the field for which the center of gravity of the Zeeman pattern lies very near to $\Delta \nu$. The center of gravity shifts to higher frequencies for increasing values of x. This corresponds to the Zeeman effect at intermediate fields and Eqs. (9)-(12) must be used to describe this case. Terms in g_i may be neglected in calculations of the Zeeman effect in the weak and intermediate field regions, since at these fields the interaction energy of the nuclear moment with the applied field is small compared to other interaction energies. The two lines $(F, m) \Leftrightarrow (F-1, m-1)$ and $(F, m-1) \Leftrightarrow (F-1, m)$ are represented by a single line in Figs. 3 and 4. Actually these lines constitute a close doublet, except in the case $m = \pm (i + \frac{1}{2})$, whose frequency separation is $2g_{i\mu_0}H/h$. This energy cannot be indicated on the scale of our curves, nor could this doubling be experimentally observed at the very small fields at which we observed the Zeeman effect of the line |F - F'| = 1.

The convergence of the frequencies of all the lines for which $\Delta m_i = \pm 1$, $\Delta m_J = 0$, to a value independent of the field can be seen from Figs. 2a and 2b. Table I lists the transitions for the case of a nuclear spin of $\frac{3}{2}$. This case is chosen because it applies to the experiments described in this paper; the extension to other cases can easily be made. In designating the states involved in a transition, the first number in the parenthesis denotes the *F* value of the state, and the second number, the *m* value. This designation does not imply that these states are characterized by a definite total angular momentum, but rather that each of these states is derived from a

definite (F, m) state by an "adiabatic" transformation. The expressions for the frequencies of the lines in Table I are derived from Eqs. (9)–(11) and are valid for all values of x. However, when $x^2 \gg 1$ a simple expansion of the terms in the frequency expressions indicates that the frequencies approach limiting values of $\Delta \nu/4$ independent of field, except for the term $g_{i\mu_0}H/h$. In the general case the limiting frequency is $\Delta \nu/(2i+1)$.



FIG. 3. The frequencies of the components of the Zeeman pattern of the line $|\Delta F| = 1$ as a function of the magnetic field. The values of x are appropriate to any atom in a ${}^{2}S_{4}$ state with a spin of $\frac{3}{2}$, but the indicated values of *H* correspond to the case of K³⁰ only.

Figures 5 and 6 give the frequencies of the lines, in units of $\Delta \nu$, associated with the transitions $\Delta m_i = \pm 1$, $\Delta m_J = 0$ as a function of the field parameter x, for K³⁹ and Li⁷, respectively. The curves have been calculated by use of the expressions in Table I. The two sets of curves are different for the two atoms, even though their nuclear spins are the same, because the term $g_{i\mu_0}H/h$, though small, is not negligible and has a different value for each nucleus. The values of g_i which have been used in these calculations are $-0.260/1837^{5}$ and $-2.167/1837^{1}$ for K³⁹ and Li⁷, respectively. The range of x covered in each of these figures is such as to include an experimentally convenient range of field, and the indicated values of field are appropriate only to the one nucleus in question. Since x itself may be expressed in terms of the field through the use of Eq. (12), $\Delta\nu$ can be calculated from the frequency observed for any one transition at some known field. The value of g_i need not be known with great precision since the contribution of the term $g_i\mu_0H/h$ to the total frequency is small. Moreover, the magnetic field need not be accurately determined since the transition frequencies vary slowly with field for large values of H, and in certain regions are practically independent of field.

For each transition $(F, m) \Leftrightarrow (F, m-1)$, there will be a pair of lines, one arising for $F=i+\frac{1}{2}$ and the other arising for $F=i-\frac{1}{2}$, provided only that m does not have the value $i+\frac{1}{2}$ and that m-1does not have the value $-(i+\frac{1}{2})$, since the m values $\pm (i + \frac{1}{2})$ do not exist for the state $F = i - \frac{1}{2}$. At a fixed field the expression for the mean frequency of each pair of such lines does not depend on the value of the nuclear g factor. These pairs of lines are identified in Figs. 5 and 6 by the fact that at low fields the two members of the pair have very nearly equal frequencies. The line originating in the state $F=i+\frac{1}{2}$ is indicated by a full line, and that originating in the state $F=i-\frac{1}{2}$ by a dotted line. Regardless of the value of the nuclear spin all of the lines arising from the transitions $\Delta m_i = \pm 1$, $\Delta m_J = 0$, except the lines $(i+\frac{1}{2}, -i-\frac{1}{2}) \Leftrightarrow (i-\frac{1}{2}, -i+\frac{1}{2})$ and $(i+\frac{1}{2}, -i+\frac{1}{2})$ $i+\frac{1}{2}$ \Leftrightarrow $(i+\frac{1}{2}, i-\frac{1}{2})$ may be arranged in such pairs. It may be seen by reference to Table I that the sum of the mean frequencies of all the pairs of lines added to the sum of the frequencies of the single lines is identically equal to $\Delta \nu$, for a constant value of x. This is true not only for our case where $i=\frac{3}{2}$, but for any atomic system described by Eqs. (9)-(12). This property permits the determination of Δv even if neither g_i

TABLE I. The transitions $\Delta m_i = \pm 1$, $\Delta m_J = 0$ for an atom in a 2S_1 state whose nuclear spin is $\frac{3}{2}$, and the expressions for the frequencies of these lines.

TRANSITIONS	Expression for frequencies
$(2,2) \leftrightarrow (2,1)$	$\frac{1}{2}\Delta\nu[(1+x)-(1+x+x^2)^{\frac{1}{2}}]+g_i\mu_0H/h$
$\begin{array}{c} (2,1) \leftrightarrow (2,0) \\ (1,1) \leftrightarrow (1,0) \end{array}$	$ \frac{\frac{1}{2}\Delta\nu[(1+x+x^2)^{\frac{1}{2}}-(1+x^2)^{\frac{1}{2}}]+g_i\mu_0H/h}{\frac{1}{2}\Delta\nu[(1+x+x^2)^{\frac{1}{2}}-(1+x^2)^{\frac{1}{2}}]-g_i\mu_0H/h} $
$\begin{array}{c} (2,0) \Leftrightarrow (2,-1) \\ (1,0) \Leftrightarrow (1,-1) \end{array}$	$\frac{\frac{1}{2}\Delta\nu[(1+x^2)^{\frac{1}{2}} - (1-x+x^2)^{\frac{1}{2}}] + g_i\mu_0H/h}{\frac{1}{2}\Delta\nu[(1+x^2)^{\frac{1}{2}} - (1-x+x^2)^{\frac{1}{2}}] - g_i\mu_0H/h}$
$(2, -2) \Leftrightarrow (1, -1)$	$\frac{1}{2}\Delta\nu[(1-x+x^2)^{\frac{1}{2}}-(1-x)]-g_i\mu_0H/h$

⁵ P. Kusch, S. Millman and I. I. Rabi, Phys. Rev. 55, 1176 (1939).

nor the value of the field is known, by the simple expedient of observing the frequencies of all the lines at some fixed field and making the appropriate summation.

If the field is known the value of g_i can be found from the frequency separation of the members of each pair of lines. Such a determination will not be very accurate since we are measuring the rather small frequency difference, $2g_{i\mu0}H/h$, between two lines whose total frequency is large compared to this difference. With the molecular-beam magnetic-resonance method for determining nuclear g values, the quantity $g_{i\mu0}H/h$ was measured directly as a first-order effect and could, therefore, be determined much more precisely than in the present experiments.

Method

In order to detect the transitions between the energy levels, produced by the oscillating magnetic field, we utilize the change in atomic magnetic moment produced by the transitions. As in the molecular-beam magnetic-resonance method, the atoms are deflected in an inhomogeneous magnetic field, then pass through a homogeneous field and are finally deflected in the opposite direction by a second inhomogeneous field to strike a detecting filament. The transitions occur within the homogeneous field region where the oscillating field is situated. Transitions cause a drop of intensity at the detector only if the moment of the atom in the second deflecting field is appreciably different from its value in the first.

This experiment is different from the previous experiments in which molecules were studied because the magnetic moment of the atom is a function of the magnetic field and therefore the deflections in the inhomogeneous fields depend not only on the gradient but also on the magnitude of these fields. For example, we consider a transition which at high fields may be regarded as one in which m_i changes by one unit (Eq. (8)). Clearly at high fields the atomic moment remains practically unchanged because m_J , and therefore the electronic moment, is the same in both states. If the second deflecting field is large $(x\gg1)$ there will be almost no difference between the paths of atoms which have made these transitions and



FIG. 4. The frequencies of the components of the Zeeman pattern of the line $|\Delta F| = 1$ as a function of the magnetic field. The values of x are appropriate to any atom in a ${}^{2}S_{4}$ state with a spin of 1, but the values of H correspond to the case of Li⁶ only.

the paths of atoms which have not. However, if the deflecting field is small $(x \sim 1)$ the difference in magnetic moment between the two states is great and the paths correspondingly different.

The magnetic moment of the atoms as a function of the magnetic field may be obtained from the Eqs. (9)-(12), since $\mu_{F,m} = -\partial W_{F,m}/\partial H$. These are:

$$\frac{\mu_{i+\frac{1}{2},m}}{\mu_{0}} = -g_{i}m$$

$$-\frac{x+2m/(2i+1)}{2[1+4mx/(2i+1)+x^{2}]^{\frac{1}{2}}}(g_{J}-g_{i}) \quad (13)$$

for $F = i + \frac{1}{2}$; $m = i - \frac{1}{2}$, $i - \frac{3}{2}$, $\cdots - (i - \frac{1}{2})$,

$$\frac{g_{i+1,\pm(i+1)}}{\mu_0} = \mp \left(\frac{g_J}{2} + g_i i\right)$$
(14)

for $F = i + \frac{1}{2}$; $m = \pm (i + \frac{1}{2})$, and

 $\frac{\mu_{i-\frac{1}{2},m}}{\mu_0} = -g_i m + \frac{x+2m/(2i+1)}{2[1+4mx/(2i+1)+x^2)]^{\frac{1}{2}}}(g_J - g_i) \quad (15)$

for $F = i - \frac{1}{2}$; $m = i - \frac{1}{2}$, $i - \frac{3}{2}$, $\cdots - (i - \frac{1}{2})$, where $\mu_{F,m}/\mu_0$ is the magnetic moment of the atom in units of the Bohr magneton, for any state

characterized by the quantum numbers F and m. Figs. 7a and 7b show a plot of these moment values for spins of $\frac{3}{2}$ and 1, respectively. For the use to which these moment values are to be put, we can safely neglect the term in mg_i . It can be seen from the formulae, and from the curves which they describe, that in cases where $\Delta \nu$ is small (0.01 cm⁻¹), the inhomogeneous field must be relatively weak, since the field corresponding to any value of x is proportional to $\Delta \nu$. For K³⁹, for example, a value of the inhomogeneous field of about 50 gauss (x = 0.3) would be appropriate for the detection of transitions for which $\Delta m_i = \pm 1$, $\Delta m_J = 0$. The resonance frequencies required for the production of transitions depend only on the magnitude of the homogeneous field and not on the values of the field in the deflecting magnets. The choice of field values in the latter merely determines the ease with which the transitions in the homogeneous field are detected.



FIG. 5. The field dependence of the frequencies of the lines associated with the spectrum of K³⁹ resulting from the transitions $\Delta m_i = \pm 1$, $\Delta m_J = 0$.

Apparatus and Experimental Procedure

The apparatus used in these experiments was the same as that used in the determination of nuclear magnetic moment by the molecularbeam magnetic-resonance method. The inhomogeneous fields were about 50 cm long. At the position of the beam, fields of about 12,000 gauss and gradients of the order of 10⁵ gauss per cm could be attained. These long magnets and high field gradients were required in the older experiments because the systems which were to be deflected had moments of the order of a nuclear magneton. In the present experiments where the moments are of the order of a Bohr magneton, such deflecting power is unnecessary; in fact, these same fields were used in the older experiments, with beams produced by evaporating the alkali metals, to remove the atomic portion of the beam and to permit the study of molecules. The reduction of deflecting power for the present experiments is easily accomplished by limiting the currents in the field coils to very small values. The field values varied from 50 to 100 gauss. The ratio of field gradient to field was about 8. While the field values in the deflecting magnets need not, in general, be critically adjusted for observing atomic transitions, special care must be taken for some particular cases. For example, for the case $i=\frac{3}{2}$ the transition $(F, 0) \Leftrightarrow (F', -1)$ will yield no change in the moment of the atom in the deflecting field if it corresponds to the value of the field parameter xof about 0.27. For such a field value the transition cited would be unobservable.

The high frequencies which were required in these experiments were produced by a tunedplate tuned-filament oscillator with Western Electric 316A tubes. The design of the oscillator followed closely that suggested by the manufacturers of the tube. The frequency of the oscillator was continuously variable and could be set to any desired value to better than one part in 20,000. All frequency measurements were made by use of a General Radio heterodyne frequency meter, Type 620-A. The accuracy of frequency determinations is governed by the precision with which one can set on a zero beat frequency and read the scale. This precision for a single setting was about one part in 20,000. In



practice, however, determinations of frequency were made by observing the position of zero beat frequency between the high frequency oscillator and several harmonics of the wave meter. The frequencies so deduced showed excellent internal consistency and the average of observations on several harmonics of the meter probably gives a frequency with a precision of better than one part in 20,000. The frequency of the quartz crystal against which the wave meter is calibrated is known to better than one part in 100,000 and therefore introduces no error within the precision of our measurements. The stability of the oscillator was so great that no detectable change in frequency occurred over a period of several minutes. The high order of stability is also evident from several examples of the Zeeman pattern to be presented later.

The oscillating magnetic field was produced, as in the previous experiments,⁶ by the passage of the oscillating current through two flattened parallel tubes in the gap of the magnet which supplied the homogeneous magnetic field. The oscillating field was approximately perpendicular to the fixed field. The presence of a component of the oscillating field parallel to the fixed field was probably due to the circumstance that the atomic beam did not traverse the field at a point midway between the two wires as well as to the fact that the current distribution in the tubes at very high frequency is considerably different from that at low frequencies. It is to be expected that both π and σ transitions would occur, even though the σ transitions will be considerably weaker than the π transitions.

The oscillating fields required to induce the transitions $\Delta F = \pm 1$ are much smaller than those used in the previous experiments on molecules. The reason for this is that the torque per gauss exerted on the electron is much greater than on the nucleus, since nuclear moments are of the order of 1/1000 as great as the electronic moment, while the change in angular momentum and the

⁶S. Millman, Phys. Rev. 55, 628 (1939).



FIG. 7. The variation of magnetic moment of an atom in a ${}^{2}S_{1}$ state with magnetic field. (a) describes the case when $i = \frac{3}{2}$ and (b) the case when i = 1.

time during which the torque acts is approximately the same for the two cases. This is a very fortunate circumstance because it obviates the necessity of using high power ultra-high frequency oscillators. Indeed, the transitions occurred with oscillating currents of only a few milliamperes.

RESULTS

 \mathbf{K}^{39}

Figures 8-11 show the weak field Zeeman patterns of the line $F \Leftrightarrow F - 1$ of K³⁹ under various experimental conditions. That of Fig. 8 was obtained for a fixed field of 0.25 gauss and an oscillating field sufficiently large to show the σ components. It is to be noted that only two of the three possible σ lines appear. The reason for the absence of the line $(2, -1) \Leftrightarrow (1, -1)$ can easily be found by an inspection of Fig. 7a, where it is seen that when the value of x in the deflecting fields is about 0.5 ($H \sim 80$ gauss for K^{39}) no change in the magnetic moment of the atom will result in this transition. Even though the calibration constants for the deflecting magnets at low field values were not known with any degree of accuracy, the above explanation was easily verified by increasing the current in the deflecting magnets by about 50 percent. This increase resulted in a field value at which all transitions were accompanied by appreciable moment changes. Fig. 9 shows the resulting pattern, with none of the expected lines missing. The fixed field value in this case, as calculated from the separations of the lines, was about 0.2gauss. The intensity of all the σ lines in Figs. 8 and 9 is low since the component of oscillating field parallel to the fixed field is guite smallcompared with the perpendicular component. The lines $(F, -1) \Leftrightarrow (F', 0)$ and $(F, 0) \Leftrightarrow (F', -1)$ are most intense since F can take on the two values, 2 and 1, and each of the lines is really an unresolved doublet. The separation between the members of a doublet in a field of 0.2 gauss is only about 100 sec.⁻¹ for K³⁹ so that the line is not measurably affected in width but is merely raised in intensity.

The half-widths of the lines in the Zeeman patterns shown in Figs. 8 and 9 are considerably greater than can be expected by use of the relationship,

$$\Delta \nu \Delta \tau \sim 1 \tag{16}$$

where $\Delta \tau$ is the time spent by the atoms in the radiation field and $\Delta \nu$ is roughly the half-width of the curve in frequency units. For potassium

atoms at a temperature of about 550°K the average time spent in a radiation field 4 cm in length is approximately 7×10^{-5} sec., so that the lines should have a half-width of the order of 1.5×10^4 sec.⁻¹. The experimental widths range from 15×10^4 to 30×10^4 sec.⁻¹ and are chiefly due to the large oscillating fields used in the observation of these curves. When the oscillating field is made extremely small, a pattern of the type shown in Fig. 10 is observed. The component of oscillating field parallel to the fixed field is so small that the σ lines are no longer observable. The two inner lines have a halfwidth of about 2×10^4 sec.⁻¹ and the two outer lines of about 4×10^4 sec.⁻¹. This well resolved Zeeman pattern was observed for a field of about 0.05 gauss. Fig. 11 shows an unresolved Zeeman pattern at a field slightly greater than that used for the pattern in Fig. 10. The lack of resolution is due to the excess of oscillating field.

In Table II are listed the expected transitions and the observed frequencies of the centers of the lines shown in Figs. 8, 9 and 10. It is seen that



FIG. 8. The Zeeman pattern of the line, $|\Delta F| = 1$, of K^{39} .

the frequency differences between the successive lines are constant within the precision of measurement of the frequencies of the lines. This is in agreement with the predictions of Eqs. (5) and (6). From the observation of a large number of curves of the type shown in Figs. 8, 9 and 10 we find for $\Delta\nu$ of the ground state of K³⁹ the value $(461.75\pm0.02)\times10^6$ sec.⁻¹ or 0.015403 cm⁻¹ if we take the velocity of light⁷ as 2.99776 $\times10^{10}$ cm/sec. The $\Delta \nu$ of K³⁹ has also been determined by observations on transitions for which $\Delta m_i = \pm 1$, $\Delta m_J = 0$, at high magnetic fields. The frequencies of these transitions as a function of the field are indicated in Fig. 5. In Table III are listed the frequencies observed for these lines at a field of 3950 gauss. The third column lists the mean frequencies of the double lines together with the frequencies of the single lines. $\Delta \nu$ is the sum of the frequencies listed in this column. In column 4 of



FIG. 9. The Zeeman pattern of the line, $|\Delta F| = 1$, of K³⁹, observed for a larger value of the inhomogeneous field than that of Fig. 8.

Table III we list the values $\nu/\Delta\nu$ appropriate to the mean frequency of each of the doublets for a field of 3950 gauss. The last column gives the calculated $\Delta\nu$. From the evaluation of three sets of similar observations we obtain for $\Delta\nu$ the value $(461.75\pm0.02)\times10^6$ sec.⁻¹. The complete agreement between the two independent methods of calculating $\Delta\nu$ indicates that the high field method of obtaining the h.f.s. separation of the ground state is as reliable as the method involving the measurement of the energies of the transitions between different *F* levels at very weak fields.

If the frequency interval between the two components of any double line is measured for a known field, the value of g_i can be obtained as explained in an earlier section. The value which we obtain from such measurements for g_i of K³⁹ is -0.261/1837. This is to be compared with the value -0.260/1837 obtained by the molecular beam magnetic resonance method. The present value is the less accurate because the field values were not very well known and the frequency

⁷ F. G. Dunnington, Rev. Mod. Phys. 11, 65 (1939).

=

difference between two lines is subject to considerable error, even though the frequencies of the lines themselves are very well known.

\mathbf{K}^{41}

The hyperfine structure separation of the ground state of K^{41} was measured in fields of 0.15 to 0.5 gauss in the same way as the $\Delta\nu$ of K^{39} was determined at low magnetic fields. The nuclear spin is the same for both isotopes of potassium and Eqs. (5) and (6) apply to K^{41} as well as to K^{39} . We find the value $(254.02\pm0.02) \times 10^6 \text{ sec.}^{-1}$ or 0.008474 cm^{-1} for the $\Delta\nu$ of K^{41} . The ratio $(\Delta\nu)_{41}/(\Delta\nu)_{39}$ is 0.55012 ± 0.00006 . This value is also the ratio of the nuclear moments of the two isotopes if we assume that hyperfine structure is due solely to the magnetic interaction of the nucleus with the atomic electrons.

Li⁶

The hyperfine structure separation of the ground state of Li⁶ was measured in fields of 0.25 to 1.5 gauss. Since the spin of this nucleus is 1, the Zeeman pattern will contain only five components, three π lines and two σ lines. Fig. 4 indicates the expected frequencies of these lines, in units of $\Delta \nu$, as a function of H. The observed Zeeman pattern for a field of 0.24 gauss is given in Fig. 12. The central component does not vary in frequency with the field for very small values of H. The frequency associated with this π line, therefore gives $\Delta \nu$ of Li⁶ without any further



FIG. 10. The Zeeman pattern of the line, $|\Delta F| = 1$, of K³⁹, observed for very low amplitude of oscillating field. Only the π components appear.



FIG. 11. The Zeeman pattern of the line, $|\Delta F| = 1$, of K³⁹, for approximately the same fixed field as in Fig. 10, but for a very much larger amplitude of oscillating field.

calculation. The same result is, of course, obtained by averaging the frequencies of the outside π lines, or by averaging the frequencies of the two σ components. Table IV contains a list of frequencies of the observed lines. The slight departure from the expected uniformity of the spacing of the lines can be explained on the basis of the observation that the outer π lines are broader than any of the inner lines. These outer

 TABLE II. Lines in the Zeeman patterns of K³⁹ shown in Figs.
 8, 9 and 10, and their observed frequencies.

	Observed frequencies (megacycles)			
TRANSITION	F1G. 8	Fig. 9	Fig. 10	
$(2, -2) \Leftrightarrow (1, -1)$	461.21		461.64	
$(2, -1) \leftrightarrow (1, -1)$	missing			
$(2, -1) \Leftrightarrow (1, 0) (2, 0) \Leftrightarrow (1, -1) $	461.58	461.64	461.72	
$(2, 0) \Leftrightarrow (1, 0)$	461.74	461.76		
$\begin{array}{c} (2, 1) \Leftrightarrow (1, 0) \\ (2, 0) \Leftrightarrow (1, 1) \end{array}$	461.91	461.92	461.79	
$(2, 1) \Leftrightarrow (1, 1)$	462.12			
$(2, 2) \Leftrightarrow (1, 1)$	462.28		461.87	

lines, therefore, overlap the adjacent σ lines to a greater degree than does the central π line. This circumstance leads to a slight shift of the outer π lines toward the center of the pattern and a shift of the σ lines away from the center. The value of $\Delta \nu$ is found to be $(228.22\pm0.01)\times10^6$ sec.⁻¹ or 0.007613 cm⁻¹.

Li⁷

The $\Delta \nu$ of Li⁷ was first measured in strong fields by observing the transitions for which $\Delta m_i = \pm 1$, $\Delta m_J = 0$. As has already been pointed out, this procedure enables one to obtain $\Delta \nu$ with the use of applied frequencies in the neighborhood of $\Delta \nu / (2i+1)$ which in the case of Li⁷ are about 200 megacycles. Fig. 6 indicates the expected frequencies of transition, in units of $\Delta \nu$, as a function of the field parameter x.

In Table V are listed the transitions for the values of the field used in these experiments. By reference to Fig. 6 it may be seen that the frequencies of transition do not vary rapidly with the field for the fields used in these experiments. The first column lists the transitions. The second column lists the corresponding frequencies of the lines at a field of 3060 gauss. Column 3 lists the frequencies of the single lines and the mean frequencies of the double lines. The sum of the frequencies in this column yields $\Delta \nu$. The fourth column lists $\nu/\Delta\nu$ calculated from the

TABLE III. The frequencies observed for the lines $\Delta m_i = \pm 1$, $\Delta m_J = 0$ for K^{30} at a field of 3950 gauss and the $\Delta \nu$ calculated from these lines. In the third column, marked ν' , are tabulated the frequencies of the single lines and the mean frequencies of the double lines. The sum of the frequencies in this column is equal to $\Delta \nu$. The quantity $\nu/\Delta \nu$ tabulated in the fourth column is that appropriate to the mean frequencies of the double lines.



FIG. 12. The Zeeman pattern of the line, $|\Delta F| = 1$, of Li⁶.

expressions listed in Table I, while the fifth column gives $\Delta \nu$ as obtained from columns 2 and 4. A similar set of data is given in the last four columns of the table. For this latter set of observations the field did not remain constant. This drift in field occurred because of the rather long time required to observe the shape of the resonance line as well as to find its frequency.

A number of sets of observations similar to those given in Table V was made. In addition observations were made on the lines $(1, 0) \Leftrightarrow (1, -1)$ and $(2, 0) \Leftrightarrow (2, -1)$ in the neighborhood of x=3.5 where the frequencies of the lines take on maximum values and therefore do not vary rapidly with *H*. From all observations we find for $\Delta \nu$ of the ground state of Li⁷ the value $(803.54\pm0.04)\times10^6$ sec.⁻¹ or 0.026805 cm⁻¹.

The lines tabulated in the second half of Table V are shown in Fig. 13. The average half-width of the lines is 4.7×10^4 sec.⁻¹. The temperature of the oven from which the lithium issued was about 1000°K, so that the most probable velocity of the lithium atoms is about 1.7×10^5 cm/sec. By use of Eq. (16) we find that the theoretical width of the lines is about 4.3×10^4

TRANSITION	y Observed Frequency (mega- cycles)	۷' (MEGA- CYCLES)	ν/Δν	Δν (MEGA- CYCLES)	TABLE IV. Lines in the Zeeman path 12, and their observed j	ern of Li ⁶ , shown in Fig. ^c requencies.
$(2, -2) \Leftrightarrow (1, -1)$	119.904	119.904			TRANSITIONS	Observed Frequency (megacycles)
$(1, 0) \Leftrightarrow (1, -1)$ $(2, 0) \Leftrightarrow (2, -1)$	117.347	116.562	0.252439	461.74	$\begin{pmatrix} \frac{3}{2}, -\frac{3}{2} \end{pmatrix} \Leftrightarrow \begin{pmatrix} \frac{1}{2}, -\frac{1}{2} \end{pmatrix}$ $\begin{pmatrix} \frac{3}{2}, -\frac{1}{2} \end{pmatrix} \Leftrightarrow \begin{pmatrix} 1 & -1 \end{pmatrix}$	227.83
$(1, 1) \Leftrightarrow (1, 0)$ $(2, 1) \Leftrightarrow (2, 0)$	114.961 113.374	114.167	0.247235	461.78	$(\frac{3}{2}, -\frac{1}{2}) \Leftrightarrow (\frac{1}{2}, -\frac{1}{2})$ $(\frac{3}{2}, -\frac{1}{2}) \Leftrightarrow (\frac{1}{2}, \frac{1}{2})$ $(\frac{3}{2}, \frac{1}{2}) \Leftrightarrow (\frac{1}{2}, -\frac{1}{2})$	228.01
$(2, 2) \Leftrightarrow (2, 1)$	111.115	111.115			$\begin{pmatrix} 2\\2\\\frac{3}{2},\frac{1}{2} \end{pmatrix} \Leftrightarrow \begin{pmatrix} 1\\2\\\frac{1}{2},\frac{1}{2} \end{pmatrix}$	228.43
	Sur	n 461.75			$\left(\frac{3}{2},\frac{3}{2}\right) \Leftrightarrow \left(\frac{1}{2},\frac{1}{2}\right)$	228.62



FIG. 13. Contours of the lines in the Paschen-Back spectrum of Li^7 corresponding to Fig. 6.

FREQUENCY (megacycles)

sec. $^{-1}$. The observed half-widths are thus in agreement with those theoretically expected.

The value of g_i which we obtain for Li⁷ is -2.18/1837. This value is to be compared to the value -2.167/1837 obtained from the molecular beam magnetic resonance method.¹ The agreement is satisfactory in view of the errors in the determination of this quantity by the present method as discussed in the case of K³⁹.

Subsequent to our measurement of the h.f.s. of the ground state of Li⁷ by the methods indicated in the preceding paragraphs, Jackson and Kuhn⁸ have obtained the value 0.0275 ± 0.0003 cm⁻¹ from observations of the splitting of the atomic lines ${}^{2}S_{i} - {}^{2}P_{i}$ and ${}^{2}S_{i} - {}^{2}P_{i}$. Their

⁸ D. A. Jackson and H. Kuhn, Proc. Roy. Soc. A173, 278 (1939).

result differs from our value of 0.026805 cm⁻¹ by considerably more than their experimental error. It, therefore, seemed worth while to measure the $\Delta \nu$ of the ground state of Li⁷ by direct observation of the Zeeman pattern of the line $F \Leftrightarrow F-1$ at very low magnetic fields. Such measurements required frequencies in the neighborhood of 800 megacycles, which lie above the upper frequency limit of the Western Electric 316A tube. However, in view of the very small amplitudes of oscillating field required for this work, we attempted to utilize the second harmonic of an oscillator whose fundamental frequency range was in the neighborhood of 400 megacycles. We were able to observe the π components, but because of the small amplitudes of the oscillating field which induced the transitions, the σ lines

TABLE V. The lines resulting from the transitions $\Delta m_i = \pm 1$, $\Delta m_J = 0$ for Li⁷ at high magnetic fields and the Δv calculated from these lines. The columns marked v contain the observed frequencies in megacycles. The column headed v lists the frequencies of the single lines and the mean frequencies of the double lines. The sum of these frequencies is equal to Δv . All observations recorded in the left-hand side of the table were made at a field of 3060 gauss. The lines on the right-hand side of the table are shown in Fig. 13, and were observed at the indicated fields.

TRANSITION	ν MC	ν' MC	$\nu/\Delta \nu$	Δ <i>ν</i> MC	۷ MC	H GAUSS	ν/Δν	$\Delta \nu$ MC
$(2, 2) \nleftrightarrow (2, 1)$	182.38	182.38	0.226937	803.46	183.18	3455	0.227985	803.46
$(2, 1) \leftrightarrow (2, 0)$	190.51	105 50	0.237075	803.58	190.52	3450	0.237087	803.58
$(1, 1) \Leftrightarrow (1, 0)$	200.65	195.58	0.248666	803.58				
$(2, 0) \Leftrightarrow (2, -1)$	199.78		0.249693	803.42	198.82	3440	0.247388	803.66
$(1, 0) \leftrightarrow (1, -1)$	209.96	204.87	0.261284	803.56	210.19	3420	0.261572	803.58
$(2, -2) \Leftrightarrow (1, -1)$	220.72	220.72	0.274705	803.66	219.79	3410	0.273570	803.42
	Sui	n 803.55	Mean 803.54				Mea	an 803.54

were not observed. Fig. 14 shows the Zeeman pattern of the line $F \Leftrightarrow F-1$ at a field of 0.15 gauss. The frequencies of the observed lines are tabulated in Table VI. The average half-width of the lines is 5×10^4 sec.⁻¹ in agreement with the theoretical expectations. The value for $\Delta \nu$ of Li⁷ which we obtain from a number of sets of observations similar to that of Fig. 14 and Table VI is $(803.54 \pm 0.03) \times 10^6$ sec.⁻¹ or 0.026805 cm⁻¹. The result is in excellent agreement with that obtained from observations at high field.

From the measured values of the $\Delta \nu$ of Li⁶ and Li⁷ we obtain for the ratio $(\Delta \nu)_7/(\Delta \nu)_6$ the value 3.5209. Using the expression for the ratio of the nuclear magnetic moments of two isotopes of the same element, we obtain:

$$\frac{\mu_7}{\mu_6} = \frac{(2i/2i+1)_7}{(2i/2i+1)_6} \frac{(\Delta\nu)_7}{(\Delta\nu)_6} = 3.9610 \pm 0.0004.$$
(17)

This is to be compared with the moment ratio of 3.963 ± 0.004 obtained from the direct moment measurements by the molecular beam magnetic resonance method.

In view of the accuracy with which the ratio of the $\Delta \nu$'s was determined with the present methods it was decided to improve the directly measured moment ratio. Accordingly resonance curves of Li⁶ and Li⁷ were obtained in LiCl by keeping the homogeneous field fixed and varying the oscillating frequency. The ratio of the g values of the two nuclei is given directly by the ratio of the frequencies at which the minima occurred. No knowledge of the field is required. In the earlier experiments the frequency remained fixed and resonance minima were observed by varying the current in the exciting coils of the homogeneous magnet. Since the value of the magnetic field depends not only upon the current in the coils but also on the previous history of the magnet it was found that errors due to nonreproducibility of the field were of the order of 0.1 percent. The results obtained with the improved procudure give for the ratio μ_7/μ_6 the value 3.9601 ± 0.0015 .

COMPARISON WITH PREVIOUS MEASUREMENTS

These results are, in the main, in excellent agreement with the results of hyperfine structure measurements by spectroscopic and atomic beam methods, which have previously been made on the same atoms. Since the present results are of far greater precision they can serve as a criterion of the spectroscopic and atomic beam methods. It appears that the precision of these experimental methods was sometimes overestimated. In the case of K^{39} , the present results agree with those of Jackson and Kuhn⁹ and of Meissner and Luft¹⁰ who used spectroscopic methods, and with the results of Millman¹¹ and of Fox and Rabi¹² who worked with the atomic beam methods of zero moments. Our result for the $\Delta \nu$ of K⁴¹ is in good agreement with that determined by Manlev¹³ by atomic beam methods. However, the ratio $(\Delta \nu)_{41}/(\Delta \nu)_{39}$ here obtained is in slight disagreement with his results. The discrepancy falls outside his limits of error which were more stringent for the ratio than for the absolute values.

For Li⁷ there exist the h.f.s. measurements by Jackson and Kuhn⁸ and the atomic beam measurements of Fox and Rabi.12 The present result is in agreement with the result of the atomic beam methods, and is lower by some three percent than the value of Jackson and Kuhn.8 Manley and



FIG. 14. The Zeeman pattern of the line, $|\Delta F| = 1$, of Li⁷. A very small amplitude of oscillating field was used, so that the σ -lines do not appear.

Millman² measured the ratio $(\Delta \nu)_7/(\Delta \nu)_6$ with atomic beam methods. Their value is lower than ours by about 2 percent.

- ¹¹ S. Millman, Phys. Rev. 47, 739 (1935).
- ¹⁹ M. Fox and I. I. Rabi, Phys. Rev. 48, 746 (1935).
 ¹³ J. H. Manley, Phys. Rev. 49, 921 (1936).

⁹ D. A. Jackson and H. Kuhn, Proc. Roy. Soc. A165, 303

^{(1938).} ¹⁰ K. W. Meissner and K. F. Luft, Zeits. f. Physik 106, 362 (1937)

TABLE	VI.	Lines	in th	e Z	leeman	þattern	of Li	' shown	in	Fig.
		14 a	ind t	heir	r observ	ed frequ	encie.	5.		0

TRANSITION	Observed frequency megacycles
$(2, -2) \Leftrightarrow (1, -1)$	803.22
$\begin{array}{c}(2, -1) \Leftrightarrow (1, 0)\\(2, 0) \Leftrightarrow (1, -1)\end{array}$	803.45
$\begin{array}{c} (2, 1) \Leftrightarrow (1, 0) \\ (2, 0) \Leftrightarrow (1, 1) \end{array}$	803.64
$(2, 2) \Leftrightarrow (1, 1)$	803.84

DISCUSSION

The measurements of the ratio μ_7/μ_6 show that the ratio of the nuclear moments of the isotopes of lithium derived from h.f.s. measurements is in excellent agreement with the directly measured ratio. Within the limits of experimental error, there is no indication that nonelectromagnetic interactions between the electron spin and the nucleus, if they exist at all, have any measurable effect on atomic energy levels. This result does not preclude the possibility of a spin independent force of the nonelectromagnetic type. Such interactions with the nuclear constituents would not be revealed by these experiments since they would merely cause an equal shift of both h.f.s. levels of a given atomic energy state and would not, therefore, affect the h.f.s. separation of the state.

The precise agreement between the values of the h.f.s. splitting as determined from observations at high and at low magnetic fields shows that the quantum-mechanical expressions for the variation of the energies of the magnetic levels of h.f.s. multiplets in external magnetic fields are accurate to at least one part in 10⁴. This is the most searching test to which these expressions have been subjected. As a consequence the procedure may be inverted and the Zeeman effect of the hyperfine structure used as a field measuring device for other purposes, such as the more exact determination of nuclear moments.

As indicated by the results of these experiments, the radiofrequency method applied to the measurement of hyperfine structure is capable of greater precision than other known methods. The resolution, or the power of distinguishing between very close levels, as here exhibited appears to represent an advance by a factor greater than 10⁴. This is shown by the ease with which a wellresolved Zeeman pattern is obtained in fields as low as 0.05 gauss. It should be possible to increase the precision of frequency measurements even further by a more refined high frequency technique. These methods should be applicable to the study of nuclear quadrupole moments of light atoms which produce only small departures from the interval rule and may, therefore, escape detection by other methods.

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