On the Dynamics of Complex Fission

We wish to report preliminary results of an investigation of the large amplitude distortions of a nucleus undergoing fission. The nucleus is assumed to have a uniform charge and mass density and a constant volume for all distortions. The radius vector r is expanded in a series of Legendre polynomials $r=R(1+\Sigma a_n P_n)$, and the values of a_0 and a_1 are adjusted to insure constancy of the volume and center of gravity. An estimate of the quantum excitation energies shows that only the first five harmonics need be considered. The potential energy of distortion, including all terms so far evaluated, is given by

$$\begin{aligned}\Delta E=4\pi R^2 O\{&0.4(1-x)a_2^2+(0.7143-0.4082x)a_3^2\\ &+(1-0.3704x)a_4^2+(1.2727-0.3306x)a_5^2\\ &-(0.03810+0.07619x)a_2^3-(0.07619+0.2503x)a_2a_3^2\\ &-(0.1143+0.3429x)a_2^2a_4-(0.05195+0.2226x)a_3^2a_4\\ &-(0.1732+0.6702x)a_2a_3a_5-(0.2171-0.2563x)a_2^4\\ &-(1.3615-1.0934x)a_2^2a_3^2+(0.09350+0.03206x)a_2^5\},\end{aligned}$$

where $2x=(3z^2e^2/5R)/(4\pi R^2 O)$ and O is the surface tension.

The following conclusions may be drawn from this formula. (1) There are large coupling terms between odd and even harmonics. These provide a possible explanation of the marked asymmetry of fission.[1] Bohr and Wheeler[2] have neglected these coupling terms. (2) The coupling between a_2 and a_4 is such as to give an equatorial bulge to the drop when it is on the road to fission.[3] The drop is pinched in near both poles; this would lead to ternary fission were it not for the excitation of the third harmonic which causes one of the preformed droplets to come off first, leaving a residual fragment which has not a sufficiently high excitation energy for its low charge number to divide further. However, it seems possible that such a ternary fission may occasionally be observed, in view of the fact that the saddle point for this mode of fission corresponds to $a_3=0$, and moreover, appears to lie at very low energies. Thus, whether fission occurs principally through the coupling of a_2 and a_3 or through the coupling of a_2 and a_4, in either case the fragments are of unequal sizes. According to the experiments, it is very improbable that the nucleus should divide into nearly equal fragments.[1, 4] (3) The coupling between a_2 and a_3 leads to an energy surface which has been investigated for a wide range in x. We have assumed a_3 to be of the order of $a_2^{3/2}$ and taken all terms in ΔE up through the fifth order in a_2. For $x=0.75$ we find a saddle point at $a_2=0.66$, $a_3=0.30$ with an activation energy $E_f=13$ Mev (assuming the surface energy of uranium to be 500 Mev); for $x=0.85$ there is a saddle point with $a_2=0.65$, $a_3=0.25$ and $E_f=6$ Mev. For such large amplitudes, the terms considered can give only a rough description of the fission process; moreover, theoretical uncertainties in the values of the nuclear radius and surface tension make any exact estimate impossible. These results are nevertheless of the right order of magnitude to explain the observed fission of uranium. (4) The energy surface for the a_2-a_4 coupling has been investigated for terms up through the fourth order in a_2 assuming that a_4 is of the order of a_2^2. For $x=0.75$ there is a saddle point at $a_2=0.47$, $a_4=0.057$ with $E_f=4.4$ Mev and for $x=0.85$ we find one at $a_2=0.31$, $a_4=0.029$ with $E_f=1.1$ Mev. (5) With all the mutual coupling terms for a_2, a_3 and a_4 included, there are two roots of the extremalizing equations which are of interest. One of these corresponds to $a_3=0$ and reproduces the results already stated in (4). The other leads to a reduction of the amplitudes and activation energies evaluated in (3). We have not yet included a sufficient number of terms to give a significant estimate of this effect, but it appears to be of the order of several Mev. In view of this fact and especially because of the modifications that may arise with the inclusion of higher order terms, we cannot conclude that the a_2-a_4 mode of fission requires less activation energy than the a_2-a_3 mode. (6) An estimate of the particle sizes to be expected from the a_2-a_3 mode of fission can be made by using the relative amplitudes of a_1, a_2 and a_3 at the saddle point. The two fission products have approximately 0.4 and 0.6 the mass of the original nucleus, this result being insensitive to the value of x. For the a_2-a_4 mode of fission the particle sizes are roughly in the ratio of 1 : 3 for binary fission and 1 : 2 : 1 for ternary fission. The first of these is consistent with the experimental energy peaks.[1]

Finally we want to emphasize that certain conclusions of this note are subject to possible modification due to the inclusion of higher order terms which are probably not small. A more detailed discussion of the dynamics of fission will be published shortly.[5]

R. D. Present
J. K. Knipp

Purdue University,
Lafayette, Indiana,
April 1, 1940.

[1] Booth, Dunning and Slack, Phys. Rev. **55**, 981 (1939); M. H. Kanner and H. H. Barschall, Phys. Rev. **57**, 372 (1940).
[2] N. Bohr and J. A. Wheeler, Phys. Rev. **56**, 426 (1939).
[3] In contradiction to the statement of Bohr and Wheeler, reference 2, p. 433, that a_4 contributes a concavity about the equatorial belt such as to lead continuously to a dumb-bell shaped figure. We do not check the higher order coefficients of their Eq. (22) and we note further that Eq. (23) is inconsistent in sign and magnitude with Eq. (22).
[4] L. A. Turner, Rev. Mod. Phys. **12**, 1 (1940), cf. table on p. 23.
[5] Related biophysical problems are being considered.

Radio Isotopes of Chromium

We have investigated the induced radioactivities of chromium by the aid of our cyclotron. The results are summarized as follows:

(I) *Deuteron bombardment.* The exposure of chromium metal to about 100 microampere hours of 3-Mev deuterons yields chemically identified chromium isotopes of half-lives 1.6 hours and about 14 days.

(II) *Slow neutron bombardment.* In this case activity induced in chromium was very weak. The following periods were established: 2.8 hr., 14 hr., 1.7 hr. The purity of the sample was chemically tested and Al (1.1 percent), Si (0.17 percent), Fe (0.7 percent), Mn(>0.02 percent) were shown as impurities. Therefore the 2.8-hour activity seems to be due to Si^{31} (2.5 hours) or Mn^{56} (2.5 hours), and the 14-hour activity is probably due to Na^{24} produced in aluminum impurity by the reaction Al-n-α-Na, as a considerable number of fast neutrons were always among slow neutrons. The period 1.7 hours is very near to the