

## A Theory of Spark Discharge

J. M. MEEK

*Department of Physics, University of California, Berkeley, California*

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The breakdown of a uniform field is considered to occur by the transition of an electron avalanche proceeding from cathode to anode into a self-propagating streamer, which develops from anode to cathode to form a conducting filament between the electrodes. A criterion is put forward for such a transition, *viz.*, a streamer will develop when the radial field about the positive space charge in an electron avalanche attains a value of the order of the external applied field. For then photoelectrons in the immediate vicinity of the avalanche will be drawn into the stem of the avalanche and will give rise to a conducting filament of plasma, and a self-propagating streamer proceeds towards the cathode. The theory thus depends on ionization by electrons and photo-ionization in the gas and dispenses with the classical assumption of ionization by positive ions in the gas or secondary actions at the cathode. On this basis

an equation for breakdown is developed, and reference to  $\alpha/p - X/p$  curves enables the potential required for breakdown to be determined. Satisfactory agreement between calculation and experiment is found in air for values of pressure times gap length down to  $p\delta \sim 100$  mm Hg  $\times$  cm. The theory does not conform absolutely to Paschen's law, but the deviations are within the present day margins of experimental error. For lower values of  $p\delta$  the deviation between calculation and experiment is explained by the fact that the density of photo-ionization becomes small, and the secondary mechanism,  $\gamma$ , is observed in this region, so that classical theory applies. The theory is indicated to be consistent with all the requirements so far established in connection with sparks for large  $p\delta$  up to the lightning discharge.

### INTRODUCTION

IN the early 1900's J. S. Townsend<sup>1</sup> evolved his now classical theory of the mechanism of spark discharge on the basis of measurements made at high values of  $X/p$ , the ratio of field strength to pressure, and low values of  $p\delta$ , the product of pressure times gap-length. His theory has been extrapolated to explain the mechanism of sparking under ordinary conditions for longer gap-lengths at atmospheric pressure and above. It has formed the guiding principle of all considerations of the sparking mechanism for nearly 40 years. Many researches have indicated that for values of  $p\delta$  near and somewhat above the minimum sparking potential the theory was adequate and relatively accurate. The theory in this region enables calculation of the sparking potentials to be made with adequate accuracy, and recently it has been shown by Schade<sup>2</sup> to be consistent with time-lag studies.

In recent years, however, it has become increasingly apparent that the theory is entirely inapplicable when extrapolated to sparks for  $p\delta$  greater than about 200 mm Hg  $\times$  cm in air, and analogous values in other gases. The reasons for this conclusion may be stated briefly as follows:

1. The formative time lags of sparks at *atmospheric pressure*, *i.e.*, the time required for the breakdown mechanism to materialize, have been measured by several diverse methods, and all lead to time intervals of the order of  $10^{-7}$  sec., or even less, for a one-cm gap at small over-voltages.<sup>2</sup> These time intervals are far below the microseconds required by positive ion movement as shown by Schade<sup>3</sup> and others.<sup>4</sup>

2. At atmospheric pressure the sparking potential has been found to be independent of cathode material to a very high degree,<sup>5</sup> while the theory above, even though somewhat insensitive to the value of  $\gamma$  at higher pressures, requires a definite dependence of the order of 10 percent on the value of  $\gamma$ .<sup>6</sup> The values of  $p\delta$  at which this de-

<sup>1</sup> J. S. Townsend, *Nature* **62**, 340 (1900); *Phil. Mag.* **1**, 198 (1901).

<sup>2</sup> P. O. Pedersen, *Ann. d. Physik* **71**, 371 (1923); J. W. Beams, *J. Frank. Inst.* **206**, 809 (1928); J. J. Torok, *Trans. A. I. E. E.* **47**, 177 (1928); **48**, 46 (1930); Buraway, *Arch. f. Elek.* **16**, 14 (1926); I. Tamm, *Arch. f. Elek.* **19**, 235 (1928); W. Rogowski, *Arch. f. Elek.* **20**, 99 (1928); F. G. Dunnington, *Phys. Rev.* **38**, 1535 (1931); H. J. White, *Phys. Rev.* **46**, 99 (1934); A. Tilles, *Phys. Rev.* **46**, 1015 (1934); R. R. Wilson, *Phys. Rev.* **49**, 1082 (1936); M. Newman, *Phys. Rev.* **52**, 652 (1937); R. Strigel, *Wiss. Veröffentl. Siemens Werken* **15**, 1 (1936).

<sup>3</sup> R. Schade, *Zeits. f. Physik* **104**, 487 (1937).

<sup>4</sup> I. Tamm, *Arch. f. Elek.* **19**, 235 (1928); W. Rogowski, *Arch. f. Elek.* **20**, 99 (1928); W. Rogowski and I. Tamm, *Arch. f. Elek.* **20**, 625 (1928); A. Tilles, *Phys. Rev.* **46**, 1015 (1934).

<sup>5</sup> L. B. Loeb, *Fundamental Processes of Electrical Discharge in Gases* (John Wiley, 1939), p. 415.

<sup>6</sup> Reference 5, p. 416 *et seq.*

pendence begins rapidly to vanish occur again at  $p\delta \sim 200$ .<sup>5</sup>

3. In longer sparks, in lightning discharge, and in positive point corona at atmospheric pressure, it is virtually certain that the character and action of the cathode is without any influence whatsoever on the process.

4. The observed appearance of sparks in measurements of Townsend's coefficient,  $\alpha$ , at higher pressures and adequate  $x$  with no measurable value of  $\gamma$  indicates in another way that the theory based on a  $\gamma$  is no longer applicable at higher values of pressure and gap length.<sup>7</sup>

5. Observations of actual sparks in short time intervals made visible by Kerr cell-shutter studies<sup>8</sup> and by cloud-track pictures of spark paths after short time intervals,<sup>9</sup> as well as the usual visual observations of sparks, indicates that breakdown at higher pressures occurs along very narrow channels or filaments. This is incompatible with the transient rearrangement of conduction currents in a gas caused by accumulations and movements of positive ions or successive waves of electron ionization when  $\gamma e^{\alpha x} > 1$ . This discrepancy is made still more glaring by the zigzag nature of the spark filament and the frequent branching observed in longer sparks.

6. While only one electron is required to initiate a spark on the Townsend theory as interpreted by Holst and Oosterhuis<sup>10</sup> it is clear that the greater the number of photons incident on the cathode by external irradiation the more rapidly should the space charge accumulations occur, so that formative time lags should on that theory decrease as the illumination intensity of the cathode increases. This has been shown to be true by Schade for low values of  $p\delta$ . However, at high  $p\delta$  observation shows no appreciable change in the *formative* time lag and of the sparking potential with illumination over a considerable range of intensity.<sup>11</sup> When the illumination

intensity is increased by a factor of  $10^5$  beyond that giving the  $10^7$  electrons/sec./cm<sup>2</sup> generally used, both formative time lag and sparking potential are lowered.<sup>12</sup> In this case the potential is lowered at a maximum by no more than 10 percent, and the formative time lag appears to be doubled when the intensity of illumination is decreased by a factor of 500.

## THEORY

The above-mentioned objections to the classical theory make it imperative to formulate a new mechanism which does not so conflict with experimental observations. The importance of streamer formation in the mechanism of spark breakdown has been discussed in a recent article by Loeb and Kip<sup>13, 13a</sup> and more extensively by Loeb in his book *Fundamental Processes of Electrical Discharge in Gases*. The breakdown process is there pictured in qualitative fashion, but no quantitative criterion is given for streamer formation and breakdown. However, it is shown that under sparking conditions the positive space charge in an electron avalanche which is originated at the cathode produces an electric field of the same order as the external applied field when the avalanche reaches the anode. This fact was also indicated previously by Slepian,<sup>14</sup> who then postulated the development of thermal ionization in the avalanche. It is further stated by Loeb that the axial field distortion produced by the positive space charge inhibits the advance of the electron avalanche towards the anode, but favors the development of a positive streamer towards the cathode. An explanation of the mechanism of streamer formation and mid-gap streamers is also given, though the fundamental criterion for their appearance is lacking. However, it is indicated that breakdown will not occur until a conducting filament exists across the gap. The single electron avalanches observed by Raether<sup>15</sup> just below breakdown do not form such a fila-

<sup>7</sup> F. H. Sanders, Phys. Rev. **44**, 1020 (1933); D. Q. Posin, Phys. Rev. **50**, 650 (1936); W. E. Bowls, Phys. Rev. **53**, 293 (1938).

<sup>8</sup> L. von Hamos, Ann. d. Physik **7**, 857 (1930); F. G. Dunnington, Phys. Rev. **38**, 1535 (1931); H. J. White, Phys. Rev. **46**, 99 (1934).

<sup>9</sup> E. Flegler and H. Raether, Zeits. f. Physik **99**, 635 (1936); **103**, 315 (1936); H. Raether, Zeits. f. Physik **96**, 567 (1936); **110**, 611 (1938); **112**, 464 (1939).

<sup>10</sup> G. Holst and E. Oosterhuis, Phil. Mag. **46**, 1117 (1923).

<sup>11</sup> H. J. White, Phys. Rev. **48**, 113 (1935); R. R. Wilson, Phys. Rev. **50**, 1082 (1936).

<sup>12</sup> H. J. White, Phys. Rev. **48**, 113 (1935).

<sup>13</sup> L. B. Loeb and A. F. Kip, J. App. Phys. **10**, 142 (1939).

<sup>13a</sup> At the request of Professor Loeb, attention is called to page 157 of this article where a careless blunder is made in the calculation of the field strength produced by space charge. However, the order of magnitude of the calculated values is not affected by correction, and the argument given in the article is by no means invalidated thereby.

<sup>14</sup> J. Slepian, Electrical World **91**, 761 (1928).

<sup>15</sup> H. Raether, Zeits. f. Physik **107**, 91 (1937).

ment but produce a charge concentration at the anode so that positive streamer propagation to the cathode is needed.

In the above-mentioned theoretical discussions, consideration appears to have been given principally to the field distortion produced by the positive space charge in the direction of the axis of the avalanche, i.e., in the direction of the external applied field. Clearly, however, the positive space charge will also produce a field distortion in the direction radial to the axis of the avalanche, i.e., perpendicular to the external field. Consideration of this fact led the writer to the following criterion for the transition of an electron avalanche to a conducting streamer, a criterion which makes quantitative calculation possible. *A streamer will develop when the radial field about the positive space charge in an electron avalanche attains a value of the order of the external applied field.* Then photoelectrons produced in the gas in the immediate vicinity of the avalanche will not only be accelerated in the direction of the external applied field but will also be drawn into the stem of the avalanche. (It has been shown by Cravath,<sup>16</sup> Dechene,<sup>17</sup> Loeb and Kip<sup>13</sup> that streamer propagation depends on photo-ionization in the gas.) In this manner the positive ion space charge attracts photoelectrons from the surrounding space in sufficient numbers to originate a self-propagating streamer. For lower values of external applied field, where the multiplication of ions in the avalanche is insufficient to produce a radial field of the same order of magnitude as the external field, photoelectrons produced in the gas proceed to the anode and are not deflected to the main avalanche.

It is clear that the exact equality of the radial field and the external applied field need not be insisted upon, and it is possible that the requisite criterion could be satisfied when the radial field is only 50 percent of the applied field. However, in the calculations the two fields have been set equal, and the application of this condition at once leads to equations for field strength for spark breakdown that closely approximate experimental observations and clarifies difficulties in the qualitative picture given by Loeb. The criterion not only enables quantitative calcula-

tions of breakdown to be made, but it also assists in the interpretation of a number of other sparking phenomena. It thus seems of interest to present the results of the analysis and its applications, together with its experimental justification.

#### DERIVATION OF EQUATION FOR BREAKDOWN

Because of the cumulative character of the ionization in an electron avalanche and the relative immobility of positive ions as compared with electrons, the bulk of the positive ions left behind by the advancing electron swarm are concentrated in a region of several ionizing free paths behind the tip of the avalanche. The region is also bounded by the periphery of the avalanche, which is conditioned by electron diffusion. Since the estimation of the field produced by such a distribution is difficult it will be assumed for convenience of calculation that the positive space charge is concentrated in a spherical volume of radius,  $r$ , equal to that of the avalanche. While the volume may be more nearly in the form of an oblate or prolate spheroid, the assumption of a perfect sphere will not affect the calculated field in order of magnitude.

The field,  $X_1$ , at the surface of a spherical volume of uniformly distributed space charge is given by

$$X_1 = \frac{Q}{r^2} = \frac{4}{3} \pi r N \epsilon,$$

where  $N$  is the number of ions per  $\text{cm}^3$  and  $\epsilon$  is the charge on an electron. To find  $N$ , one recalls that owing to cumulative ionization the number,  $n$ , of positive ions formed when the avalanche has progressed a distance,  $x$ , is  $n = e^{\alpha x}$ , where  $\alpha$  is the Townsend coefficient for ionization by electrons. In a distance  $dx$  the number of ions created is  $dn = \alpha e^{\alpha x} dx$ . These ions are assumed to be contained in a cylindrical volume of radius,  $r$ , and of length,  $dx$ , so that

$$N = \alpha e^{\alpha x} / \pi r^2.$$

According to Raether<sup>15</sup> the radius,  $r$ , is related to the distance,  $x$ , of avalanche travel by means of the diffusion equation  $r^2 = 2Dt = 2Dx/v$ , where  $v$  is the velocity of advance of the avalanche and has been measured by Raether<sup>15</sup> and White.<sup>18</sup>

<sup>16</sup> A. M. Cravath, Phys. Rev. **47**, 254 (1935).

<sup>17</sup> C. Dechene, J. de phys. et rad. **7**, 533 (1935).

<sup>18</sup> H. J. White, Phys. Rev. **46**, 99 (1934).

Since  $v=KX$ , where  $K$  is the mobility and  $X$  is the applied field, we have

$$X_1 = \frac{4}{3} \frac{\alpha e^{\alpha x} \epsilon}{r} = \frac{4}{3} \frac{\alpha e^{\alpha x} \epsilon}{(2Dx/KX)^{\frac{1}{2}}}$$

Both  $D$  and  $K$  are functions of  $X/p$  and are approximately related by the expression

$$\frac{D}{K} = \frac{P}{N\epsilon} \frac{c_1^2}{c^2},$$

where  $P$  is atmospheric pressure,  $N\epsilon$  is the Faraday constant,  $c$  is the speed of thermal agitation of electrons ( $1.2 \times 10^7$  cm per sec. at  $20^\circ\text{C}$ ), and  $c_1$  is the square root of the mean-squared velocity of the electron in the applied field. Townsend<sup>19</sup> has shown that  $c_1$  is considerably larger than  $c$ . According to Compton<sup>20</sup> the value of  $c_1$  is given by

$$c_1^2 = 1.33 \frac{\epsilon X \lambda}{m \sqrt{f}},$$

where  $\lambda$  is the mean free path of the electron,  $X$  is the applied electric field,  $m$  is the mass of an electron,  $p$  is the gas pressure, and  $f$  is a factor which depends on the fractional loss of energy per collision. If  $\lambda_0$  is the mean free path of an electron at a pressure of 760 mm Hg, then the value of  $c_1$  for a pressure,  $p$ , is given by

$$c_1^2 = 1010 \frac{\epsilon X \lambda_0}{m p \sqrt{f}}.$$

The values of  $\lambda_0$  and  $f$  obviously depend on  $X/p$  and are not known for the high values of  $X/p$  which are necessary for spark breakdown. But inasmuch as the theory will be found insensitive to slight errors in the value of  $\lambda_0/\sqrt{f}$  it is con-

TABLE I. Breakdown potentials corresponding to different values of  $p$  and  $\delta$ .

$\frac{p\delta}{\text{MM} \times \text{CM}}$	BREAKDOWN POTENTIAL (KV)		
	$\delta=0.1$	$\delta=1.0$	$\delta=10.0$
7600	245	248	249
760	31.5	32.2	32.9
380	18.0	18.6	18.9

<sup>19</sup> J. S. Townsend, *Electricity in Gases* (Oxford University Press, 1914), p. 122ff.

<sup>20</sup> K. T. Compton and I. Langmuir, *Rev. Mod. Phys.* **2**, 220 (1930).

sidered reasonable to calculate its value from known data based on experiments with fields close to those which produce sparking. Raether<sup>15</sup> found that the speed of advance of an electron avalanche was  $1.25 \times 10^7$  cm per sec. in a field  $X/p=41$  volts per cm per mm. Substitution of such values in the well-known Langevin equation for electron mobility, *viz.*,  $v=0.815 \epsilon \lambda X/mc_1$ , enables one to calculate that  $\lambda_0 \sqrt{f}=5.7 \times 10^{-6}$ . The mean free path  $\lambda_0$  is taken as  $3.6 \times 10^{-5}$  from curves given by Brose and Saayman<sup>21</sup> and corresponds to an electron energy of three volts. Whence we have  $f=0.025$  and  $c_1=2.01 \times 10^7(X/p)^{\frac{1}{2}}$ . Then we have

$$X_1 = \frac{4}{3} \frac{\alpha e^{\alpha x} \epsilon}{(2Pc_1^2x/N\epsilon c^2X)^{\frac{1}{2}}} \quad (1)$$

$$= 5.28 \times 10^{-7} \frac{\alpha e^{\alpha x}}{(x/p)^{\frac{1}{2}}} \text{ volts/cm.}$$

Now for  $x=\delta$ , the gap length for a plane parallel gap, application of the criterion for streamer formation, that  $X_1=X$ , gives

$$X(\delta/p)^{\frac{1}{2}} = 5.28 \times 10^{-7} \alpha e^{\alpha \delta},$$

which may be written in logarithmic form as

$$\frac{\alpha}{p} \delta + \log_e \frac{\alpha}{p} = 14.46 + \log_e \frac{X}{p} - \frac{1}{2} \log_e p \delta + \log_e \delta. \quad (2)$$

Reference to curves of the Townsend type which relate  $\alpha/p$  with  $X/p$  now enables the value of  $X/p$  required for sparking to be calculated for given  $\delta$  and  $p$ . In the case of a one-cm gap in air at atmospheric pressure the breakdown equation is satisfied for  $X/p=42.4$  ( $\alpha=18.6$ ), *i.e.*,  $X=32.2$  kilovolts per cm,\* which is in close agreement with the observed value of 31.5 kilovolts per cm.<sup>22</sup>

#### COMPARISON BETWEEN CALCULATED AND MEASURED VALUES OF BREAKDOWN POTENTIAL

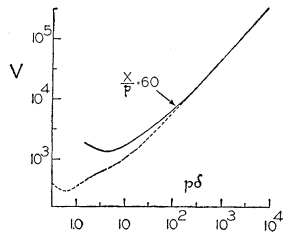
The form of the breakdown Eq. (2) shows that it is in agreement with Paschen's law other than

<sup>21</sup> H. L. Brose and E. H. Saayman, *Ann. d. Physik* **5**, 797 (1930).

\* If we set the criterion for streamer formation as  $X_1=0.2X$  instead of  $X_1=X$ , the breakdown field strength evaluated is 31.8 kilovolts per cm.

<sup>22</sup> W. Spath, *Arch. f. Elek.* **12**, 331 (1923).

FIG. 1. Experimental and calculated curves to show relationship between breakdown voltage and  $p\delta$ .



for the term,  $\log_e \delta$ . This term has little effect, however, on the calculations, as shown by Table I for breakdown potential corresponding to different values of  $p$  and  $\delta$ . The deviations, for the same value of  $p\delta$ , are within the present margins of experimental error in measurement, and it may be that for more accurate measurements such differences will be observed. For it should be pointed out that the derivation of Paschen's law is based on theoretical considerations analogous to those of Townsend. Such considerations make the mechanism of spark breakdown dependent on the total number of ions formed in the gap and, since the multiplicative factors in ion production depend on  $X/p$ , Paschen's law results. However, on the new theory, it is seen that the breakdown mechanism is not dependent on the total number of ions formed but depends on the ion density in an electron avalanche and the resultant space-charge field. In consequence, we would not expect that Paschen's law would be strictly obeyed. In fact, it has previously been shown<sup>23</sup> that with space charge distortion Paschen's law is only approximately correct, and that the deviation in breakdown potential is of the same order of magnitude as that derived here. So that unless the precision of experimental determinations of sparking potentials in the future indicates Paschen's law to hold beyond the prediction of the present theory, the deviations of the latter from Paschen's law are not thought to invalidate the theory in any way.

Both experimental and calculated curves to show the relationship between the voltage required to cause breakdown and the product,  $p\delta$ , of pressure times gap-length are plotted on logarithmic scale in Fig. 1. The calculated curve is based on values for the primary ionization

coefficient,  $\alpha$ , in air as given by Sanders<sup>24</sup> for  $X/p$  up to 100; for higher  $X/p$  the values of  $\alpha$  are taken from Posin's observations in nitrogen.<sup>25</sup> The experimental curve, shown dotted, is that given by Whitehead<sup>26</sup> as the mean of the results given by a number of independent workers. It will be observed that the calculated curve is in agreement with that experimentally determined for values of  $p\delta$  from 10,000 down to about 100 mm $\times$ cm. For values of  $p\delta < 100$ , the calculated values are higher than those observed, and the deviation increases steadily with decreasing  $p\delta$  and thus with increasing  $X/p$ . The deviation occurs in the region of  $X/p = 60$  which is about the minimum value of  $X/p$  for which the second Townsend coefficient is observed.<sup>27</sup> Thus for values of  $p\delta < 100$  the deviation can be accounted for by the presence of other sources of ionization in the gap. This is in agreement with calculations made in the past on a knowledge of the Townsend coefficients, and which have yielded approximately correct values of breakdown potential corresponding to low values of  $p\delta$ . The deviation between the two curves then indicates the region in which breakdown by streamer formation gives way to classical theory and the Townsend type of mechanism.

Preliminary calculations of the breakdown strength of other gases, such as hydrogen, have been made and give values in agreement with those observed experimentally.

## CONCLUSIONS

It is seen that the above theory postulates an entirely different mechanism of breakdown for longer gaps and higher pressures than that given by classical theory.

1. The proposed mechanism depends on photo-ionization and electron multiplication in the gas (Townsend's  $\alpha$ ). This photo-ionization is justified by recent observations on the effectiveness of such ionization under the existing conditions.<sup>16,17</sup>

2. It requires that the density of positive ion space charge created in an electron avalanche is

<sup>24</sup> F. H. Sanders, *Phys. Rev.* **44**, 1020 (1933).

<sup>25</sup> D. Q. Posin, *Phys. Rev.* **50**, 650 (1936).

<sup>26</sup> S. Whitehead, *Dielectric Phenomena* (D. Van Nostrand & Company, New York, 1927), p. 42.

<sup>27</sup> W. E. Bowls, *Phys. Rev.* **53**, 293 (1936).

<sup>23</sup> R. N. Varney, H. J. White, L. B. Loeb and D. Q. Posin, *Phys. Rev.* **48**, 818 (1935).

sufficient to draw in photoelectrons and their progeny, produced by ionization by collision, in a measure which renders the positive space charge a conducting plasma, and permits the further development of the streamer. This is only possible when the field strength and gas pressure are such as to allow a sufficient density of photo-ionization in the neighborhood of the streamer head to permit self-propagation.

3. It shows that the development of long sparks is completely devoid of any dependence on cathode material, in conformity with observation.

4. It is in full accord with time-lag studies in that it involves formative time lags of  $\sim 10^{-7}$  sec. or less and explains the fact that when a potential is applied to a gap in excess of the minimum required to cause breakdown midgap streamers develop and the formative time lag is reduced.<sup>8</sup> For in this case an electron avalanche which originates at the cathode will have developed the requisite density of positive ions to cause the initiation of a streamer before the anode is reached. The reduced distance which the avalanche has to travel before it produces a streamer makes it probable that several streamers will be initiated simultaneously. A conducting filament across the gap will be formed by the junction of such streamers, and results in a corresponding reduction in time of formation of the spark.

5. At the sparking threshold the spark is initiated by a single electron liberated from the cathode. The chance of a spark then depends on (a) the chance of a single electron avalanche which produces at the anode or in midgap the requisite space charge, and (b) the success which this space charge has in photo-ionizing the ambient gas and in attracting sufficient electrons to ensure self-propagation of a streamer. This accounts for the statistical time lags, and for the observed indefinite character of the sparking potential. That the sparking threshold is at all definite under these conditions is explained by the cumulative nature of electron ionization, i.e.,  $i = i_0 e^{\alpha x}$ , and also that  $\alpha/p = f(X/p)$  where  $f(X/p)$  is of the form  $Ae^{BX/p}$  or  $C(X/p - D)^2$ .

6. The theory is completely consistent with the observations of H. Raether on the development of sparks in a cloud chamber. In fact,

Raether's photographs clearly indicate the passage of an electron avalanche from cathode to anode followed by a streamer which proceeds from the anode to the cathode to form a conducting filament bridging the gap. Simultaneously and quite independently, Loeb, on the basis of corona studies, and Raether,<sup>28</sup> on the basis of cloud-chamber observations, postulated a qualitative picture of the sparking mechanism which is here rendered quantitative.

7. It can be shown that the field distortion produced by cumulative ionization with high densities of photoelectric current gives a lowering of sparking potential in rough quantitative agreement with the lowering observed by White,<sup>11</sup> Rogowski and Wallraff.<sup>29</sup>

8. It clearly accounts for the filamentary character of spark breakdown at longer gaps and higher pressures and explains both the zigzag character of the spark and the branching observed. This differentiates the sparks under this mechanism from those in the Townsend regime ( $p\delta < 100$  in air).

9. It explains the fact that in the study of the Townsend coefficients at lower  $X/p$  and higher  $p\delta$  sparking occurs before values of the second coefficient,  $\gamma$ , can be observed.

10. The slight deviation of the theory from Paschen's law is not inconsistent with experimental data in this field of investigation.

11. While at present the inexact character of the quantitative criterion for streamer formation leaves considerable latitude owing to the exponential nature of electron multiplication by ionization by collision, the equation gives numerical results in as good agreement with experiment as those based on Townsend's theory, where a hypothetical value of  $\gamma$  has to be assumed, since it cannot be observed. Numerical agreement of this character between theory and experiment must not be taken too seriously as a confirmation of any particular theory in view of the latitude given by the exponential term and the lack of adequate experimental data.

12. The theory has been extended to the explanation of sparks of all lengths at higher  $p\delta$

<sup>28</sup> H. Raether, *Zeits. f. Physik* **112**, 464 (1939).

<sup>29</sup> W. Rogowski and H. Wallraff, *Zeits. f. Physik* **102**, 183 (1937).

in air and the necessary conditions for sparking have been established by Loeb. A more exhaustive discussion of the application of the theory will be given in an article to appear at some future date.

13. It can be shown that the theory leads to an explanation of the breakdown of unsymmetrical gaps where the field distribution is known, and that quantitative calculations can be made. The theory may be applied to either surge-impulse or static breakdown.

14. It is found that the self-propagating streamer mechanism, as modified for longer sparks, will adequately account for the pilot

streamer in Schonland's mechanism<sup>30</sup> for the lightning discharge, and it clarifies the theory proposed by the writer<sup>31</sup> for the "stepping" observed in lightning and spark discharges.

The writer wishes to express his appreciation to Professor L. B. Loeb, whose original qualitative theory stimulated this work and whose suggestions and criticisms have contributed much to the development of the present theory. He also thanks the Commonwealth Fund for the grant of a Fellowship, during the tenure of which this work was carried out.

<sup>30</sup> B. F. J. Schonland, Proc. Roy. Soc. **A164**, 132 (1938).

<sup>31</sup> J. M. Meek, Phys. Rev. **55**, 972 (1939).

## Ferromagnetic Anisotropy, Magnetization at Saturation, and Superstructure in Ni<sub>3</sub>Fe and Nearby Compositions\*

EUGENE M. GRABBE†

*Sloane Physics Laboratory, Yale University, New Haven, Connecticut*

(Received February 13, 1940)

The order-disorder transformation involving Ni<sub>3</sub>Fe affects both the ferromagnetic anisotropy and the saturation magnetization. Iron-nickel alloys in the range 65 to 80 percent nickel have been investigated, using spheroidal specimens. With superlattice formation the anisotropy becomes more like that of pure nickel with (111) as the direction of easiest magnetization. The change is largest near Ni<sub>3</sub>Fe. The saturation magnetization increases with ordering; the greatest observed increase was 5.8 percent for an alloy very near Ni<sub>3</sub>Fe. Different rates of cooling from above the critical temperature (about 490°C) affect the saturation magnetization. These changes are attributed to changes in degree of local ordering effects. Long distance order, induced by baking for long periods, greatly influences both anisotropy and magnetization at saturation.

### INTRODUCTION

**I**RON-NICKEL alloys in the composition range 65 to 80 percent nickel have unusual properties which are sensitive to heat treatment. They develop extremely high permeabilities when cooled rapidly from above 600°C. Baking below this temperature or cooling slowly does not produce uncommonly high permeabilities. Attempts to explain these characteristics in terms of an order-disorder transformation involving Ni<sub>3</sub>Fe have long been hampered by the

lack of conclusive evidence for the existence of this superlattice.<sup>1</sup>

Changes in resistance, which occur in these alloys with proper baking after quenching, are not as marked as in many order-disorder transformations. This is to be expected, however, since the differences in size and chemical properties between iron and nickel atoms are not great. Specific heat measurements have not been conclusive because the anomaly, due to the magnetic transformation, occurs near the critical ordering temperature. X-ray analyses have, until after the experiments here reported were com-

\* Part of a dissertation presented to the Faculty of the Graduate School of Yale University in candidacy for the degree of Doctor of Philosophy.

† Now at the General Laboratories, United States Rubber Company, Passaic, New Jersey.

<sup>1</sup> O. Dahl, Zeits. f. Metallkunde **28**, 133-138 (1936); S. Kaya, J. Fac. Sci., Hokkaido Imp. Univ. **2**, 29-53 (1938); F. E. Haworth, Phys. Rev. **54**, 693-698 (1938).