Anomalous Scattering of Fast Neutrons

M. R. MacPhail
*Palmer Physical Laboratory, Princeton University, Princeton, New Jersey**

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The cross sections of a number of elements for neutron scattering were measured as a function of neutron energy throughout a range of 0.5 Mev. Anomalous cross sections were found in the cases of magnesium and aluminum. It was possible to account quantitatively for the anomaly in magnesium on the assumption that the effect was due to resonance in Mg^{25}.

Introduction

The first measurements of nuclear cross sections for fast neutron scattering were made by Dunning in 1934.[1] Using neutrons from a mixture of radon and beryllium whose energies ranged from about 0.1 to nearly 14 Mev, he concluded that the cross section of a nucleus of atomic weight A was proportional to $A^{\frac{2}{3}}$. This cross section was, of course, an average with respect to neutron energy. More recently, Ladenburg and Kanner,[2] and Kikuchi and Aoki[2a] remeasured the cross sections of a large number of nuclei, using the nearly homogeneous neutrons produced in the deuteron-deuteron reaction. They found that the cross section for these neutrons was not proportional to $A^{\frac{2}{3}}$, and in fact was not a monotonic function of the atomic weight. Evidently then, even for fast neutrons, nuclear cross sections must have been appreciably dependent on neutron energy. It therefore seemed worth while to measure throughout as wide an energy range as possible the cross sections of a number of elements in order to verify the expected energy dependence, and, more important, to see if there were anomalous scattering. The experimental results were reported some time ago.[3] Aoki[4] had previously reported a similar effect in silicon, and recently in more extended experiments[5] found a number of additional anomalies. Other work has also been reported by Zinn, Seely, and Cohen.[6]

The Apparatus

The neutrons were produced by the action of 100-kv deuterons on targets of heavy ice. The deuteron source and discharge tube were similar to the source and discharge tube described by Zinn and Seely.[7, 8] The ion beam was unanalyzed because it was thought undesirable to have a contamination target of variable strength formed where the discarded beam would hit the walls of the discharge tube. Ion currents used were from 600 to 1000 microamperes, the upper limit being set by the amount of heat which could be dissipated at the target. This was a cylindrical copper box cooled with liquid air, on the front face of which heavy water was condensed. The 100-kv accelerating voltage was supplied by an x-ray transformer-rectifier set.

For counting neutrons, a system consisting of a paraffin-lined ionization chamber, linear amplifier,[9] and thyratron counter[10] was used. The ionization chamber and linear amplifier were mounted on a light table which could be easily rolled about on wheels. A Lauritsen electroscope[11] lined with paraffin was used to measure the neutron intensity.

The arrangement of the apparatus is shown schematically in Fig. 1. The target, scatterer, ionization chamber, and electroscope are represented, respectively, by T, S, I, and E. Scatterers were cylindrical and were slightly larger in di-

* Now with the Standard Oil Company of Venezuela, Caracas.
[1] J. R. Dunning, Phys. Rev. **45**, 586 (1934).
[2] R. Ladenburg and M. H. Kanner, Phys. Rev. **52**, 911 (1937).
[2a] S. Kikuchi and H. Aoki, Sci. Papers of the Inst. of Phys. and Chem. Research **34**, 865 (1938).
[3] M. MacPhail and J. Giarratana, Phys. Rev. **56**, 207 (1939).
[4] H. Aoki, Phys. Rev. **55**, 795 (1939).
[5] H. Aoki, Proc. Phys. Math. Soc. Japan **21**, 232 (1939).
[6] Zinn, Seely, and Cohen, Phys. Rev. **56**, 260 (1939).
[7] W. H. Zinn, Phys. Rev. **52**, 655 (1937).
[8] W. H. Zinn and S. Seely, Phys. Rev. **52**, 919 (1937).
[9] J. R. Dunning, Rev. Sci. Inst. **5**, 387 (1934).
[10] J. Giarratana, Rev. Sci. Inst. **8**, 390 (1937).
[11] C. C. Lauritsen and T. Lauritsen, Rev. Sci. Inst. **8**, 438 (1937).

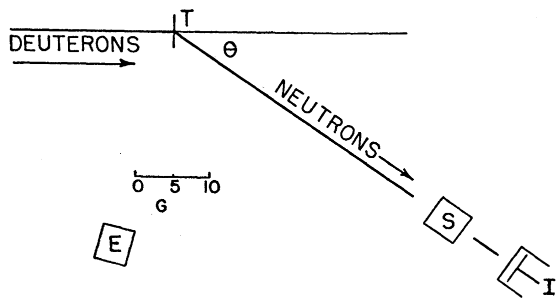

FIG. 1. Arrangement of neutron source T, electroscope E, scatterer S, and ionization chamber I.

ameter than the ionization chamber (4.2 cm) and between 4 and 6 cm in length.

The neutron energy was varied by rotating the counter and scatterer about the target, that is, by varying the angle θ between the paths of the neutrons and the deuterons. The following relation obtained from the equations of conservation of energy and momentum gives the dependence of neutron energy E_n on the deuteron energy E_D and angle θ:

$$E_n-(E_DE_n/2)^{\frac{1}{2}}\cos\theta-(E_D+3Q)/4=0.$$

Q, the energy released in the reaction $D^2+D^2 \rightarrow He^3+n^1+Q$, was taken to be 3.32 Mev.[12] The dependence of the neutron energy on θ is shown graphically in Fig. 2 for deuteron energies of 0.10 and 0.05 Mev which are the energies of the D^+ and $D_2{}^+$ ions, respectively. Because of the presence of both kinds in the beam and because of the slowing down of the deuterons in the ice target, the neutrons will have a spread of energy corresponding to that of the deuterons from 0 to 0.10 Mev.

Experimental Procedure

The cross section for a particular scatterer was calculated from the measured transmission, i.e., the ratio of the number of neutrons indicated by the counter with the scatterer in front of it to that indicated with the scatterer removed, the times being such that the electroscope deflections in each case were the same. A correction was made for the number of neutrons deflected by the scatterer into the ionization chamber on the assumption that the scattering was isotropic. For the dimensions indicated in Fig. 1 and a measured transmission t, this was $0.01(1-t)$ and was to be subtracted from t. A second small correction due to the finite width of the neutron energy spectrum will be given later. From the corrected transmission t_0 thus obtained, the cross section σ was calculated by means of the formula $\sigma=(MA/Nm)\log(1/t_0)$ where A is the area of a face of the scatterer, m its mass, M its atomic or molecular weight, and N Avogadro's number. The exponential law of absorption on which this formula rests was tested by measuring the transmission as a function of length through scatterers of aluminum. The results of two tests are given in Fig. 3.

The intensity of scattered neutrons coming from the walls and floor of the room was estimated by measuring neutron intensity as a function of the distance D of the counter from the target. The results of two tests made at 0° and 90° to the deuteron beam are shown graphically in Fig. 4. All measurements of transmissions were done at a distance of 56 cm from the target for which $D^{-2}=320\times10^{-6}$ cm^{-2}. The intercept at $D^{-2}=0$, i.e., the proportion of neutrons scattered by the walls and floor, is less than 3 percent of the number at 56 cm coming from the target. The effect of these scattered neutrons would be to decrease the cross section from 0 to 3 percent depending on their intensity. But as the intenisty

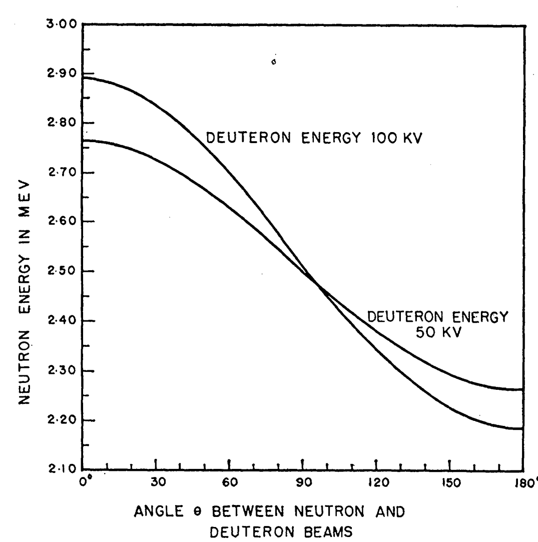

FIG. 2. Neutron energy as a function of the angle of emission for a thin target.

[12] L. G. Bonner, Nature **143**, 681 (1939).

of the scattered neutrons is probably about the same for all values of θ, a correction, if it could be made, would only increase the cross sections at each energy by a small amount.

Because of the finite width of the energy spectrum and the finite angle subtended at the target by the neutron counter, the measured transmission is an average value, the average being taken first with respect to neutron energy and then over the face of the ionization chamber. A simple calculation gives

$$t(\theta)=t_0(E_0)+\frac{1}{2}\left[\frac{\eta^2}{4}\frac{d^2E_0}{d\theta^2}\cdot\frac{dt_0}{dE_0}+(h^2+k^2)\frac{d^2t_0}{dE_0{}^2}\right]+\cdots \quad (1)$$

where $t(\theta)$ is the transmission measured at angle θ (the angle between the direction of the deuterons and the line through the target and the center of the ionization chamber), $t_0(E_0)$ the transmission for homogeneous neutrons of energy E_0, E_0 the average energy of the neutrons emitted at angle θ, η the angle subtended by the radius of the ionization chamber at the target,

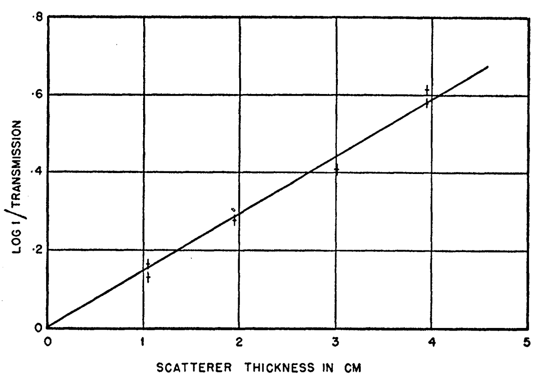

FIG. 3. Experimental test of the exponential law of absorption for the case of aluminum.

$h=\frac{1}{2}\eta dE_0/d\theta$ the root-mean-square variation of neutron energy across the ionization chamber, and k the root-mean-square variation of the neutron energy about the average, E_0, due to the lack of homogeneity of the neutron beam at a given angle θ. The term in square brackets is to a first approximation the effect on the transmission of finite resolving power determined by h and k. We shall neglect terms beyond it which is equivalent to assuming that the transmission-

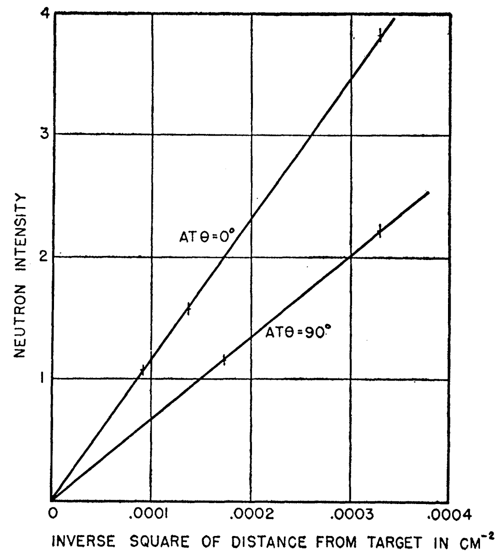

FIG. 4. Testing of the inverse square law; if scattering from the walls of the room were appreciable, the lines would not pass through the origin.

energy curves show no fine structure. Replacing t_0 by t in the square brackets, we can then solve (1) for t_0 in terms of the measurable quantities, $t(\theta)$ and its first two derivatives; i.e.,

$$t_0(E_0)=t(\theta)-\left[\frac{\eta^2}{8}\frac{d^2E_0}{d\theta^2}\cdot\frac{dt}{dE_0}+\frac{1}{2}(h^2+k^2)\frac{d^2t}{dE_0{}^2}\right]. \quad (2)$$

The first term in the square brackets turns out to be 10^{-4} or less, so is negligible. The other term represents the combined effects of an ionization chamber of finite size and of inhomogeneous neutrons. It is seen to sharpen the transmission curves by raising the peaks and lowering the hollows where $d^2t/dE_0{}^2$ is, respectively, negative and positive. This is of course what one would expect.

To obtain E_0 and k as functions of θ, it is necessary to calculate the neutron spectrum. Thus, if the number of neutrons emitted at angle θ with energies between E and $E+dE$ is $f(E)dE$, then

$$DE_0=\int_0^\infty Ef(E)dE$$

and

$$Dk^2=\int_0^\infty (E-E_0)^2f(E)dE, \quad \left[D=\int_0^\infty f(E)dE\right].$$

The function $f(E)$ was calculated on the assump-

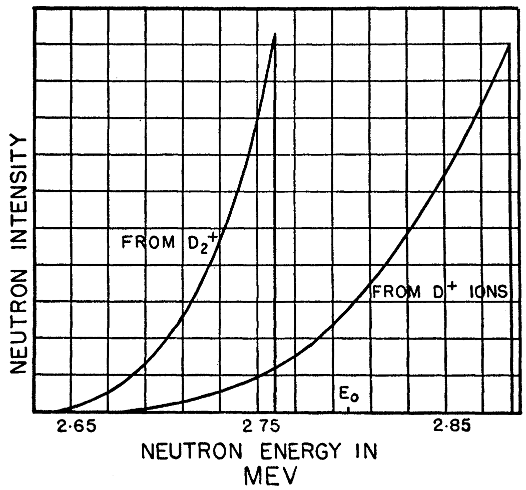

FIG. 5. Distribution in energy of the neutrons coming off the target in the forward direction calculated for 100 kv accelerating potential for the cases (*a*) pure molecular beam, (*b*) pure atomic beam.

tion that the ice target was thick (an effort was made always to fulfill this condition experimentally), and that the unanalyzed beam of deuterons was 20 percent atomic and 80 percent molecular. In addition, the 1 Mev group of neutrons reported by Bonner[12] and others was not taken into consideration because of general uncertainty regarding it. The excitation function used was that given by Baldinger, Huber, and Staub.[13] By way of an example, the neutron spectrum calculated in this way for $\theta=0°$ is given in Fig. 5. Values of E_0 and k for $\theta=0°$ and certain additional angles are listed in the following table:

θ	0°	58°	76°	92°	109°	129°
E_0 (Mev)	2.80	2.65	2.57	2.49	2.41	2.34
k (Mev)	0.065	0.035	0.020	0.009	0.010	0.025

The other quantities in square brackets in (2), namely, $h=\eta dE_0/2d\theta$, $d^2E_0/d\theta^2$, dt/dE_0, and d^2t/dE_0^2, were found graphically. As mentioned before, the correction was entirely due to the term $-\frac{1}{2}(h^2+k^2)d^2t/dE_0^2$. In absolute magnitude it was in every case between 0 and 0.008; that is, it was of the order of 1 percent or less of t.

The cross section for each material and for each chosen neutron energy was calculated from the corresponding transmission after the corrections mentioned above had been applied. This value of the transmission was the average of from five to twenty measurements made on different days. The probable error of the average transmission was calculated from the deviations of the individual values about their means. In Table I are given as functions of neutron energy the average transmission (corrected), its probable error, and the corresponding cross section in units of 10^{-24} cm^2.

The cross sections given in Table I are plotted against neutron energy in Fig. 6. The coordinates of the center of each rectangle are the most probable values of the cross section and the

[13] Baldinger, Huber, and Staub, Helv. Phys. Acta **11**, 245 (1938).

TABLE I. *Average corrected transmission coefficients, their probable error (P.E.) and cross section in units of 10^{-24} cm^2 as functions of neutron energy.*

E (MEV)	2.80	2.65	2.57	2.49	2.41	2.34
			Aluminum (thickness 10.6 g cm^{-2})			
Trans.	0.556	0.496	0.496	0.598	0.596	0.556
P.E.	0.004	0.005	0.004	0.003	0.006	0.006
σ_{Al}	2.48	2.94	2.94	2.18	2.19	2.49
			Magnesium (thickness 8.75 g cm^{-2})			
Trans.	0.602	0.576	0.630	0.684	0.657	0.623
P.E.	0.007	0.006	0.007	0.004	0.006	0.010
σ_{Mg}	2.34	2.54	2.14	1.76	1.94	2.19
			Carbon (thickness 7.80 g cm^{-2})			
Trans.	0.543	0.566	0.585	0.585	0.580	0.579
P.E.	0.006	0.007	0.005	0.005	0.003	0.010
σ_C	1.57	1.45	1.38	1.38	1.39	1.41
			Sodium (thickness 5.73 g cm^{-2})			
Trans.	0.700	0.686	0.668	0.668	0.668	0.663
P.E.	0.004	0.007	0.007	0.005	0.006	0.006
σ_{Na}	2.38	2.50	2.69	2.69	2.69	2.74
			Sodium azide (NaN_3) *(thickness 6.78 g cm^{-2})*			
Trans.	0.680	0.671	0.650	0.665	0.671	0.656
P.E.	0.004	0.007	0.006	0.006	0.002	0.013
σ_{NaN_3}	6.13	6.35	6.86	6.50	6.35	6.72
σ_N	1.25	1.28	1.39	1.27	1.22	1.33

corresponding energy. The height of the rectangles for a given element is the experimental probable error in σ. This was calculated from the average observed probable error of the corresponding transmissions. Widths of the rectangles indicate the amounts by which the neutron energy would change due to a change in proportion of atomic deuterons in the beam from 10 to 30 percent about a mean of 20 percent. The side of the rectangle corresponding to 10 percent atoms is the one nearer the energy of 2.50 Mev.

There are two additional possible sources of error in the neutron energies:

1. An error dQ in the Q value for the D–D reaction would cause the cross-section curves to be shifted horizontally by approximately $3dQ/4$. Bonner's estimate for dQ is ± 0.04 Mev.[12]

2. Fluctuations of the accelerating voltage. This voltage was measured frequently and believed to be 100 ± 5 kv. The effect on the neutron energy would be similar to but smaller than that produced by fluctuations in the proportion of atoms in the deuteron beam.

The results of Aoki[5] and of Zinn and others[6] for the elements reported here have been added to Fig. 6 after they were recalculated with $Q=3.32$ Mev and allowances made for the widths of the neutron spectra. Each of their values is represented by the chemical symbol of the element, the former's surrounded by a circle and the latters' by a triangle.

A Theory of Anomalous Scattering of Fast Neutrons

We shall now attempt to discuss on the basis of present theory the anomalous behavior of the magnesium cross section around 2.5 Mev and later make a few remarks about the case of aluminum.

What happens when magnesium is bombarded by fast neutrons is complicated by the presence of three isotopes of masses 24, 25, and 26 and respective proportions 0.78, 0.11, and 0.11; but because of the greater abundance of Mg^{24}, we shall attribute the anomaly in cross section to an excited level in the intermediate Mg^{25} nucleus formed from Mg^{24}. The possible reactions to be considered are elastic and inelastic scattering and the following disintegrations and capture:

1. $Mg^{24}+n^1\rightarrow Na^{24}+H^1-4.0$ Mev,
2. $Mg^{24}+n^1\rightarrow Ne^{21}+He^4-2.3$ Mev,
3. $Mg^{24}+n^1\rightarrow Mg^{25}+h\nu$ (7.0 Mev).

With 2.5-Mev neutrons, the first disintegration is energetically impossible, and the second highly improbable because of the small amount of surplus energy for the α-particle. The third can be

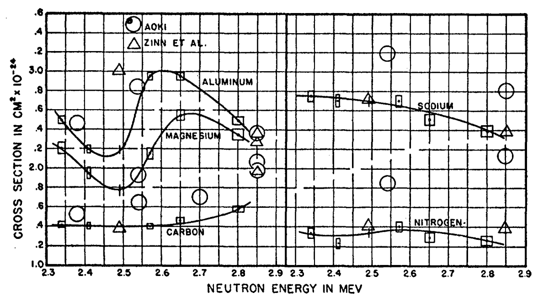

FIG. 6. Observed scattering cross sections as a function of neutron energy. The rectangles represent the present observations, their heights and widths corresponding to the range of error in cross section and energy determinations. The circles give the results of Aoki, the triangles those of Zinn, Seely, and Cohen (both recalculated; see text).

neglected too, being about 10^{-4} times as probable as scattering.[14]

Inelastic scattering also is apparently small. The cross section for this process has been given by Kikuchi and others[15] as 0.08×10^{-24} cm² or only about 3 percent of the total cross section. We therefore conclude that the only important process to be considered is elastic scattering by the nucleus Mg^{24}.

The cross section for elastic scattering of particles of zero spin by nuclei also of zero spin is[16]

$$\sigma=4\pi\lambda\!\!\!^{-2}\sum_n(2n+1)\sin^2\delta_n, \tag{3}$$

where $\lambda\!\!\!^{-}=\hbar/(2ME)^{\frac{1}{2}}$. M is the reduced mass of the incident particle, E its kinetic energy, and n its orbital quantum number in the field of the nucleus. The δ's are phase shifts of the outgoing or scattered waves with respect to an incident wave unaffected by the scattering nucleus. If,

[14] H. A. Bethe, Rev. Mod. Phys. **9**, 160 (1937), Table XXVIII.
[15] Kikuchi, Aoki, and Husimi, Proc. Phys. Math. Soc. Japan **18**, 115 (1936).
[16] N. E. Mott and H. S. W. Massey, *The Theory of Atomic Collisions*, p. 24.

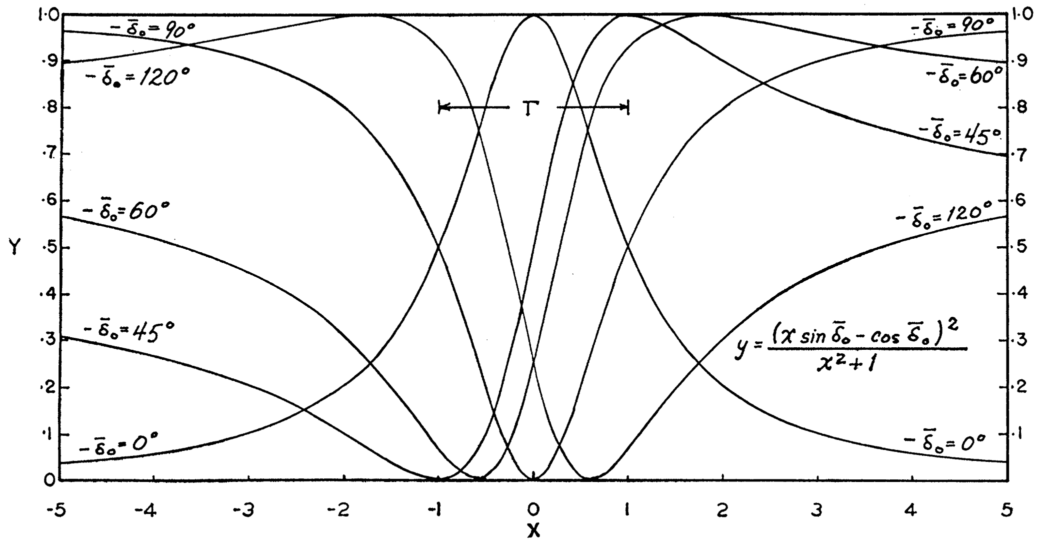

FIG. 7. If the elastic scattering cross section is multiplied by neutron energy, theory predicts in certain simple cases that the result is a function of energy which is composed of a constant part and a part which varies with energy as $y=\sin^2\delta$, where $\delta=\bar{\delta}-\text{arc cot }(E-E_0)/\frac{1}{2}\Gamma_0=\bar{\delta}-\text{arc cot }x$. The theoretical curves given here for y as a function of x are compared with the observations to find the value of $\bar{\delta}$ which gives the best agreement.

however, the incident particles have spin $\frac{1}{2}$, the nuclei still having spin 0, the cross section will be

$$\sigma=4\pi\lambda\!\!\!^{-2}\sum_{s=\pm\frac{1}{2},\,n=0,\,1,\,\cdots}(n+\tfrac{1}{2})\sin^2\delta_{ns}. \tag{4}$$

For orbital quantum number $n=0$ there is only one phase shift to consider, $\delta_0=\delta_{0,\frac{1}{2}}=\delta_{0,-\frac{1}{2}}$.

The phase shifts occurring in (4) are in principle determinable by solving the wave equation for the compound nucleus temporarily formed in the scattering process. We are interested in the case when this compound nucleus possesses a certain semi-stable level which is narrow in comparison both with the kinetic energy of the neutron and with the distance between levels. We denote by n, s the orbital and spin quantum numbers of the neutron which is required to form this state from the original nucleus, by E_{ns} the energy which the incident neutron should have to be in resonance with this state, and by Γ_{ns} the width of the resonance level. The mean lifetime, T, of the compound state is connected with Γ_{ns} by the relation $T=\hbar/\Gamma_{ns}$. According to the theory of nuclear scattering,[17, 18] the neutron wave corresponding to the given orbital and spin quantum numbers will be shifted in phase with respect to an unperturbed wave by an amount which depends on the neutron energy E and which is given by

$$\delta_{ns}=\bar{\delta}_{ns}-\text{arc cot }(E-E_{ns})/\tfrac{1}{2}\Gamma_{ns} \tag{5}$$

provided the neutron energy does not come into the neighborhood of another resonance level. The constant $\bar{\delta}_{ns}$ expresses the influence of all other levels of the compound nucleus which belong to the given spin and orbital quantum numbers.

It is evident from (5) what will happen as the neutron energy is increased. Considerably below the resonance energy we shall have a phase shift $\delta_{ns}=\bar{\delta}_{ns}$. As E increases, the phase shift rises more and more rapidly until we reach the center of the resonance level, where $E=E_{ns}$ and $\delta_{ns}=\bar{\delta}_{ns}+\pi/2$; the phase shift then continues to increase, but more and more slowly, reaching finally the value $\bar{\delta}_{ns}+\pi$. This is true of course only of the phase shift corresponding to the given orbital and spin quantum numbers; the variation with energy of the other phase shifts may be neglected if we are dealing with the scattering in the neighborhood of a single isolated resonance level. The actual scattering cross section, according to (4), is the product of the slowly varying factor $4\pi\lambda\!\!\!^{-2}$ (in-

[17] G. Breit and F. L. Yost, Phys. Rev. **48**, 203 (1935).
[18] P. L. Kapur and R. Peierls, Proc. Roy. Soc. **166**, 277 (1938).

versely proportional to neutron energy) and a sum of terms due to neutrons of various orbital and spin quantum numbers. Of these terms only one will change appreciably through the resonance region and be responsible for the variation of cross section with energy. This term is (except for the constant factor $n+\frac{1}{2}$)

$$y=\sin^2 \delta_{ns}.$$

It may either increase or decrease as we approach the resonance level, according as the constant $\bar{\delta}_{ns}$ happens for the particular nucleus to be less than or greater than $\pi/2$. In fact, from the manner in which the cross section varies near resonance we can even estimate the actual value of $\bar{\delta}_{ns}$ with some accuracy. Curves are drawn in Fig. 7 showing the variation with energy of the term $y=\sin^2 \delta_{ns}$ for a number of values of $\bar{\delta}_{ns}$. The independent variable in these curves is $x=(E-E_{ns})/\frac{1}{2}\Gamma_{ns}$; since it varies linearly with energy, the theoretical curves can be compared as regards *shape* (quite without any reference to the *absolute magnitude* of the scale of independent and dependent variables) with a plot of the observed cross section as a function of energy. It was found in this way that $\bar{\delta}_{ns}=-60°$ gave a theoretical variation of cross section with energy in closest accord with the observations on magnesium reported here.

By comparing on an *absolute* scale the theoretical and observed scattering, we can determine in addition to $\bar{\delta}_{ns}$ (i) the value of the quantum number n (ii) which appears in the statistical factor $n+\frac{1}{2}$, the energy E_{ns} (iii) and width Γ_{ns} (iv) of the resonance level, and the essentially constant contribution (v) to the scattering due to deflection of neutrons with other spin and orbital quantum numbers. Conversely, we may say that we have four adjustable constants and our choice of a certain integer n with the help of which to fit the theoretical formula to the observations. To determine n, we note that the factor y varies as we go through resonance between a minimum value of 0 and a maximum value of 1, so that the scattering cross section changes by the amount

$$\begin{array}{lll} 4\pi\bar{\lambda}^2 & \text{if} & n=0, \\ 4\pi\bar{\lambda}^2(n+\frac{1}{2}) & \text{if} & n=1,2,\cdots \end{array} \qquad (6)$$

(provided we neglect the slight variation of $\bar{\lambda}^2$ itself with energy) its minimum value being determined by the contribution (v).

We shall apply (6) to the experimental results for magnesium, but must first multiply $4\pi\bar{\lambda}^2$ by the relative abundance 0.78 of the isotope Mg^{24}. With a neutron energy of 2.55 Mev we have $0.78\times 4\pi\bar{\lambda}^2=0.84\times 10^{-24}$ cm². Therefore the total variation in cross section through resonance will be

$$0.84\times 10^{-24}\ \text{cm}^2 \quad \text{if} \quad n=0. \qquad (7)$$

$$0.84\times 10^{-24}(2n+1)/2 \quad \text{if} \quad n=1,2,\cdots, \qquad (8)$$

according to theory. The observed value taken from Fig. 6 is 0.80×10^{-24} cm². Thus all values of n other than $n=0$ seem to be entirely excluded. On the other hand, with $n=0$, we obtain a real check between theory and observation.

The constant $\bar{\delta}_{ns}$ is to be designated as $\bar{\delta}_0$ in the present case and has already been estimated to be $-60°$. The resonance energy E_0 and level width Γ_0 are conveniently found from the energies E_1 and E_2 at which the observed cross section reaches, respectively, its maximum and minimum values by the help of the relations

$$E_0=\tfrac{1}{2}(E_1+E_2)+\tfrac{1}{2}(E_1-E_2)\cos 2\bar{\delta}_0, \qquad (9)$$

$$\Gamma_0=(E_2-E_1)\sin 2\bar{\delta}_0, \qquad (10)$$

which follow directly from Eqs. (4) and (5). Observing that $E_1=2.67$ Mev and $E_2=2.50$ Mev (see Fig. 6) we complete our determination of the

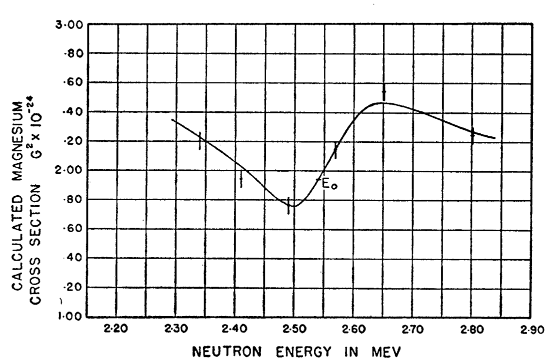

FIG. 8. The vertical lines represent the present observations on magnesium for six different energies and are compared with the smooth theoretical curve calculated with the best values of the four adjustable parameters discussed in the text. These parameters allow no freedom as to the vertical scale of the theoretical curve; for the increment in cross section between minimum and maximum theory gives 0.84×10^{-24} cm², observation gives 0.80×10^{-24} cm².

TABLE II. *The complexity of the states of the compound nucleus* Al^{28}. *A few of the lowest states are classified according to parity, total angular momentum, orbital angular momentum and multiplicity.*

	Total angular momentum			
Parity	0	1	2	
Even	^5D	$^5D\ ^7D\ ^7G$	$^5S\ ^5D\ ^7D\ ^5G\ ^7G$	etc.
Odd	^7F	$^5P\ ^5F\ ^7F$	$^5P\ ^7P\ ^5F\ ^7F\ ^7H$	

parameters which enter the theoretical formula for the cross section near resonance. They are four in number (since $n=0$ is not really an adjustable constant):

(i) $\bar{\delta}_0=-60°$,
(iii) $E_0=2.54$ Mev,
(iv) $\Gamma_0=0.15$ Mev,
(v) Cross section's minimum value

$$=1.76\times 10^{-24}\ \text{cm}^2. \quad (11)$$

The value found for Γ corresponds to a mean life of the given level of the compound nucleus of 4.5×10^{-21} sec. On the other hand, the time for a 2.5-Mev neutron to travel a nuclear diameter (say 10^{-12} cm) is 4.5×10^{-22} sec. That is, the neutron spends about 10 times as long inside the nucleus as it would take to travel straight across it with no interaction.

With the constants given in (11), the cross section was calculated as a function of energy. In this calculation the variation with energy of the factor $4\pi\lambda^2$ in Eq. (4) was taken into account. The results are plotted in Fig. 8. The six experimental values indicated by the vertical strokes are added for reference and show satisfactory agreement with the theoretical formula containing the four adjustable parameters.

The conclusion, based on the foregoing interpretation, is that the anomalous scattering found in magnesium is due to an S level in Mg^{25} at a height of 9.5 Mev above ground. This height was calculated from the masses of the neutron, of Mg^{24}, and of Mg^{25} as well as from the neutron energy at resonance.

Because there is but one aluminum isotope, Al^{27}, the situation is much better in this respect than for magnesium. The problem is however greatly complicated by the large nuclear spin, 5/2,[19] of Al^{27}. The simplification in the case of magnesium was due to the fact that only one orbital quantum number n could be assigned to a state of given parity and given total angular momentum. On the other hand, the complexity of the states of the compound nucleus, Al^{28}, is indicated in Table II in which a few of the lowest ones are classified according to parity, total angular momentum, orbital angular momentum, and multiplicity (given by the superscript). The scattering cross section for a particular state is now not only determined by the corresponding values of n but also by the transition probabilities between the different components of that state. These are unfortunately unknown.

This method of treating elastic neutron scattering is accordingly limited in its applicability to nuclei of spin zero. It would however be possible to extend it to light nuclei of spin $\frac{1}{2}$ for which S states might be expected to be most probable. If $n=0$, the variation of cross section near resonance would then be given by $3\pi\lambda^2 \sin^2 \delta_{03}$ or by $\pi\lambda^2 \sin^2 \delta_{01}$ according as the state was a triplet or singlet. The δ's would of course still be determined by Eq. (5). This is a case which it will be useful to test as more experimental results are obtained.

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[19] S. Millman and P. Kusch, Phys. Rev. **56**, 303 (1939).