ible. The straightness of the lead absorption curve is therefore an indication of the maximum possible proportion of radiation with energy appreciably different from 0.511 Mev.

A second indication is the fact that the energy of the Li^7 gamma-radiation (0.459 Mev) measured here agrees with the value 0.455 ± 0.015 Mev obtained by RRH in an entirely different way. The probable error in the whole determination was figured from estimates of: first, the errors in measuring the absorption coefficients of the positron radiation and of the soft component of the lithium radiation; second, the errors in reading slopes and absorption coefficients from Heitler's curves; and third, the error given by RRH for the energy of the excited level. The over-all probable error is taken as the square root of the sum of the squares of the individual probable errors. It is about the same as the error that would have been made in determining the energy of the lithium level if there had been 10 percent as many 0.28-Mev quanta as 0.511-Mev quanta in the radiation from N^{13} and its positrons.

This upper limit of 10 percent is on the low side of Lyman's 10 ± 7.5 percent but since it was obtained in such a devious way, it must be considered only as supplementary evidence that very little 0.28-Mev radiation is emitted.

Mr. R. E. Warren and Mr. D. L. Bobroff have aided in taking data and making drawings. Professor G. Breit has given valuable discussions of the nuclear processes involved. The Wisconsin Alumni Research Foundation and the Penrose Fund of the American Philosophical Society have furnished financial support.

APRIL 1, 1940 PHYSICAL REVIEW VOLUME 57

Positive Excess and Electron Component in the Cosmic-Ray Spectrum

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(Received January 25, 1940)

The sea-level cosmic-ray energy spectrum has been determined, using a 30-cm counter controlled cloud chamber in a 12,400-oersted magnetic field. The spectrum of mesotrons alone is obtained by inserting a 10-cm lead filter in the counter train, the total spectrum is obtained with no filtering, and the spectrum of the soft (electron) component is then the difference between these two spectra. An excess of positive particles exists both with and without the lead, the ratio of positives to negatives being 1.21 ± 0.08 (standard error) in the former, and 1.18 ± 0.08 in the latter case. Comparison of the spectra with and without lead shows the presence of an absorbable component (electrons) in the energy region 2 to 8×10^8 ev but no absorbable particles of higher energy. The spectra have been corrected for the distortion at low energies caused by the magnetic field. The corrected mesotron spectrum possesses a maximum and a rapid decrease below the maximum while the total spectrum shows no maximum but a continuous increase in number with decreasing energy.

IN June, 1939, at the Chicago cosmic-ray symposium H. Jones[1] reported on the energy distribution of mesotrons. The energy spectrum was obtained with the large cosmic-ray magnet at the University of Chicago, using a 10-cm lead filter to remove electrons. The results showed a greater number of positive than negative particles, the excess, amounting to 29 percent, being spread rather uniformly throughout the spectrum. Such an excess might mean either a greater absorption of negatives in the lead or a real positive excess in the energy spectrum incident on the magnet. Blackett's[2] work on the unshielded energy spectrum showed a small positive excess which was interpreted by him as being probably of no significance. Ringuet,[3] using a lead absorber, found a large positive excess at high energies but no excess without the lead. Pair production of mesotrons would of course result in an equality in regard to sign, and such an equality

[1] H. Jones, Rev. Mod. Phys. **11**, 235 (1939).

[2] P. M. S. Blackett, Proc. Roy. Soc. **A159**, 1 (1937).

[3] L. Leprince-Ringuet and J. Crussard, J. de phys. et rad. **8**, 207 (1937).

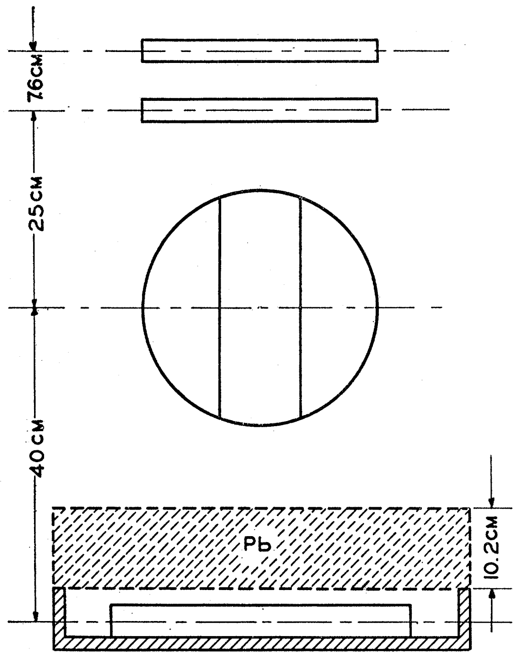

FIG. 1. Diagram of apparatus.

is usually assumed. However, explanation of the observed excess in terms of absorption of negatives is very difficult, for the lead absorbers used should have practically no effect on the high energy rays which are involved in these experiments. Because of the open nature of this question it seemed desirable to us to continue the experimental work, having two objects in view, (1) to remove as far as possible any instrumental asymmetry which might give rise to a spurious effect and (2) to determine the relation of the lead absorber to the positive excess.

Apparatus

The apparatus is shown diagrammatically in Fig. 1. Details of construction and operation of the cloud chamber and electromagnet have been given elsewhere.[4] Expansions of the 30-cm chamber are controlled by the triple coincidence G–M counters, the average interval between expansions being of the order of 2 min. When the lead filter is in place, only those particles with a residual range greater than 10-cm lead, namely, the mesotrons, are recorded. With the filter removed, the absorbable particles (the electrons) as well as the mesotrons are obtained.

It is of great importance that the effect of any systematic curvature introduced by distortion be removed. If gas motions in the chamber tend to curve tracks more in one direction than the other, a spurious excess of one sign would of course develop. Before taking the present series of photographs, therefore, we have attempted to reduce and balance the distortions as much as possible. Accurate control of the motion of the chamber diaphragm by material used, placement, and pressure regulation, has been found to be of primary importance in controlling distortion. Reference 4 gives a more complete description of these measures, and Fig. 2 shows the distortions now present in the chamber. Here are plotted the observed curvatures (caused by distortion) of 110 tracks photographed with no magnetic field during the course of the present experiment. d is the central displacement for a 20-cm track length, while E is the energy which would be inferred from d if the 12,400-oersted field were present. The distribution of d's about the correct value zero gives the probable error in d which is shown in the figure—0.012 cm. This probable error applied to the measurements with magnetic field represents an error of 6 percent at 10^9 ev and 100 percent at 1.6×10^{10} ev, thus fixing the maximum detectable energy at the latter value. The curve shows no asymmetry which might cause an apparent excess of particles of one sign.

Measurement of the distortions with zero field,

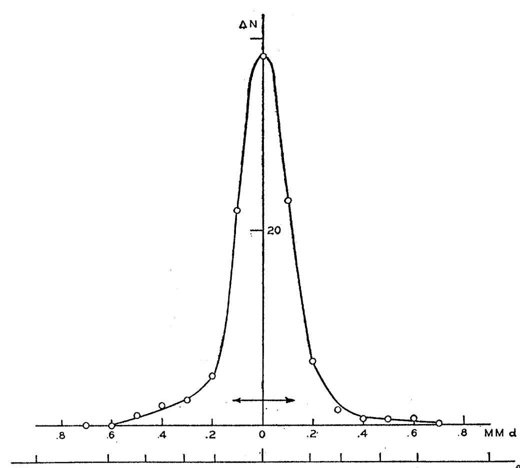

FIG. 2. Curvature of zero field tracks.

[4] H. Jones and D. Hughes, Rev. Sci. Inst. **11**, 79 (1940).

of course, does not take into account those additional distortions which may be present only when the magnet is on, for instance those arising from convection currents caused by heating. To reduce convection currents, the temperature of the chamber has been controlled by reducing the exciting current of the magnet so that the pole pieces and chamber can be kept quite accurately at room temperature by the cooling oil. While the field strength has been lowered only from 16,000 to 12,400 oersteds, the power consumption and heat produced is reduced by 75 percent. This temperature control is effective in stopping the mass motion of gas that sometimes develops when the apparatus is not at temperature equilibrium.

However, to average out any asymmetry that might be present in spite of the symmetrical shape of Fig. 2, the magnetic field has been periodically reversed throughout the experiment. This procedure should nullify the asymmetrical effect of any distortions which are present with the magnetic field but absent without it.

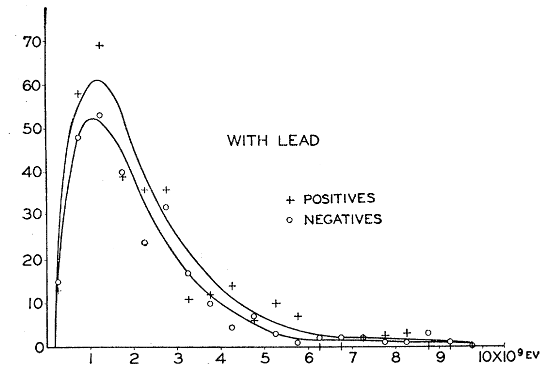

FIG. 3. Positive-negative spectra with lead absorber.

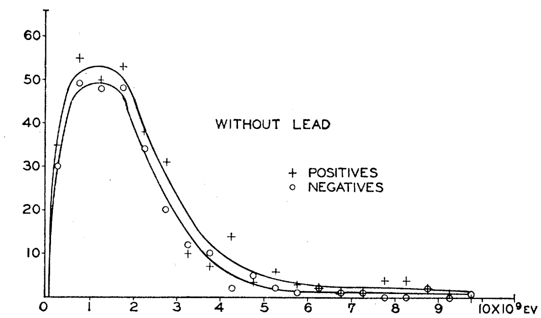

FIG. 4. Positive-negative spectra without absorber.

THE POSITIVE EXCESS

A total of 1334 tracks with magnetic field has been measured, 674 with, and 660 without the lead absorber. The results form two separate energy spectra, which are given in Table I. Here are listed the number of positive and negative particles in each 5×10^8 ev energy interval for both the unfiltered and the filtered spectrum.

TABLE I. *Energy distribution of tracks.*

Range in 10^9 ev		With lead +	With lead −	With lead Total	Without lead +	Without lead −	Without lead Total
0	to 0.5	13	15	28	35	30	65
0.5	1.0	58	48	106	55	49	104
1.0	1.5	69	53	122	50	48	98
1.5	2.0	39	40	79	53	48	101
2.0	2.5	36	24	60	38	34	72
2.5	3.0	36	32	68	31	20	51
3.0	3.5	17	11	28	10	12	22
3.5	4.0	10	12	22	7	10	17
4.0	4.5	14	7	21	14	2	16
4.5	5.0	7	6	13	3	5	8
5.0	5.5	10	3	13	6	2	8
5.5	6.0	7	1	8	3	1	4
6.0	6.5	0	2	2	2	2	4
6.5	7.0	0	2	2	1	1	2
7.0	7.5	2	2	4	1	1	2
7.5	8.0	2	1	3	4	0	4
8.0	8.5	3	1	4	4	0	4
8.5	9.0	0	3	3	2	2	4
9.0	9.5	0	1	1	1	0	1
9.5	10.0	0	0	0	0	1	1
10.0	∞			87			72
		323	264	674	320	268	660

Considering the total number of positives and negatives for each spectrum, it is seen that the positive excess is present both with and without lead, the ratio (R) of positives to negatives having the following values:

$$\text{With lead: } R=1.21\pm0.08 \text{ (standard error)}$$
$$\text{Without lead: } R=1.18\pm0.08.$$

Thus the value is the same within experimental error whether the absorber is present or not. The presence of the excess with lead confirms Jones' result, and the equality of R in both cases shows that the excess is real, i.e., is present in the sea-level cosmic-ray spectrum, and is not of instrumental origin.

If the tracks are grouped according to the direction of the magnetic field ("direct" or "reversed"), it is found that both with and without lead, the positive excess is smaller with the field reversed. This difference is less than the experimental error but may be indicative of an asymmetry which occurs only with the magnet on, but which is averaged out by the field reversals.

The positive-negative spectra are compared graphically for the data with lead absorber in Fig. 3 and for that without the absorber in Fig. 4. Although the separation of the positive and the negative curves is not great, it is evident in each case that the positive excess is spread throughout the spectrum rather than localized in any one

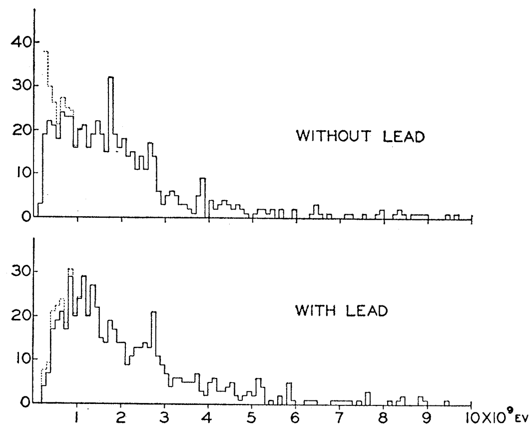

FIG. 5. Positives and negatives combined; 10^8 ev energy intervals.

region of it. This rough constancy of R, which was also found in Jones' work, is not in accord with Ringuet's finding that the excess increases with energy.

The experimental results, of course, give no information concerning the origin of the observed excess. It is well known that the primary rays which give rise to the mesotrons are predominantly positive, and it may be that this initial excess of positives is in some way propagated to sea-level.

THE ELECTRON COMPONENT

If the positives and negatives are grouped together, we have two energy spectra, one of the mesotrons alone and another of the total spectrum. The raw data comprising these are shown by the solid line in the block diagrams of Fig. 5. The presence of an absorbable component at energies less than 10^9 is immediately obvious upon comparing the two curves. Both spectra are somewhat distorted below 10^9 by magnetic deflection of rays, but the ratio of the ordinates of the two curves at any energy is unaffected by such distortion.

Comparison of the spectra with and without lead shows the location in the spectrum of the rays absorbed by the lead. The counting rate *in the magnetic field* is increased by 10 percent when the lead is removed. This means that of those rays capable of passing the magnetic barrier and setting off the counters, about 10 percent are absorbed by the lead. (The counting rate increases by 39 percent with removal of the lead when the magnet is off, hence most of this 39 percent soft group is of energy below the magnetic cut-off value.) In order to compare the spectra, therefore, it is necessary to adjust the areas under the two curves, the area under the unshielded spectrum being made 10 percent greater than the other.

The adjusted spectra are shown together in Fig. 6 (energy intervals of 5×10^8 ev), and the low energy end in more detail in Fig. 7 (energy intervals of 1×10^8 ev). It is seen that there is no significant difference between the spectra for high

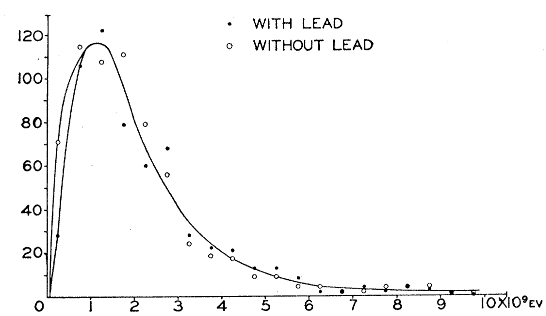

FIG. 6. Spectra with areas adjusted; 5×10^8 ev energy intervals.

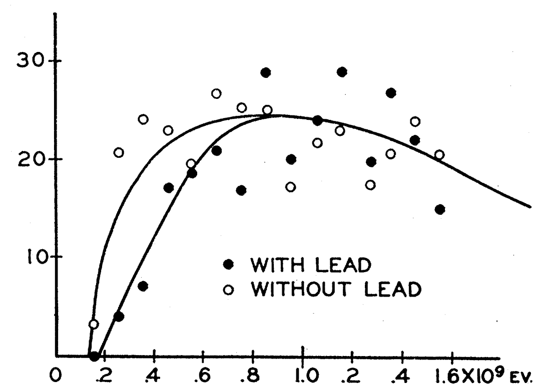

FIG. 7. Low energy region of Fig. 6; 10^8 ev energy intervals.

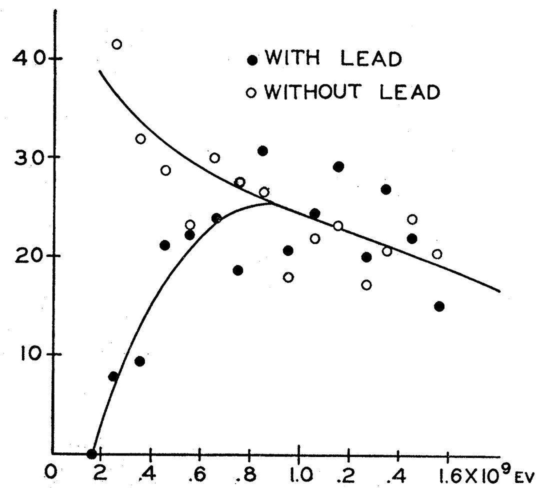

FIG. 8. Low energy spectra corrected for magnetic distortion.

energies, while for energies less than about 8×10^8 ev there is a definite absorbable group. It must be remembered that the difference between the two curves at low energies is in reality greater than it appears, for at such energies both curves are lowered by magnetic deflection. An actual count of the tracks in the spectra of Figs. 5 and 6 shows that for energies less than 8×10^8 ev there are 186 tracks in the unshielded spectrum and 134 in the shielded, while for energies greater than this value, the numbers are 506 and 505, respectively. Thus, there are 52 particles absorbed in the low energy region of the spectrum and none, within experimental error, at higher energies.

The rays stopped by the lead must represent electrons of sufficient energy to pass the magnetic barrier. They cannot be mesotrons, losing energy by ionization only, for a mesotron with residual range less than 10–14 cm lead has less than the magnetic cut-off energy. If the supposition of the preceding sentence is untrue—if mesotrons in the interval 2 to 8×10^8 ev lose energy by some process in addition to ionization—then the absorbable group may be composed in part of mesotrons. Analysis shows that the number of particles removed by scattering in the lead block is less than $\frac{1}{4}$ percent and hence does not affect the results appreciably. The scattering is small because the upper counters are close together and at a great distance from the lower counter, because the lower counter is larger than the upper two, and because the lead block is very close to the lower counter.

It is possible to calculate the fraction of the rays of a given energy which are deflected away from the chamber and hence to correct the low energy end of the spectra for this magnetic deflection. This has been done by tracing the paths of representative rays through the apparatus, finding the "allowed cone" and, taking into account the zenith angle distribution, calculating the number of rays at each energy which pass through the chamber. The correction is inappreciable for energies greater than 1.2×10^9 ev and increases with decreasing energy until it becomes 100 percent between 1 and 2×10^8 ev. The dotted line of Fig. 5 shows the raw data corrected for the magnetic deflection, and Fig. 8 the corrected low energy region of Fig. 7. The spectrum of the penetrating rays changes only slightly with the correction, still showing a maximum and a rapid decrease below the maximum. The total spectrum on the other hand shows no maximum at all, but a continuous increase in number with decreasing energy. Because the correction becomes indeterminate between 1 and 2×10^8, the extreme low energy ends of the spectra are unknown.

Counting rates taken with no magnetic field support the correctness of this calculation. The change in the area under each spectrum due to the calculated correction should, of course, be equal to the change in the measured counting rate when the field is removed. The counting rate without lead increases by 28 percent when the field is removed, while that with lead increases by only 3 percent, which is in accord with the calculated increases in area.

The results thus show that there are very few electrons of energies greater than about 8×10^8, while from this energy to about 2×10^8 there is a transition region in which the predominating particles change from mesotrons to electrons, which other work has shown are dominant at lower energies. Blackett and Wilson's experiments on the energy loss in metal plates has shown the sudden decrease in the electron spectrum, and Wilson has placed the transition region at 1 to 3×10^8 ev. The present work, involving a larger number of photographs, shows the same behavior but indicates that the electron component extends to appreciably higher values than that given by Wilson.

It must be emphasized that this experiment, using a high field, was not primarily intended for work in this transition energy region. More accurate information, not involving the necessity of so stringent a correction, could be obtained by using an experimental set-up with a lower field such that the magnetic correction would set in at an energy lower than the one of primary interest.

It is not only a pleasure but a privilege for me to acknowledge at this point the ever-present help and inspiring guidance of Professor A. H. Compton which have made this work possible. Dr. Haydn Jones has been directly responsible for the construction of most of the apparatus used, and I wish to express my great indebtedness to him. In all the work of taking the photographs and measuring the tracks Mr. F. Leslie Code has been of very great assistance. The help of Mr. Ralph Meagher is also gratefully acknowledged.

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A Theoretical Study of the Diffuse Scattering of X-Rays by Crystals*

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(Received January 22, 1940)

A new derivation is made of the intensity expression for the coherent scattering of x-rays by a small crystal. The accepted intensity formula for the diffuse scattering—the well-known Debye formula—is shown to be incorrect and is replaced by a more complicated expression. According to the revised theory the intensity of the diffuse scattering varies much more rapidly with the scattering direction and exhibits a series of diffraction maxima. These maxima are found in the directions $\mathbf{k}_m$ satisfying equations of the type $(1+\lambda\tau_{\min})\mathbf{k}_m=\mathbf{k}_0+\mathbf{B}_H$, where $\mathbf{k}_0$ is the direction of incidence (with $k_m=k_0=1/\lambda$, the reciprocal wave-length) and $\mathbf{B}_H$ a reciprocal lattice vector. The consequences of the theory are discussed in detail.

Introduction

When a beam of parallel and monochromatic x-rays falls on a crystal scattering processes take place. Disregarding the thermal agitation of the atoms in the crystal lattice the intensity of the coherent part of the scattered radiation is given by the well-known Laue expression

$$J_0=Sf^2\prod_i\frac{\sin^2\left[N^{\frac{1}{3}}\pi(\mathbf{k}-\mathbf{k}_0)\cdot\mathbf{a}_i\right]}{\sin^2\left[\pi(\mathbf{k}-\mathbf{k}_0)\cdot\mathbf{a}_i\right]}. \qquad (1)$$

The equation holds for a small crystal (linear dimensions of the order 10^{-4} cm, for instance) containing N atoms with one atom per unit cell. S is the familiar J. J. Thomson formula for the intensity of scattering from a single free electron, f is the atomic scattering power and $\mathbf{a}_i$ $(i=1,2,3)$ are the edges of the unit cell. The vectors $\mathbf{k}_0$ and $\mathbf{k}$, where $k_0=k=1/\lambda$, represent the directions of incidence and of scattering. According to Eq. (1) there are sharp intensity maxima in the Laue directions which are given by

$$\mathbf{k}-\mathbf{k}_0=\mathbf{B}_H\equiv H_1\mathbf{b}_1+H_2\mathbf{b}_2+H_3\mathbf{b}_3, \qquad (2)$$

where H_i designate three integers, $\mathbf{b}_i$ the vector set reciprocal to $\mathbf{a}_i$. For other directions the intensity is zero.

In the famous paper of P. Debye[1] the effect of the thermal vibrations was studied theoretically, and it was found that Eq. (1) had to be replaced by

$$J=J_1+J_2,\quad J_1=J_0e^{-2M},\quad J_2=NSf^2(1-e^{-2M}), \qquad (3)$$

where M is a function of temperature and scattering angle. While the first term like the original Laue expression exhibits sharp maxima

* The results given in this article were presented in a paper before the American Physical Society at the Chicago meeting December 1, 1939.

[1] P. Debye, Ann. d. Physik **43**, 49 (1914).