The Dispersion of Supersonic Waves in Cylindrical Rods of Polycrystalline Silver, Nickel, and Magnesium*

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The velocities of elastic waves in polycrystalline rods of silver, nickel, and magnesium were measured at supersonic frequencies ranging from 120 to 750 kc. This was done by mounting the rods in a horizontal position so that one end was supported by a loop of silk thread and the other end passed through a diaphragm into a special box which was filled with transformer oil and which contained the quartz crystal source of supersound. Lycopodium powder scattered along the rod formed wave patterns when the rod was vibrating resonantly and hence wave-lengths could be measured directly. The products of these wave-lengths and the corresponding frequencies as read on the wave meter yield the required velocities. The dispersion theory of Giebe and Blechschmidt was tested by calculating theoretical curves for four of the six rods and comparing these with the corresponding experimental curves. The theory is found adequate to account satisfactorily only for the low frequency dispersion. One reason for its failure in other respects is discussed.

INTRODUCTION

THE work of Giebe, Scheibe, Blechschmidt, Röhrich and Schoeneck during the last ten years has shown that the velocity of propagation of elastic waves through a homogeneous solid medium ceases to be constant if the frequency of these waves is increased sufficiently. Dispersion of longitudinal waves occurs in the frequency range where the wave-length is comparable to the transverse dimensions of the specimen. This dispersion has been demonstrated and measured quantitatively by Giebe and Scheibe for quartz rods,[1] Giebe and Blechschmidt for tubes, rods, and bars of nickel and nickel alloys,[2] Röhrich for cylindrical rods of copper, aluminum, brass, glass, and steel,[3] and by Schoeneck for cylindrical rods of polycrystalline zinc and single-crystalline zinc, cadmium, and tin.[4]

Long before any means were available to test it experimentally, Lord Rayleigh developed a formula which predicted dispersion of longitudinal waves in solid rods at supersonic frequencies. In every instance the work that has been done to date has yielded a dispersion much larger than expected on the basis of this formula. In 1933 Giebe and Blechschmidt published a new theory of supersonic dispersion in solid tubes, rods, and rectangular bars. In general they found that their theory satisfactorily accounted for their experimental results. Schoeneck, however, found that whereas the new theory agreed more closely with his results than did the Rayleigh formula, the calculated and experimental curves were not in good agreement beyond 300 kc.

The present investigation was undertaken with the following objectives: to obtain absolute measurements of the velocities at supersonic frequencies of elastic waves in rods of polycrystalline silver, nickel, and magnesium; to study the dispersion of these waves and to compare the experimental results with the theory of Giebe and Blechschmidt.

SUPERSONIC DISPERSION THEORY

Here we are concerned just with that part of the Giebe and Blechschmidt dispersion theory which pertains to circular cylindrical rods. According to this theory, which treats longitudinal vibrations only, such a rod is viewed as composed of two separate mechanical systems each of which possesses a characteristic set of resonant frequencies independent of the set belonging to the other. When the rod is set into vibration, the resonant frequencies found experimentally are considered to arise as the result of coupling between the two oscillating mechanical systems.

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[1] E. Giebe and A. Scheibe, Ann. d. Physik (5) **9**, 93 and 137 (1931).
[2] E. Giebe and E. Blechschmidt, Ann. d. Physik (5) **18**, 417 and 457 (1933).
[3] K. Röhrich, Zeits f. Physik **73**, 813 (1932).
[4] H. Schoeneck, Zeits f. Physik **92**, 390 (1934).

The fundamental equation of the theory is:

$$(F_1^2-F^2)(F_2^2-F^2)=q^2F^4, \qquad (1)$$

where F represents the set of characteristic frequencies of the coupled system, F_1 and F_2 represent the sets of characteristic frequencies of the two component systems (considered separately) and q is a constant. In the case of cylindrical rods it is reasonable to take for the set F_1 the harmonics in the direction of its axis of a very long straight rod of infinitesimal cross-sectional area, *viz.*, the well-known set given by:

$$kf_x=(k/2x)(E/\rho)^{\frac{1}{2}}, \quad (k=1,2,3,\cdots)$$

(f_x is the fundamental; E = Young's modulus; ρ = density; x = length of rod) and for the set F_2 the radial vibrations of a circular disk of infinitesimal thickness. The set of these radial vibrations is given by the following expression:

$$f_r=\frac{\zeta}{2\pi r}\left[\frac{E}{\rho(1-\mu^2)}\right]^{\frac{1}{2}},$$

where r is the radius of the rod, μ is Poisson's ratio and the ζ's are the roots of the equation:

$$\zeta J_1'(\zeta)+\mu J_1(\zeta)=0,$$

where J_1 is the Bessel function of the first order. Furthermore, it is assumed that practically all of the coupling between the two systems occurs between the whole set of axial vibrations on the one hand and the lowest member of the infinite set of radial vibrations on the other.

One of the most striking conclusions of this theoretical method is the prediction not of just one, but two independent sets of physically real characteristic frequencies for longitudinal vibrations. To date there is very slight evidence of the actual existence of the "second" series in the case of cylindrical rods, but nothing comparable with the complete experimental verification Giebe and Blechschmidt obtained for tubes, where an analogous situation exists and many members of both series were found.

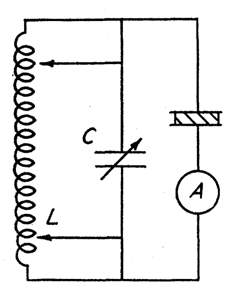

FIG. 1. Diagram of the crystal circuit.

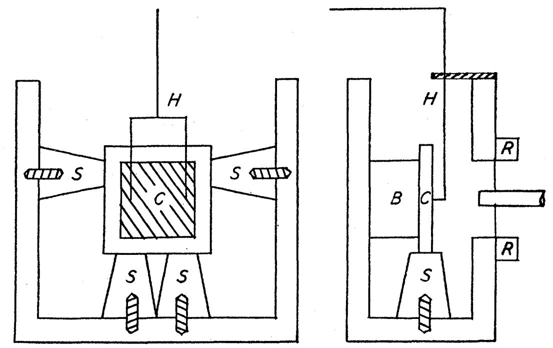

FIG. 2. Two sections of the crystal holder. S, stand-off insulators; H, high potential lead; C, quartz crystal; R, steel ring; B, brass cylinder.

Of equal physical interest is the prediction of a so-called "dead zone"—i.e., a frequency region within which it is impossible to excite the rod into longitudinal vibration. This prediction was substantiated by Giebe and Blechschmidt in the case of tubes, but so far it has not been verified for cylindrical rods. Upon examining the theory to find out just how the "dead zone" arises out of it, we discover that it is conditioned precisely by the simplifying assumption mentioned above that the only truly operative radial frequency is the fundamental.[5] If this restriction is removed, the "dead zone" vanishes. It will be seen that the present work agrees with all that has previously been published in exhibiting experimental points that fall within this supposedly forbidden region. Thus the suspicion grows that the "dead zone" really does not exist for solid cylindrical rods.

Apparatus

The frequency source was a simple Hartley oscillator containing a Taylor Type T-40 power tube. Both capacity and inductance were variable; the available capacity ran from about 60 to 2400 $\mu\mu$f and the maximum inductance was about one millihenry. The oscillator covered the frequency range from about 120 kc to 850 kc with a maximum power output of 50 watts.

To obtain the high voltages and fairly large

[5] It should be noted here that in the case of tubes, the radial series F_2 contains only *one* member.

power required to drive the quartz crystal so that it would radiate supersonic energy strongly outside its resonant frequencies, a tuned circuit which contained the crystal was coupled inductively to the oscillator output. This simple resonant circuit is shown in Fig. 1. The variable condenser C had a maximum capacity of 50 $\mu\mu$f and was built to withstand 12,000 volts. L represents either of two large variable coils. One of these had an inductance of approximately 3.8 millihenries and was used for the lower half of the frequency range. The other had an inductance of about one millihenry and covered the higher frequency range.

The quartz crystal was obtained from the American Optical Company. It was X-cut, of dimensions $(5\times 50\times 50)$ mm^3. A thin layer of silver was evaporated onto each of the large square faces and thickened with a layer of copper by electroplating. Enough of the double layer was removed to expose a border five millimeters wide around the edge of each square face.

The crystal holder was designed so that the rod which was to be studied could be mounted horizontally (a condition made necessary by the use of lycopodium powder along the rod to form standing wave patterns). It consisted of a box with supports inside for the crystal and a diaphragm in the front wall through which one end of the horizontal rod projected. Fig. 2 shows diagrams of two sections of the holder.

The box itself was made of $\frac{1}{4}''$ steel plate and had outside dimensions $(13\times 13\times 6.3)$ cm^3. The square front wall had a hole $1\frac{3}{8}''$ in diameter cut out at its center. The box was deliberately large to prevent any possibility of sparking from the high potential face of the crystal (or the lead to it) to the walls.

In order to support the crystal and to keep it away from the walls and bottom, four stand-off insulators were mounted inside the box. A brass cylinder about 4 cm in diameter and 2.5 cm long was mounted with its axis through the center of the rear box wall; one end of this cylinder provided a firm wall against which the quartz crystal was held by the pressure of the high potential lead. This lead was in the form of a plane two-pronged fork made of radio bus bar.

A ring, cut out of $\frac{1}{4}''$ steel plate, of inner diameter the same ($1\frac{3}{8}''$) as that of the hole in the front wall of the box and outer diameter about $\frac{3}{4}''$ greater, could be screwed very tightly to the outside of the front wall. A sheet of Goodrich Koroseal (0.007″ thick) was secured between the ring and the wall by pressure of the ring.

The holder was filled with transformer oil to serve the threefold purpose of protecting the crystal from shattering, providing a suitable conductor of the supersonic radiation from the crystal face to the end of the test rod and preventing sparking at high voltage.

All frequency measurements were made with a Type 224 wave meter.

Although many substances have been used successfully by other experimenters in forming standing wave patterns along solid rods, in the present study lycopodium powder was found to be most satisfactory.

Procedure

In order to secure maximum freedom of motion of the lycopodium powder the cylindrical rod to be studied was freed of any grease or dirt on its surface by washing it with ordinary soap and very hot water. It was thoroughly dried and handled by means of paper towels to prevent contact of the hands. One end was carefully inserted in a hole at the center of the diaphragm and the other end was supported by a loop of silk thread. Then transformer oil was poured into the crystal holder and a layer of lycopodium powder scattered uniformly along the upper surface of the test rod.

The procedure adopted to obtain a single measurement was standard for all regions throughout the entire frequency range and may be described as follows. L and C (Fig. 1) and the oscillator tank capacity were adjusted until the frequency desired was obtained with a reading on the crystal milliammeter (A in Fig. 1) of about 50. The surface of the rod was carefully observed while the frequency was continuously varied—slowly and in such a direction that the crystal current increased—by altering the oscillator capacity. If activity in the rod was evidenced at any time during the process by the shifting or falling off of groups of lycopodium

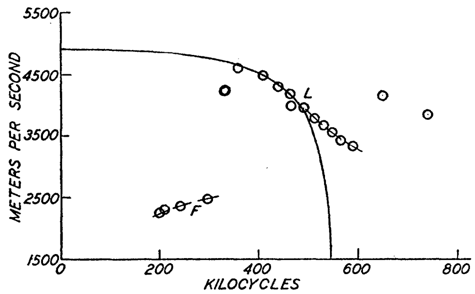

FIG. 3. Dispersion curves for the larger magnesium rod. Length: 14.98 cm. Mean diameter: 5.895 mm. *L* and *F* denote the longitudinal and flexural branches, respectively. The full line denotes the Giebe and Blechschmidt theoretical curve.

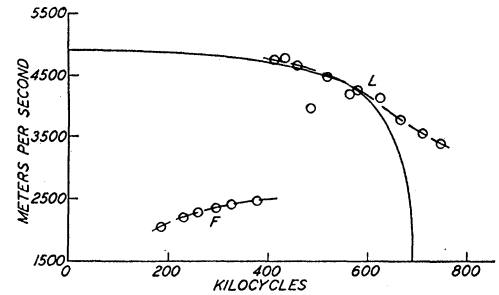

FIG. 4. Dispersion curves for the smaller magnesium rod. Length: 16.42 cm. Mean diameter: 4.615 mm. *L* and *F* denote the longitudinal and flexural branches, respectively. The full line denotes the Giebe and Blechschmidt theoretical curve.

spores, the frequency was allowed to remain fixed and the "free" end of the rod agitated either by a simple electromagnetic hammer or by the end of a wooden rule held in the hand. If a clear wave pattern formed, it was measured with a steel rule supported by small pillars at each end. If the pattern could not be brought out well, the rod was powdered again and the frequency variation was continued until a measurable pattern could be produced.

In order to conduct the run in a systematic fashion, the oscillator was adjusted at the start to yield its lowest frequency and as the run proceeded the frequency was increased by steps until the upper limit was reached. Reference to the dispersion curve obtained by plotting the measurements usually indicated the desirability of further examination at a few frequencies and the run was completed by making additional measurements in the neighborhoods of these frequencies.

Results and Conclusions

Qualitatively, the findings of Röhrich and Schoeneck were corroborated throughout, both with respect to the phenomena which were observed during the taking of readings and the general appearance of the dispersion curves.

The taking of readings, as both these investigators found, was not a rapid task nor so easy as one might have expected. It was almost always made difficult by the fact that a pure elastic vibration of one type—*viz.*, longitudinal, torsional, or flexural—cannot be elicited with complete exclusion of both of the other two types. This resulted in the simultaneous appearance of parts of two or three wave patterns and the superposition of these frequently made any measurement impossible. Thus a clear pattern could be formed only when a vibration of one of the three kinds was excited much more strongly than the other two. In such cases, indeed, it was often possible to make a wave-length measurement whose error did not exceed 1 percent. In this connection, it was found in the case of all the rods that it was practically impossible to excite anything but flexural vibrations at frequencies below a certain value—about 250 kc for the silver rods and about 350 kc for the nickel and magnesium rods. Above these values longitudinal and torsional vibrations were much more readily excited than flexural.

The various curious phenomena described by Röhrich appeared at times. The most frequently recurring of these was the simultaneous formation of two patterns representing two different types of vibration; both in the same region on the rod, one on each side of a line in the upper surface parallel to the axis of the rod. Not infrequently a clear pattern formed which had so large a spacing between successive heaps of powder that the only reasonable assumption was that it represented a vibration whose whole wave-length was denoted by the interval instead of its half-wave-length; that is, it would seem as if for some reason the heaps at the alternate nodes had been shaken off. Once in a while the long narrow heaps were inclined obliquely to the axis of the rod instead of being formed at right angles to it as they ought to have been. Occa-

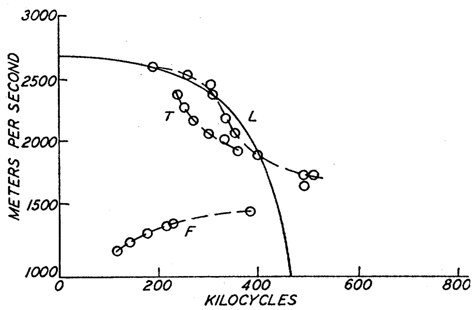

FIG. 5. Dispersion curves for the smaller silver rod. Length: 25.78 cm. Mean diameter: 4.057 mm. L, T, and F denote the longitudinal, torsional, and flexural branches, respectively. The full line denotes the Giebe and Blechschmidt theoretical curve.

sionally, also, eddies formed; little clusters of lycopodium spores at certain spots along the rod whirled about points near their centers, but had no translational motion as a whole.

The general appearance of the curves plotted for an entire run of any rod is essentially the same for all the rods (cf., for example, Fig. 6) and is similar to that of the curves obtained by Röhrich and Schoeneck. Typically this is as follows. Three branches are present, representing longitudinal, torsional, and flexural velocities, respectively. The longitudinal branch starts (experimentally) at middle frequencies (250 kc to 350 kc) and falls quite rapidly; it always lies higher than the other two branches and if the low frequency end of it is extrapolated back to audible frequencies a value is reached, invariably somewhat higher than the audible velocity given in standard tables. The torsional branch starts at about the same place as the longitudinal, always lies below it and usually runs nearly parallel to it. The flexural branch starts at low (supersonic) frequencies, always lies far below the other two branches at the beginning and slowly rises as the frequency increases. Finally, the indication in every instance is that at high frequencies all three branches tend to approach a common value.

Giebe and Blechschmidt theoretical curves were computed for four of the six rods and these are drawn on the same graphs with the experimental curves. This was done by solving equation (1) for F^2 with the quadratic formula and by choosing the root which has the negative sign in front of the radical. The quantities x and r were obtained from the rod itself by measuring its length and radius, respectively. Values had to be assumed for Poisson's ratio (μ) and for the audible velocity of axial waves. Then the values of q and ζ corresponding to the assumed value of μ were taken from the table in Giebe and Blechschmidt's article (cf. footnote 2 above).

Certain special comments pertaining to the individual rods are in order.

Magnesium

The two rods studied were of pure magnesium, obtained in the form of sticks and turned down on the lathe. The graph for the larger rod (see Fig. 3) shows: (a) well-defined flexural and longitudinal branches but only a single point that presumably represents a torsional velocity; (b) the agreement with the theoretical curve is good out to 490 kc; (c) experimental points in the theoretical "dead zone"; (d) two points at high frequency which cannot be accounted for by the theory in its present form. In the article referred to above, Giebe and Blechschmidt mention finding similar anomalous points in and beyond the "dead zone."

The graph for the smaller rod (see Fig. 4) shows: (a) a greater "spread" of the longitudinal points than is found for any of the other five rods; (this probably results in part from the fact that this rod is so light that the ordinary agitation required to bring out a good wave pattern caused enough of the lycopodium layer to be displaced so that frequently only small chains of nodes could be measured); (b) only one presumably torsional point; (c) smaller dispersion for a given frequency than the other

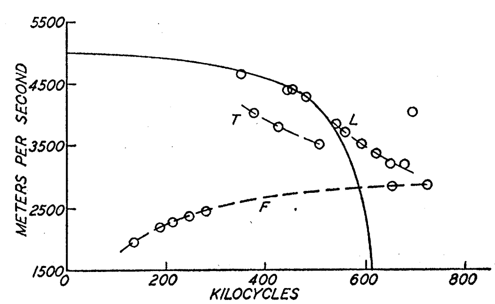

FIG. 6. Dispersion curves for the smaller nickel rod. Length: 25.45 cm. Mean diameter: 5.500 mm. L, T, and F denote the longitudinal, torsional, and flexural branches, respectively. The full line denotes the Giebe and Blechschmidt theoretical curve.

magnesium rod displays; (d) good agreement between the theoretical curve and the most reasonable experimental curve out to 610 kc.

The theoretical curves for the magnesium rods were calculated with $\mu=0.25$ and V_0 = velocity at low audible frequencies = 4900 m/sec.

Silver

The two rods studied were of hard drawn silver, pure to within one part in two thousand. The graph for the smaller silver rod (see Fig. 5) shows: (a) fairly good agreement with the theoretical curve out to 400 kc, although the experimental curve has a change of curvature, whereas the theoretical one cannot have a change of curvature; (b) experimental points in the theoretical "dead zone"; (c) no evidence of "second series" velocities; (d) less dispersion for a given frequency than occurs for the other silver rod (whose graph is not shown). The theoretical curve was calculated with $\mu=0.39$ and $V_0=2700$ m/sec.

Nickel

The two rods studied were of commercial nickel, about 99 percent pure. The graph for the smaller nickel rod (see Fig. 6) shows: (a) less dispersion for a given frequency than occurs for the other nickel rod (whose graph is not shown); (b) good agreement with the theoretical curve out to 530 kc; (c) experimental points in the theoretical "dead zone"; (d) one unaccountable point at high frequency like the two in Fig. 3. The theoretical curve was calculated with $\mu=0.30$ and $V_0=4980$ m/sec.

To summarize, it is seen that the agreement between the experimental results and the Giebe and Blechschmidt theory is quite good out almost to the "cut-off"—i.e., the frequency at which the so-called "dead zone" begins. Nevertheless, there is no evidence whatsoever for the existence of such a region and nothing more than material for speculation about the presence of anomalous dispersion. It seems clear that the theory in its present state is able to account satisfactorily only for the low frequency dispersion. In the opinion of the authors, further development of the theory is called for, starting with the removal of the simplifying assumption discussed above.

We wish to express our sincere thanks to Professor R. B. Lindsay for suggesting the problem and for his help throughout the investigation.

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The Variation of the Principal Elastic Moduli of Cu_3Au with Temperature

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Single crystals of a copper-gold alloy containing 24.92 atomic percent gold were prepared in a vacuum furnace. The crystals were brought to the ordered state at room temperature by an annealing procedure similar to that described by Sykes and Evans. The elastic moduli were measured by the method of the composite piezoelectric oscillator. The elastic moduli are tabulated as a function of temperature over the range 20°C to 450°C. At the critical temperature, 387.5°C, there is a discontinuity in each modulus-*vs.*-temperature curve. The results indicate that the elastic constants are closely related to the degree of local order.

INTRODUCTION

In recent years there has been an increasing interest in binary alloy systems which exhibit superlattice formation near simple stoichiometric concentrations.[1] In the copper-gold system, ordered superstructures are observed near the

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[1] F. C. Nix and W. Shockley, Rev. Mod. Phys. **10**, 1 (1938). G. Borelius, Zeits. f. Electrochem. **45**, 16 (1939).