# Magnetic Resonance for Nonrotating Fields

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A treatment of the magnetic resonance is given for a particle with spin $\frac{1}{2}$ in a constant field $H_0$ and under the action of an arbitrary alternating field with circular frequency $\omega$ perpendicular to $H_0$. A method of finding a solution, valid at any time, is given which converges the better the smaller the deviations from a rotating field or the larger $H_0$. It is shown that in the lowest order correction the shape of the resonance curve is unchanged but that it is shifted by a percentage amount $H_1^2/16\, H_0^2$ where $H_1$ is the effective amplitude of the oscillating field. This also involves a correction in the values of the magnetic moments thus obtained towards smaller values which however in all practical cases is negligibly small.

THE principle of the magnetic resonance has been known since considerable time and it has already led to several important applications. In the outstanding work of Rabi and his collaborators[1] on molecular beams it has not only been used for the most precise determination of the magnetic moments of many nuclei but it has also revealed entirely new features like the quadrupole moment of the deuteron. Furthermore, by applying it to polarized neutron beams, it has recently allowed Alvarez and one of us[2] a quantitative measurement of the magnetic moment of the neutron.

The basic idea of the magnetic resonance is very simple and consists in the remark that in causing transitions of the orientation of a moment a weak alternating magnetic field is particularly effective when its frequency is in resonance with the frequency with which the moment precesses in a strong and constant perpendicular field. While the special case of a weak field rotating around the strong field can be easily treated there lies an essentially more difficult problem in the mathematical theory of a field, for which the polarization is no more circular but generally elliptic; this includes the case, commonly used, in which the alternating field performs a simple linear oscillation. It is the purpose of this paper to solve this problem by a method of successive approximations which converges the better the smaller the eccentricity and the greater the ratio of the constant field to the magnitude of the varying field; we here restrict ourselves for convenience to the treatment of a particle with angular momentum $\frac{1}{2}$.**

Let $c_{\frac{1}{2}}$ and $c_{-\frac{1}{2}}$ be the probability amplitudes for the $z$ component of the moment to have the values $m=\frac{1}{2}$ and $m=-\frac{1}{2}$, respectively. They satisfy the Schroedinger equation

$$-\frac{\hbar}{i}\dot{c}_m = -\mu \sum_{m'=-\frac{1}{2}}^{\frac{1}{2}} (\mathbf{H}\boldsymbol{\sigma}_{mm'})c_{m'}, \tag{1}$$

where $\mathbf{H}$ is the vector of the magnetic field with components $H_x$, $H_y$, $H_z$, $\mu$ the magnetic moment of the particle and $\boldsymbol{\sigma}_{mm'}$ a matrix vector the components of which are the spin matrices of Pauli.† Written out Eq. (1) becomes

$$(\hbar/i)\dot{c}_{\frac{1}{2}} = \mu[(H_x+iH_y)c_{-\frac{1}{2}}+H_z c_{\frac{1}{2}}], \tag{2a}$$

$$(\hbar/i)\dot{c}_{-\frac{1}{2}} = \mu[(H_x-iH_y)c_{\frac{1}{2}}-H_z c_{-\frac{1}{2}}]. \tag{2b}$$

These two equations can be reduced into one for the only quantity

$$u = c_{\frac{1}{2}}/c_{-\frac{1}{2}}, \tag{3}$$

which is physically important, since it determines directly the probabilities $P_m(m=\pm\frac{1}{2})$ to find any one of the two possible values of $m$ in the form

$$P_{\frac{1}{2}} = |u|^2/(1+|u|^2), \tag{4a}$$

$$P_{-\frac{1}{2}} = 1/(1+|u|^2). \tag{4b}$$

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[1] I. I. Rabi, S. Millman, P. Kusch and J. R. Zacharias, Phys. Rev. 55, 526 (1939).

[2] L. W. Alvarez and F. Bloch, Phys. Rev. 57, 111 (1940).

** As the unit of the angular momentum we shall throughout use the quantity $\hbar = h/2\pi$; $h$ = Planck's constant.

† With this form of Eq. (1) we have decided that positive or negative values of $\mu$, respectively, mean, that the orientation of the magnetic moment of the particle is parallel or opposite to that of its angular momentum.

The equation for $u$, following from (2) is

$$(\hbar/i)\dot{u}=2\mu H_z u+\mu(H_x+iH_y)-\mu(H_x-iH_y)u^2. \quad (5)$$

We shall now assume, that $H_z$ has the constant value

$$H_z=H_0. \quad (6)$$

The general case of an alternating field with circular frequency $\omega$, perpendicular to it is given by

$$H_x=H_1\cos(\omega t+\varphi_1), \quad (7a)$$

$$H_y=H_z\cos(\omega t+\varphi_2), \quad (7b)$$

where the amplitudes $H_1$, $H_2$ and the phases $\varphi_1$, $\varphi_2$ are left quite arbitrary. We will now introduce the following convenient abbreviations

$$\frac{(H_1e^{i\varphi_1}+iH_2e^{i\varphi_2})\mu}{4\hbar\omega}=|f_1|e^{i\psi_1}, \quad (8a)$$

$$\frac{(H_1e^{i\varphi_1}-iH_2e^{i\varphi_2})\mu}{4\hbar\omega}=|f_2|e^{i\psi_2}, \quad (8b)$$

$$2\omega t+\psi_1+\psi_2=\tau, \quad (9)$$

$$\frac{1}{4}\left(1-\frac{2H_0\mu}{\hbar\omega}\right)=\delta, \quad (10)$$

$$a=\frac{1}{|f_1|}[(|f_1|^2+\delta^2)^{\frac{1}{2}}+\delta],$$
$$1/a=\frac{1}{|f_1|}[(|f_1|^2+\delta^2)^{\frac{1}{2}}-\delta], \quad (11)$$

$$\rho=|f_1|(a+1/a)=2(|f_1|^2+\delta^2)^{\frac{1}{2}}, \quad (12)$$

$$\epsilon=\frac{|f_1f_2|}{2(|f_1|^2+\delta^2)^{\frac{1}{2}}}. \quad (13)$$

From Eqs. (7) and (8) it also follows that

$$H_x=\frac{2\hbar\omega}{\mu}[|f_1|\cos(\omega t+\psi_1)+|f_2|\cos(\omega t+\psi_2)],$$

$$H_y=\frac{2\hbar\omega}{\mu}[|f_1|\sin(\omega t+\psi_1)-|f_2|\sin(\omega t+\psi_2)],$$

so that $(2\hbar\omega/\mu)|f_{1,2}|$ and $\psi_{1,2}$ have the significance of being the magnitudes and phases, respectively, of two fields, one rotating clockwise, the other rotating counterclockwise, by superposition of which the general alternating field (7) can also be formed. In order to facilitate the further discussion we shall replace the function $u$ of Eq. (5) by a new function $z$ through the relation

$$u=e^{i(\omega t+\psi_1)}\frac{ae^{i(z-\rho\tau)}+1/a}{1-e^{i(z-\rho\tau)}}. \quad (14)$$

With the above abbreviations it then follows from (5) that $z$ has to satisfy the equation

$$\frac{dz}{d\tau}=\epsilon\{e^{iz}[e^{-i\tau(1+\rho)}-a^2e^{i\tau(1-\rho)}]+e^{-iz}[e^{-i\tau(1-\rho)}-(1/a^2)e^{i\tau(1+\rho)}]-2[e^{i\tau}+e^{-i\tau}]\}. \quad (15)$$

The essential difficulty in solving this equation arises from the right-hand side, the importance of which is measured by the quantity $\epsilon$. We see from (13) that it reaches its maximum value $\epsilon_{\max}=|f_2|/2$ for $\delta=0$, i.e., when the frequency $\omega$ equals the Larmor frequency $2H_0\mu/\hbar$; we write for the value of $H_0$, for which this resonance condition is fulfilled

$$H_r=\hbar\omega/2\mu. \quad (16)$$

We have then with ($8^b$)

$$\epsilon_{\max}=\frac{|f_2|}{2}=\frac{1}{16}\frac{[H_1^2+H_2^2-2H_1H_2\sin(\varphi_1-\varphi_2)]^{\frac{1}{2}}}{|H_r|}. \quad (17)$$

This vanishes if and only if $H_1=H_2$ and $\varphi_1=\varphi_2+\pi/2$, i.e., according to (7) if the alternating field rotates counterclockwise or clockwise, respectively, in the $x$-$y$ plane, according to whether the frequency $\omega$ is positive or negative i.e., whether $H_r$ and $\mu$ have the same or the opposite sign.

For $\epsilon=0$ we have immediately $z=\text{const}=\rho\tau_0$,

$$u=e^{i(\omega t+\psi_1)}\frac{ae^{i\rho(\tau-\tau_0)}+1/a}{1-e^{i\rho(\tau-\tau_0)}},$$

$$|u|^2=\frac{a^2+1/a^2+2\cos\rho(\tau-\tau_0)}{2[1-\cos\rho(\tau-\tau_0)]}$$

and we obtain from Eq. ($4^b$) the familiar expression

$$P_{-\frac{1}{2}}=\frac{4\sin^2\rho(\tau-\tau_0)/2}{(a+1/a)^2}=\frac{\sin^2[(|f_1|^2+\delta^2)^{\frac{1}{2}}(\tau-\tau_0)]}{1+\delta^2/|f_1|^2}=\frac{\sin^2\left[H_1\mu\left\{1+\left(\frac{H_0-H_r}{H_1}\right)^2\right\}^{\frac{1}{2}}(t-t_0)/\hbar\right]}{1+\left(\frac{H_0-H_r}{H_1}\right)^2}, \tag{18}$$

using ($8^a$), (10), (11) and (12) and writing $\tau_0=2\omega t_0+\psi_1+\psi_2$ so that $t=t_0$ is the time for which the value $m=1/2$ is found with certainty.

The rigorous solution of (15) for $\epsilon\neq 0$ cannot be given in terms of elementary functions. We want to show, however, that a series expansion of the solution in powers of $\epsilon$ can be obtained which is valid for any value of the argument $\tau$ and the few first terms of which give a good approximation if $\epsilon\ll 1$. One sees from (17) that to have $\epsilon$ sufficiently small it is not necessary that the alternating field deviates only little from a field rotating in the $x$-$y$ plane in the sense, described above but that $\epsilon$ will be always small for arbitrary given values of $H_1$, $H_2$ and $\varphi_1$, $\varphi_2$ if only $|H_r|$ is sufficiently large. In the example of a field, oscillating in the $x$ direction ($H_2=0$) it is only necessary that its amplitude $H_1$ is sufficiently small compared to the constant field at resonance $H_r$.*

It would seem at first, that the method of successive approximations would immediately yield a good power series expansion of $z$ in terms of $\epsilon$. If, however, one substitutes in the usual way the solution of the previous approximation in the right-hand side in order to find the next approximation by an integral one sees, that already in the second approximation there occur terms linear in $\tau$ which for sufficiently large values of $\tau$ would seem to make the solution invalid. One has to admit a linear increase of $z$ with $\tau$ with a coefficient however, which again in each approximation will be more accurately determined. All other terms can be seen to be oscillatory in $\tau$ with amplitudes that are the smaller, the smaller $\epsilon$. We thus write

$$z=y+\lambda\tau, \tag{19}$$

where the coefficient $\lambda$ has to be determined in such a way, that $y$ contains only oscillatory terms. With

$$\alpha=\rho-\lambda \tag{20}$$

we can then write (15) as an integral equation for $y$ in the form

$$y=-\lambda\tau+\epsilon\int\{e^{iy}[e^{-i(1+\alpha)\tau}-a^2e^{i(1-\alpha)\tau}]+e^{-iy}[e^{-i(1-\alpha)\tau}-1/a^2e^{i(1+\alpha)\tau}]-2[e^{i\tau}+e^{-i\tau}]\}d\tau. \tag{21}$$

As in (18) we shall determine the one constant of integration for $y$ in such a way that $P_{-\frac{1}{2}}$ vanishes for $\tau=\tau_0$. For this it is evidently necessary from ($4^b$) that $u(\tau_0)=\infty$ or from (14) that $z(\tau_0)=\rho(\tau_0)$ i.e., with (19) and (20)

$$y(\tau_0)=\alpha\tau_0. \tag{22}$$

We shall now write

$$y(\tau)=\alpha\tau_0+\eta \tag{23}$$

so that we have the initial condition $\eta(\tau_0)=0$. If we further introduce

$$T=\tau-\tau_0 \tag{24}$$

we obtain for $\eta$ the integral equation

$$\eta=-\lambda T+\epsilon\int_0^T\Big\{e^{i\eta}[e^{-i\tau_0-i(1+\alpha)T}-a^2e^{i\tau_0+i(1-\alpha)T}]+e^{-i\eta}\left[e^{-i\tau_0-i(1-\alpha)T}-\frac{1}{a^2}e^{i\tau_0+i(1+\alpha)T}\right]-2[e^{i\tau_0+iT}+e^{-i\tau_0-iT}]\Big\}dT, \tag{25}$$

* This is incidentally the same condition as the one which guarantees high resolving power of the magnetic resonance method.

in which the limits of integration are evidently chosen so as to satisfy the required initial condition for $\eta(T)$ that $\eta(0)=0$. Using ($4^b$), (14), (19), (23), and (24) one can readily express $P_{-\frac{1}{2}}$ in terms of $\eta$ and its conjugate complex $\eta^*$ by the formula

$$P_{-\frac{1}{2}}=\frac{1}{2\cosh k}\frac{\cosh\left[i(\eta-\eta^*)/2\right]-\cos\left[\alpha T-(\eta+\eta^*)/2\right]}{\cosh\left[k+i(\eta-\eta^*)/2\right]} \tag{26}$$

with

$$a=e^k. \tag{27}$$

In order to obtain for $\eta$ a power series expansion in $\epsilon$ we write

$$\eta=\epsilon\eta_1+\epsilon^2\eta_2+\cdots, \tag{28a}$$

$$\lambda=\epsilon\lambda_1+\epsilon^2\lambda_2+\cdots. \tag{28b}$$

Substituting in (25) and equating terms with the same power of $\epsilon$ on both sides of the equation we find for the terms linear in $\epsilon$

$$\begin{aligned}\eta&=-\lambda_1T+\int_0^T\left\{e^{-i\tau_0}\left[e^{-i(1+\alpha)T}+e^{-i(1-\alpha)T}-2e^{-iT}\right]-e^{i\tau_0}\left[a^2e^{i(1-\alpha)T}+\frac{1}{a^2}e^{i(1+\alpha)T}+2e^{iT}\right]\right\}dT,\\&=ie^{-i\tau_0}\left[\frac{e^{-i(1+\alpha)T}-1}{1+\alpha}+\frac{e^{-i(1-\alpha)T}-1}{1-\alpha}-2(e^{-iT}-1)\right]+ie^{i\tau_0}\left[a^2\frac{e^{i(1-\alpha)T}-1}{1-\alpha}+\frac{1}{a^2}\frac{e^{i(1+\alpha)T}-1}{1+\alpha}+2(e^{iT}-1)\right].\end{aligned} \tag{29}$$

We have here taken $\lambda_1=0$ since indeed with this choice $\eta_1$ has only terms which are oscillatory in $T$ (or constant) but no terms that increase arbitrarily as $T$ increases.

For terms quadratic in $\epsilon$ we have

$$\begin{aligned}\eta_2&=-\lambda_2T+i\int_0^T\eta_1\left\{e^{-i\tau_0}\left[e^{-i(1+\alpha)T}-e^{-i(1-\alpha)T}\right]-e^{i\tau_0}\left[a^2e^{i(1-\alpha)T}-\frac{1}{a^2}e^{i(1+\alpha)T}\right]\right\}dT,\\&=-\lambda_2T+\int_0^T\left\{\left[\frac{e^{-i(1+\alpha)T}-1}{1+\alpha}+\frac{e^{-i(1-\alpha)T}-1}{1-\alpha}-2(e^{-iT}-1)\right]\left[a^2e^{i(1-\alpha)T}-\frac{1}{a^2}e^{i(1+\alpha)T}\right]\right\}dT\\&\quad-\int_0^T\left\{\left[a^2\frac{e^{i(1-\alpha)T}-1}{1-\alpha}+\frac{1}{a^2}\frac{e^{i(1+\alpha)T}-1}{1+\alpha}+2(e^{iT}-1)\right]\left[e^{-i(1+\alpha)T}-e^{-i(1-\alpha)T}\right]\right\}dT\\&\quad-e^{-2i\tau_0}\int_0^T\left\{\left[\frac{e^{-i(1+\alpha)T}-1}{1+\alpha}+\frac{e^{-i(1-\alpha)T}-1}{1-\alpha}-2(e^{-iT}-1)\right]\left[e^{-i(1+\alpha)T}-e^{-i(1-\alpha)T}\right]\right\}dT\\&\quad+e^{2i\tau_0}\int_0^T\left\{\left[a^2\frac{e^{i(1-\alpha)T}-1}{1-\alpha}+\frac{1}{a^2}\frac{e^{i(1+\alpha)T}-1}{1+\alpha}+2(e^{iT}-1)\right]\left[a^2e^{i(1-\alpha)T}-\frac{1}{a^2}e^{i(1+\alpha)T}\right]\right\}dT.\end{aligned} \tag{30}$$

We first note, that the terms of $\eta_2$, linear in $T$ are of the form

$$\left(-\lambda_2-\frac{2}{a^2}\frac{1}{1+\alpha}+\frac{2a^2}{1-\alpha}\right)T,$$

which, in order to vanish require

$$\lambda_2=2\left(\frac{a^2}{1-\alpha}-\frac{1}{a^2}\frac{1}{1+\alpha}\right). \tag{31}$$

The remaining terms of $\eta_2$ are again oscillatory (or constant). We shall explicitly write down those which are dominant for $\alpha \ll 1$ since they are of particular interest for our later purposes. They are

$$\eta_2' = \frac{2}{i\alpha}\left[(a^2+1)(e^{-i\alpha T}-1) + (1/a^2+1)(e^{i\alpha T}-1)\right]. \quad (32)$$

Proceeding in the same manner one can evidently obtain all further terms in the expansions (28). We shall, however, break off the expansion with the terms quadratic in $\epsilon$ which we have obtained so far and proceed to calculate those corrections in the expression for $P_{-\frac{1}{2}}$, which arise in this approximation, introducing all those further simplifications which are justified from the physical angle of the problem.

Inserting ($28^a$) in (26) and keeping the linear and quadratic terms in $\epsilon$ we get

$$P_{-\frac{1}{2}} = \frac{1}{2\cosh^2 k}\Bigg\{1-\cos\alpha T - \frac{\epsilon}{2}\left[(\eta_1+\eta_1^*)\sin\alpha T + i(\eta_1-\eta_1^*)(1-\cos\alpha T)\tanh k\right] + \frac{\epsilon^2}{2}\Bigg[\eta_1\eta_1^*\cos\alpha T - (\eta_2+\eta_2^*)\sin\alpha T - i(\eta_2-\eta_2^*)(1-\cos\alpha T)\tanh k - \frac{1}{2}(\eta_1-\eta_1^*)^2(1-\cos\alpha T)\tanh^2 k + \frac{i}{2}(\eta_1^2-\eta_1^{*2})\sin\alpha T\tanh k\Bigg]\Bigg\}. \quad (33)$$

This formula can now be considerably simplified for all practical purposes. We notice first that through the expressions (29) and (30) for $\eta_1$ and $\eta_2$ $P_{-\frac{1}{2}}$ contains terms periodic in $\tau_0$ which quantity by Eq. (9) sets the time $t_0$ at which the particle according to our initial conditions is sure to be found in a state with $m=\frac{1}{2}$. Since in the magnetic resonance method one generally deals with a continuous stream of particles one will not observe $P_{-\frac{1}{2}}$ but rather its average over $\tau_0$, so that one is justified in replacing the expression $P_{-\frac{1}{2}}$ by its average $\bar{P}_{-\frac{1}{2}}$ which one obtains by omitting in (33) all terms which oscillate with $\tau_0$. Since $\eta_1$ and $\eta_1^*$ are of this character this means that the terms of (33) which are linear in $\epsilon$ can be omitted.

Of the remaining terms quadratic in $\epsilon$ we shall now for further simplification keep only those which become dominant in the limit of high frequencies $\omega$ or, according to (16) for large values of the quantity $H_r$. In order to separate these terms we have to consider the order of magnitude of the quantity $\alpha$, entering in (29) and (30). According to (12), (20), ($28^b$), (31) and considering that $\lambda_1=0$ we see that in our approximation $\alpha$ is to be determined from the equation

$$\alpha = 2(|f_1|^2+\delta^2)^{\frac{1}{2}} - 2\epsilon^2\left(\frac{a^2}{1-\alpha} - \frac{1}{a^2}\frac{1}{1+\alpha}\right). \quad (34)$$

Near resonance, where $\delta$ is of the order of magnitude of $|f_1|$ we see from (11) that $a$ is of the order of magnitude of unity. The first term of (34) is then of the order of magnitude of $|f_1|$ which, from ($8^a$) is seen to be inversely proportional to $\omega$ or to $H_r$. Since near resonance $\epsilon$ is of the order of magnitude of $|f_2|$ which again is inversely proportional to $H_r$ we can approximately write

$$\alpha = 2(|f_1|^2+\delta^2)^{\frac{1}{2}} - 2\epsilon^2(a^2-1/a^2) \quad (35)$$

or with (11) and (13)

$$\alpha = 2(|f_1|^2+\delta^2)^{\frac{1}{2}}\left(1 - \frac{\delta|f_2|^2}{|f_1|^2+\delta^2}\right). \quad (36)$$

Thus for large values of $H_r$ $\alpha$ approaches zero like $1/H_r$. While according to (29) $\eta_1$ approaches a finite limit for $\alpha\rightarrow 0$ $\eta_2$ approaches the value $\eta_2'$, given by Eq. (32) which for small values of $\alpha$ (but $\alpha T$ arbitrary!) is proportional to $H_r$. Since $\epsilon^2$ is itself proportional to $1/H_r^2$ we obtain $\bar{P}_{-\frac{1}{2}}$ correctly to within terms of the order $1/H_r$ if we write instead of (33)

$$\bar{P}_{-\frac{1}{2}} = \frac{1}{2\cosh^2 k}\left\{1-\cos\alpha T - \frac{\epsilon^2}{2}\left[(\eta_2'+\eta_2'^*)\sin\alpha T + i(\eta_2'-\eta_2'^*)(1-\cos\alpha T)\tanh k\right]\right\} \quad (34)$$

or with (32) and (27)

$$\bar{P}_{-\frac{1}{2}}=\frac{4\sin^2\alpha T/2}{(a+1/a)^2}\left[1+\frac{4\epsilon^2}{\alpha}(a^2-1/a^2)\right]. \quad (37)$$

Since this formula is claimed to be correct only including correction terms of the order $1/H_r$ we can in $\epsilon^2/\alpha$ replace $\alpha$ by its first term $2(|f_1|^2+\delta^2)^{\frac{1}{2}}$ of formula (35). Or with (11) and (13)

$$P_{-\frac{1}{2}}=\frac{\sin^2\alpha T/2}{1+\delta^2/|f_1|^2}\left[1+\frac{2\delta|f_2|^2}{|f_1|^2+\delta^2}\right]. \quad (38)$$

This formula can be written in a form, very similar to (18) by introducing the "effective" $\delta$, defined as

$$\delta^*=\delta-|f_2|^2. \quad (39)$$

We have indeed, except for higher order corrections in $1/H_r$ which we have neglected anyway

$$\alpha=2(|f_1|^2+\delta^{*2})^{\frac{1}{2}}$$

$$\cong 2(|f_1|^2+\delta^2)^{\frac{1}{2}}\left[1-\frac{\delta|f_2|^2}{|f_1|^2+\delta^2}\right] \quad (40a)$$

and

$$\frac{1}{1+\delta^{*2}/|f_1|^2}\cong\frac{1}{1+\delta^2/|f_1|^2}\left[1+\frac{2\delta|f_2|^2}{|f_1|^2+\delta^2}\right]. \quad (40b)$$

In this approximation we have thus

$$\bar{P}_{-\frac{1}{2}}=\frac{\sin^2(|f_1|^2+\delta^{*2})^{\frac{1}{2}}T}{1+\delta^{*2}/|f_1|^2}, \quad (41)$$

which differs from (18) only by the replacement of $\delta$ by $\delta^*$. This can be written still more conveniently using (8), (10), and (16), introducing the "effective" quantities

$$H_1^*=\tfrac{1}{2}[H_1^2+H_2^2+2H_1H_2\sin(\varphi_1-\varphi_2)]^{\frac{1}{2}}, \quad (42)$$

$$H_r^*=H_r\left[1-\frac{H_1^2+H_2^2-2H_1H_2\sin(\varphi_1-\varphi_2)}{16H_r^{*2}}\right], \quad (43)$$

and writing $T=2\omega(t-t_0)$ in the form

$$\bar{P}_{-\frac{1}{2}}=\frac{\sin^2 H_1^*\mu\left[1+\left(\frac{H_0-H_r^*}{H_1^*}\right)^2\right]^{\frac{1}{2}}(t-t_0)/\hbar}{1+\left(\frac{H_0-H_r^*}{H_1^*}\right)^2}, \quad (44)$$

which has the same form as the corresponding expression in (18) for a rotating field with the only difference, that the magnitude $H_1$ of the rotating field and the resonance value $H_r$ of the constant field $H_0$ have to be replaced by their "effective values," given by (42) and (43).

In the usual case of an oscillating field in the $x$ direction with an amplitude $H_1$ one has $H_2=0$,

$$H_1^*=H_1/2 \quad \text{and} \quad H_r^*=H_r(1-H_1^2/16H_r^{*2}).$$

The magnetic moment of the particle is then, according to (16) and except for higher order corrections in $1/H_r^*$ related with the resonance value $H_r^*$ of the constant field by the equation

$$\mu=\frac{\hbar\omega}{2H_r^*}\left(1-\frac{H_1^2}{16H_r^{*2}}\right). \quad (45)$$

Although in the determinations of the magnetic moments by the magnetic resonance method the second term in the bracket of (45) has always been neglected we see that the correction, which it involves is in all practical cases extremely slight. Even in the case, where the ratio of the oscillating field amplitude to the constant field at resonance is as much as $\frac{1}{10}$ it amounts to less than 1 percent and in the cases of high precision determinations of magnetic moments, where that ratio has been chosen about 1/100 it is perfectly negligible. It seemed to us worth while to ascertain this fact in view of the importance of these determinations.