On the Yield of Nuclear Reactions with Heavy Elements

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The cross sections for different kinds of nuclear reactions are calculated as functions of the energy of the bombarding particles by means of statistical methods. Their application is restricted to heavy elements ($A>50$) and to bombarding energies greater than 1 Mev. The excitation curves of several (p,n)-reactions have been measured for elements with A between 60 and 115; it is found that the measured cross sections and their dependence on the energy suggests a nuclear radius of $R=1.3\times10^{-13}\times A^{\frac{1}{3}}$ cm for these elements. Section I gives a complete discussion of the calculated cross sections. Section II and III contain the derivations of these expressions. Section IV describes the new experimental material and its implications for the theory.

I. INTRODUCTION AND DISCUSSION OF RESULTS

THE present paper deals with the excitation functions of nuclear reactions insofar as one is allowed to disregard resonance effects and other features due to the properties of individual nuclear states. Our considerations are therefore restricted to reactions of fairly fast particles on heavy nuclei in order to have many quantum states involved simultaneously. Such reactions as the capture of slow neutrons or the absorption of γ-rays with an energy less than a few Mev must be excluded. The use of statistical methods for the description of nuclear processes arising from fast particles has been suggested by several authors[1–5] and this paper presents a more detailed application of these methods to special problems.

According to Bohr, nuclear reactions may be thought of as taking place in two stages: (a) the formation of a compound nucleus; (b) the disintegration of the compound nucleus into a residual nucleus and an ejected particle. This division enables one to express the cross section of a nuclear reaction in the following way: Consider a reaction $Y(a,b)Y'$ (in short an "(a,b)-reaction"), that is a process in which a nucleus Y is bombarded by a particle a and a particle b is ejected from the compound nucleus, leaving a nucleus Y'. The cross section is given by

$$\sigma(a,b)=\sigma_a(\epsilon)\eta_b(E). \tag{1}$$

Here $\sigma_a(\epsilon)$ is the cross section for the formation of a compound state by bombarding with a particle a of energy ϵ and η_b is the relative probability of emission of a particle b by the compound nucleus $Y+a$ which is excited with the energy E. Here $E=\epsilon+E_a$ where E_a is the binding energy of a in $Y+a$. E_a is defined as the energy which must be supplied to the lowest state of the compound $Y+a$ in order to just dissociate it into Y and a.

The cross section σ_a in turn can be split into factors

$$\sigma_a(\epsilon)=S_a(\epsilon)\cdot\xi_a(\epsilon). \tag{2}$$

Here $S_a(\epsilon)$ can be described as the cross section for reaching the "surface" of the nucleus and ξ_a is the probability that the particle a interchanges energy with the nucleus thus forming a compound state, whereas $1-\xi_a$ is the probability of an elastic reflection. Both magnitudes are more accurately defined and discussed in Section II.

For uncharged particles the "penetration cross section" S_a is equal to the geometrical cross section πR^2 of the nucleus if $\lambda\!\!\!^{-}=\lambda/2\pi$ is much smaller than the nuclear radius R, where λ is the wave-length of the particles. This condition is equivalent to $\epsilon\gg0.2/(R^2\times10^{24})$ where ϵ is the energy of the particle (neutron) in Mev and R is measured in cm. S_a increases with lower energies and is equal to $\pi\lambda\!\!\!^{-2}$ for $\lambda\!\!\!^{-}\gg R$. For charged particles S_a is much smaller due to the repulsive Coulomb force and is determined by the penetration

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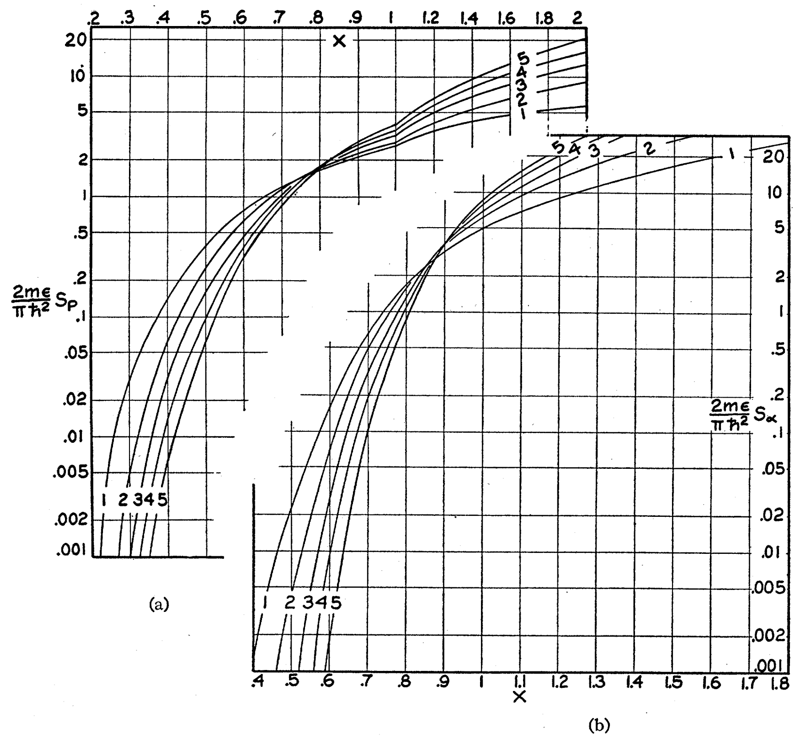

FIG. 1. Penetration cross sections S_p for protons (upper curves) and S_α for α-particles (lower curves) as functions of the energy ϵ. If ϵ is measured in Mev and the cross sections in cm^2 the relations are

$$(2m\epsilon/\pi\hbar^2)S_p = 1.52\epsilon\times10^{26}S_p. \quad x = 6.9\ \times10^{12}\epsilon r_0 A^{\frac{1}{3}}/Z.$$

and

$$(2m\epsilon/\pi\hbar^2)S_\alpha = 6.08\epsilon\times10^{26}S_\alpha. \quad x = 3.45\times10^{12}\epsilon r_0 A^{\frac{1}{3}}/Z.$$

The graphs are computed for $r_0 = 1.3\times10^{-13}$ cm. Curves 1, 2, 3, 4, 5 belong to $Z = 20, 30, 40, 50, 60$, respectively. The scale of x in the upper curves is changed for $x > 1$.

probability through the potential barrier of the Coulomb field. The calculated values depend very strongly upon the assumed magnitude of the nuclear radii. It is generally assumed that the nuclear radius can be represented by the formula $R = r_0 A^{\frac{1}{3}}$ where A is the atomic number and r_0 is a constant having the dimensions of a length. The value of r_0 has been determined by several authors[6] from evidence on the Coulomb energy of light nuclei ($A < 25$). Bethe has found $r_0 = 1.47\times10^{-13}$ cm and Barkas $r_0 = 1.43\times10^{-13}$ cm by analyzing the mass difference between isobars. These values, however, do not fit very well the experiments on the excitation functions of (p,n)-reactions for elements with $A = 60$ and higher which are described in Section IV. These experiments indicate that a smaller radius of $r_0 = 1.3\times10^{-13}$ cm would account for the results much better. One cannot exclude the possibility that r_0 depends on the atomic number and assumes smaller values in the middle of the periodic system.

In the present computations $r_0 = 1.3\times10^{-13}$ cm is used since this value represents best the observed cross sections for bombardments with charged particles. The computed curves and most of the conclusions drawn from them in this paper would remain approximately valid also if further investigations show that the interpretation and the determination of the nuclear radius from the curves were erroneous.

The variation in S_a with ϵ and the nuclear charge Z for protons and α-particles is shown in Fig. 1(a) and (b). The curves are calculated by

[6] H. A. Bethe, Phys. Rev. **54**, 436 (1938); W. H. Barkas, Phys. Rev. **55**, 691 (1939); E. Wigner, Phys. Rev. **56**, 519 (1939).

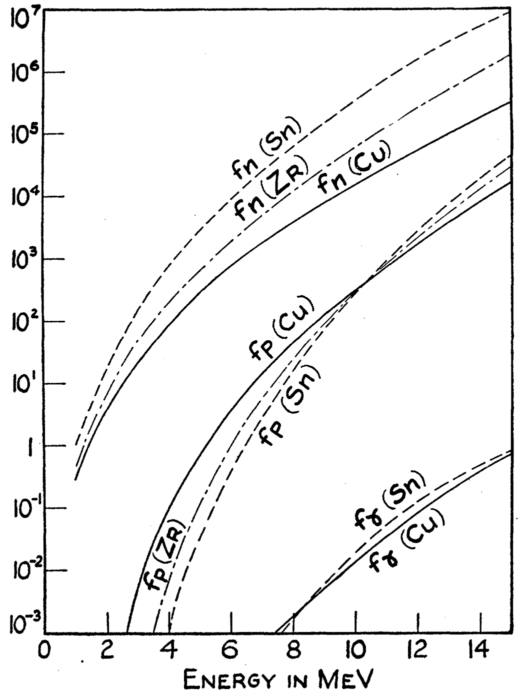

FIG. 2. The functions f_n, f_p and f_γ are given as functions of energy for the compound nuclei Cu, Zr and Sn. These values must be multiplied by 2 or 0.5 if the residual nucleus is odd-odd or even-even, respectively. Expression (21) for the level density is used in the computation.

the W-K-B approximation method (see Section II) and may deviate from the real value by about 10 percent. $S_a/\pi\lambda\!\!\!^{-2}$ is plotted as a function of $x=\epsilon/B$ where B is the Coulomb barrier height. ($B=Ze^2/r_0A^{\frac{1}{3}}=(Z/r_0A^{\frac{1}{3}})\times 1.45$ Mev if r_0 is measured in units of 10^{-13} cm.) The W-K-B method sometimes gives kinks in the excitation curves which arise from the inaccuracy of the method. These kinks are flattened out in Fig. 1(a) and (b) but the changes do not amount to more than 10 percent.

The same curves can be used for other values of r_0 in the following way: the value of $2m\epsilon S_a/\pi\hbar^2$ is roughly independent of r_0 for $x>1$ and assumes the value $(2m\epsilon S_a/\pi\hbar^2)^{(r_0'/r_0)^{\frac{1}{2}}}$ for $x\leqslant 1$ if r_0 is changed to r_0'. The value of the barrier B varies as $Br_0/r_0'=B'$, so that x corresponds to different energies for different r_0. This transformation is good within 15 percent for radii 1.2×10^{-13} cm $<r_0<1.8\times 10^{-13}$ cm. In the computation of Fig. 1(a) and (b) the atomic weight A has been put equal to that of the most abundant isotope occurring with the charge Z. The value of R is not known well enough to distinguish here between the isotopes.

The sticking probability $\xi_a(\epsilon)$ is assumed to be roughly independent of the nature of the particle. In Section II reasons are given for assuming $\xi_a \lesssim \epsilon^{\frac{1}{2}}$ for $\epsilon<1$ (ϵ expressed in Mev) and $\xi_a\sim 1$ for $\epsilon>1$. These formulae should give only a general trend; the experiments performed with protons indicate fluctuations of at least a factor 4. The probability ξ_p has been found to lie between 0.3 and 1.3 with the above values of S_p.

The relative probability η_b of the decay of the compound nucleus by emission of a particle b is considered to be independent of the way the compound nucleus is formed. This assumption is justified because of the rapid dissipation of the energy of the incident particle among the constituents of the compound nucleus. The properties of the state created are thus to a good approximation independent of the way the energy is supplied.

Under this assumption η_b is given by

$$\eta_b=\Gamma_b/\sum_{b'}\Gamma_{b'}. \tag{3}$$

Here Γ_b is the emission probability per unit time of the particle b by the compound nucleus; the sum is to be taken over all particles b' which

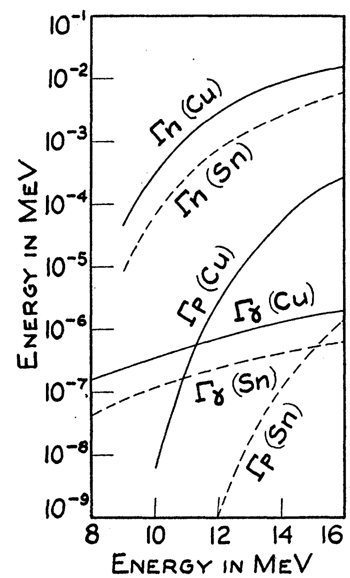

FIG. 3. Neutron, proton and radiation width as functions of the excitation energy of the compound nucleus. The binding energies E_p and E_n are put equal to 8 Mev.

can be emitted. In the following Γ_b is expressed in energy units ($\hbar\times$emission probability) and represents the partial widths of the compound level for the emission of b without specification of the state in which the residual nucleus is left.

According to Section III the emission probabilities can be written as

$$\Gamma_b = f_b(E-E_b)/\omega_c(E),$$

where $\omega_c(E)$ is the level density of the compound nucleus at the excitation energy E and f_b is a function only of the difference between E and the binding energy E_b of the particle b in the compound nucleus, which again is defined as the energy which must be supplied to the lowest state of the compound nucleus in order to dissociate it into Y' and b with Y' in its normal state. $E-E_b$ is the maximum energy $\epsilon_{b\,\max}$ the particle b can attain; but it does not attain this in most of the emission processes since the residual nucleus is generally left excited. (Of course, for photons $E_b=0$.) The functions f_b are dimensionless and do not depend in our approxi-

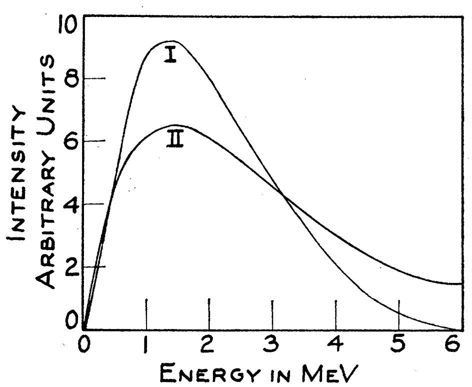

FIG. 4. Curve I: Energy distribution of neutrons leaving a compound nucleus ($A=64$) with a maximum energy of 6 Mev. Curve II: Maxwell distribution corresponding to the appropriate nuclear temperature.

mation on the properties of the compound nucleus. $f_b(E-E_b)$ is the ratio between the width Γ_b and the level separation in the compound nucleus at an excitation energy E. It is now simpler to write instead of Eq. (3):

$$\eta_b = f_b(E-E_b)/\sum_{b'} f_{b'}(E-E_{b'}). \tag{3a}$$

If the functions $f_b(\epsilon_{b\,\max})$ are given, the cross section for an (a,b)-reaction induced by a particle a of energy ϵ can be readily calculated by means of the formula:

$$\sigma(a,b) = S_a(\epsilon)\cdot\xi_a\cdot\frac{f_b(\epsilon-T(a,b))}{\sum_{b'} f_{b'}(\epsilon-T(a,b'))}. \tag{4}$$

Here $T(a,b)=E_b-E_a$ is the threshold of the (a,b)-reaction. ξ_a can roughly be put equal to unity for $\epsilon>1$ Mev. The sum $\sum_{b'}$ is to be taken over all possible reactions (a,b'). In this paper

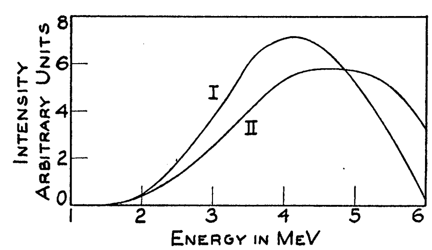

FIG. 5. Energy distribution of protons leaving a compound nucleus ($Z=29$) with a maximum energy of 6 Mev. Curves I and II are derived with two different expressions $\omega^{(1)}$ and $\omega^{(2)}$ for the level density of the residual nucleus. (See Section III.)

we consider only processes which lead to the emission of neutrons, protons or gamma-rays. Processes which involve the expulsion of α-particles are found only for light nuclei to which these considerations should be applied with great care. The f_b's corresponding to n, p, γ-emission, f_n, f_p, f_γ, can be calculated by means of formula (19) and are plotted in Fig. 2 for Cu($Z=29$), Zr($Z=40$) and Sn($Z=50$) as functions of energy. The values of the f_b for other Z's can be found from Fig. 2 by interpolation. The accuracy of these curves is not expected to be very good because of the uncertainty in the assumptions about level densities. However, an error of more than a factor two is improbable.

The strong increase of the particle widths with larger E is due to the increasing number of states in which the residual nucleus can be left and represents the energy dependence of the evaporation probability. f_p is reduced by the effect of the potential barrier. The values of f_γ which are given should be considered as a very rough estimate of the orders of magnitude involved. The accuracy in this case is certainly lower than in the computation of f_p and f_n. The real breadths Γ_b are plotted for Sn and

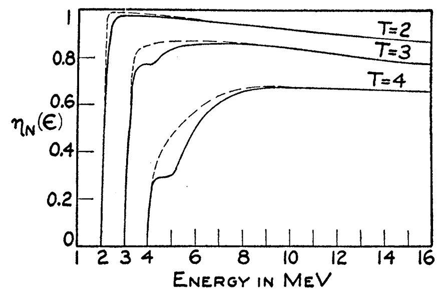

FIG. 6. The relative probability η_n for neutron emission as a function of the energy ϵ of the bombarding proton plotted for three different thresholds T of the (p,n)-reaction for $Z=29$. The heavy lines are computed with $\omega^{(1)}$ the broken lines with $\omega^{(2)}$ as expression for the level density. (See Section III.)

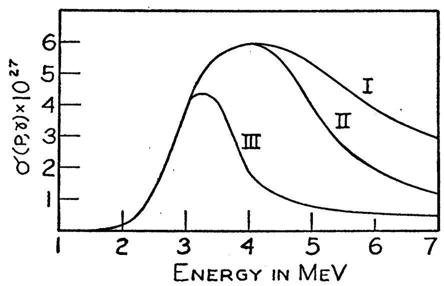

FIG. 7. The (p,γ) cross section for $Z=29$ as function of the proton energy. Curve I for infinitely high (p,n)-threshold, Curves II and III for $T(p,n)=4$ and 3 Mev, respectively.

Zn in Fig. 3, assuming the binding energy $E_n=E_p=8$ Mev.

We now discuss the cross sections of several types of nuclear reactions and their energy dependence as computed from the expression (4):

A. Reactions induced by neutron bombardment

In order that the formulae be valid it is necessary that the energy of the neutron be so high that many states of the compound nucleus are simultaneously excited. The minimum energy for this condition is probably about 1 Mev for $A>50$.

1. (n,n)-reactions.—The most probable process after the formation of the compound nucleus is the reemission of the neutron. It is seen in Fig. 3 that the neutron width is greater than the radiation width for $\epsilon>1$ Mev. It is also greater than the proton width provided that the binding energy E_p of the proton in the compound nucleus is not much lower than the binding energy E_n of the neutron. E_n-E_p is certainly not less than 0.7 Mev because of the stability of the bombarded nucleus against β-decay. We obtain therefore $\eta_n\cong1$ and for the cross section:

$$\sigma(n,n)\cong\pi R^2 \quad \text{for} \quad \epsilon>1 \text{ Mev.}$$

The energy distribution of the outgoing neutrons is given by Eq. (20). It is similar to a Maxwell distribution and is shown in Fig. 4 for 6-Mev neutrons bombarding Cu.

2. $(n,2n)$-reactions.—If the residual nucleus is left after an (n,n)-reaction in a state with an excitation energy above the binding energy of a neutron, a second neutron is emitted by the residual nucleus. The residual nucleus could, of course, also emit its excitation energy in the form of γ-rays but the probability of this process is small compared to the $(n,2n)$-reaction a few hundred kilovolts above the threshold of the latter reaction. The cross section for the $(n,2n)$-reaction can then easily be calculated from the energy distribution $I(\epsilon)d\epsilon$ of the outgoing neutrons. It is

$$\sigma(n,2n)=\pi R^2\int_0^{\epsilon'} I(\epsilon)d\epsilon \Big/ \int_0^{\epsilon_{n\,\max}} I(\epsilon)d\epsilon$$

where $\epsilon'=\epsilon_{n\,\max}-T(n,2n)$ is the maximum energy of an outgoing neutron for which the residual nucleus is left able to emit another neutron. One obtains approximately

$$\sigma(n,2n)\cong\pi R^2[1-(1+\Delta\epsilon/\Theta)e^{-\Delta\epsilon/\Theta}].$$

Here $\Delta\epsilon$ is the excess energy of the primary neutron above the threshold of the $(n,2n)$ process: $\Delta\epsilon=\epsilon-E_n'$; E_n' is the binding energy of a neutron to the residual nucleus. Θ is given by $\Theta=2(5\epsilon/A)^{\frac{1}{2}}$ Mev if ϵ is expressed in Mev. This formula is based upon a Maxwellian distribution of the secondary neutrons in the (n,n) process corresponding to a temperature Θ as derived in reference 4. This is a good approximation for the high initial neutron energies necessary in the $(n,2n)$ process.

3. (n,p)-reactions.—The yield of this reaction is naturally very small because of the strong competition of the (n,n) process. The cross section is given by

$$\sigma(n,p)=\pi R^2\frac{f_p(\epsilon-T(n,p))}{f_p(\epsilon-T(n,p))+f_n(\epsilon)}.$$

Here $T(n,p)$ is the threshold of the reaction. The energy distribution of the outgoing protons is plotted for a maximum energy of 6 Mev in Fig. 5 according to formula (20). The form of the distribution curve depends strongly on the level density of the residual nucleus as shown in Fig. 5. Any experimental evidence would be of great value since our knowledge concerning level densities is very vague.

4. (n,γ)-reactions.—The capture of a neutron is an extremely improbable process for neutron energies above 1 Mev. Some figures are given in Table I, which possess the same inaccuracy as the radiation widths.

B. Reactions induced by charged particles

The requirement that many states are simultaneously excited is practically always fulfilled since a charged particle penetrates nuclei above $A=50$ to a measurable amount only at energies above 2 Mev. For the same reason ξ can be put equal to unity throughout and the cross section for an (a,b) process is $\sigma(a,b)=S_a(\epsilon)\eta_b$. The reactions induced by deuterons are not discussed here. The comparatively small binding energy permits a break-up of the projectile before it has entered into the nucleus (Oppenheimer-Phillips process) so that methods based upon the formation of a compound nucleus cannot safely be applied.

1. (p,n)-reactions.—This is in general the most probable reaction induced by a proton. It must be remembered however that near the threshold the processes (p,p) and (p,γ) may be comparatively stronger. The (p,γ) competition is negligible a few hundred kilovolts above the threshold. The factor η_b as a function of the energy of the bombarding proton is given by

$$\eta_n=\frac{f_n(\epsilon-T(p,n))}{f_n(\epsilon-T(p,n))+f_p(\epsilon)}$$

and the calculated values for Cu for different thresholds are plotted in Fig. 6. We note that

TABLE I. *Values of the capture cross section for neutrons in cm.²*

	$\epsilon=1$ MEV	2 MEV	4 MEV	6 MEV	9 MEV
Cu	2.2×10^{-26}	3.8×10^{-27}	1.0×10^{-27}	6.0×10^{-28}	4.4×10^{-28}
Sn	5.5×10^{-26}	1.1×10^{-26}	2.1×10^{-27}	8.8×10^{-28}	5.5×10^{-28}

η_n is never greater than about 0.8 for a (p,n)-reaction with a threshold of 3 Mev in the region of copper. The higher Z is, the less probable is the escape of a proton and the closer η_n is to unity. Fig. 4 is computed for a $Y(n,p)Y'$ reaction where Y' is an even-odd or an odd-even nucleus. If Y' is odd-odd the values for η_n are higher and must be replaced by η_n' according to the formula

$$1/\eta_n'=(3\eta_n+1)/4\eta_n,$$

where η_n is taken from Fig. 6. This difference arises from the fact that a less stable nucleus has a higher level density and therefore a higher statistical weight than a stable nucleus. The curves are plotted for two different expressions for the level density of the residual nucleus. η_n depends strongly on the particular properties of the level density if the bombarding energy is near the threshold.

2. $(p,2n)$-reactions.—If the energy of the proton is large enough, the residual nucleus of the (p,n)-reaction might be so highly excited that it emits another neutron rather than a γ-ray. The threshold of this process is given by $T(p,2n)=E_n+E_n'-E_p$ and is expected to be about ~10 Mev. E_n' is the binding energy of a neutron to the residual nucleus. The cross section can be computed in the same way as $\sigma(n,2n)$. We obtain:

$$\sigma(p,2n)=S_p(\epsilon)\eta_n[1-(1+\Delta\epsilon/\Theta)e^{-\Delta\epsilon/\theta}]. \quad (5)$$

$\Delta\epsilon$ is the excess of ϵ above the threshold $T(p,2n)$. Evidently the yield of the (p,n)-reaction decreases for $\epsilon>T(p,2n)$ and it is

$$\sigma(p,n)=S_p(\epsilon)\eta_n-\sigma(p,2n).$$

3. (p,γ)-reactions.—The capture of protons is a reaction which is important if the energy of the proton is below or in the neighborhood of the threshold $T(p,n)$ of the (p,n) reaction. The only competing process then is the (p,p)-reaction. The (p,γ) cross section is given by

$$\sigma(p,\gamma)=S_p\frac{f_\gamma(\epsilon+E_p)}{f_\gamma(\epsilon+E_p)+f_n(\epsilon-T(p,n))+f_p(\epsilon)}.$$

Fig. 7 shows the (p,γ) cross section as a function of ϵ for Zn assuming that the binding energy E_p is 8 Mev. This curve is to be considered only a qualitative picture of the effect expected. It is, however, in fair agreement with measurements of Strain[7] who found a cross section of roughly 10^{-27} cm^2 for the reaction $Ni^{61}(p,\gamma)Cu^{62}$ at 4 Mev.

4. (p,p)-reactions.—The inelastic cross section of protons is given by $\sigma_p-\sigma(p,\gamma)$ if the threshold of the (p,n)-reaction is not yet reached. Above the threshold it drops rapidly to very small values because of the high (p,n) competition. The value of η_p is given by $1-\eta_n$ in a region where $f_\gamma \ll f_n$ and can be taken directly from Fig. 6 for Cu and neighboring elements. The energy distribution is the same as for (n,p)-reactions (Fig. 5). The (p,p)-reaction plays an important role in the excitation of nuclei by inelastic collisions. Barnes and Aradine[8] have found that the isomeric state of In^{115} can be excited by protons of 7 Mev bombarding In and have measured a cross section for this process of about 3×10^{-29} cm^2. The excitation of the isomeric state is in most of the collisions performed by lifting the nucleus into a higher excited state from which it performs radiative transitions partly back to the ground state and partly to the isomeric state. A collision which brings the nucleus directly into the isomeric state is a rare event because of the many other states in which the nucleus could be left. Thus the above value is a lower limit for the excitation cross section of In by proton bombardment. The calculated cross section for a (p,p)-reaction for 7 Mev is 0.6×10^{-29} cm^2. This shows that at least part of the observed effect may be attributed to inelastic collisions, but that the rest may have to be explained by other excitation processes.[9]

5. (α,n)-reactions.—The most common reaction induced by α-particles is the emission of a neutron. If the threshold of the (α,p)-reaction is higher or close to the (α,n) threshold the proton competition is small and we get $\eta_n\sim1$ and a cross section equal to S_α. If, however, $T(\alpha,n) > T(\alpha,p)$ the (α,p) competition can be appreciable. η_n can be calculated along the same lines as for (p,n)-reactions. If the α-energy is sufficiently high the compound nucleus is able to emit two neutrons, and the (α,n) cross section is decreased by the $(\alpha,2n)$ competition. It is then

$$\sigma(\alpha,n)=S_\alpha\eta_n-\sigma(\alpha,2n).$$

6. (α,2n)-reactions.—The threshold of the $(\alpha,2n)$ reaction is hard to predict owing to the lack of knowledge of the binding energy of an α-particle. This value is about four times as uncertain as the binding energy E_p and E_n of the elementary particles since we may write: $E_\alpha=E_n+E_n'+E_p+E_p'-28$ Mev. Here E_n' and E_p' are the energies necessary to remove the second neutron or proton from the compound nucleus, respectively, the value 28 Mev is introduced for the mass defect of the α-particle. More information could be obtained by measuring the threshold of the $(\alpha,2n)$ reaction. According to the relation $T(\alpha,2n)=E_n+E_n'-E_\alpha$ the uncertainty of E_α would be reduced.

If we assume roughly $E_p=E_p'=E_n=E_n'=9$ Mev we would obtain $T(\alpha,2n)\sim10$ Mev. The cross section $\sigma(\alpha,2n)$ is given by formula (5) if S_p is replaced by S_α and $\Delta\epsilon=\epsilon-T(\alpha,2n)$. α-particles of 16 Mev should therefore give $(\alpha,2n)$-yields comparable to the (α,n)-yields.

7. (α,γ)-reactions.—The capture of an α-particle becomes quite probable below the threshold of the (α,n)-reaction. A quantitative treatment is difficult because of the lack of knowledge of the binding energy E_α and thus of the excitation energy of the compound nucleus. The behavior of the (α,γ)-reaction is qualitatively similar to that of the (p,γ)-reaction.

8. (α,p)-reactions.—If the energy ϵ of the α-particle is lower than the (α,n) threshold, the main competing process is the (α,γ) reaction. Quantitative predictions can so far only be made if $\epsilon>T(\alpha,n)$. The strongest competing process is then the neutron emission and we may write

$$\eta_p=\frac{f_p(\epsilon-T(\alpha,p))}{f_p(\epsilon-T(\alpha,p))+f_n(\epsilon-T(\alpha,n))}.$$

9. (α,α)-reactions.—This type of reaction should be excluded since we have assumed that the emission of an α-particle by the compound nucleus is under all circumstances extremely improbable and has not been observed with heavy nuclei ($A>60$) outside of the radioactive group. The cross sections obtained by an application of our formulas are very small indeed. Even

[7] C. V. Strain, Phys. Rev. **54**, 1021 (1938).
[8] S. W. Barnes and P. W. Aradine, Phys. Rev. **55**, 50 (1939).
[9] For other excitation possibilities see V. F. Weisskopf, Phys. Rev. **53**, 1018 (1938).

for an α-particle energy of 16 Mev we get $\sigma(\alpha,\alpha)\leqq 3\times 10^{-29}$ cm^2 for Cu and $\sigma(\alpha,\alpha)\leqq 4\times 10^{-33}$ cm^2 for Sn. Here it is assumed that the binding energy of the α-particle in the compound nucleus is equal to that of the neutron. If the latter is larger by 4 Mev the cross section is about ten times as large. This small value of $\sigma(\alpha,\alpha)$ seems to exclude the possibility of exciting In^{115} to In^{115*} by means of an (α,α)-reaction. Since this excitation does take place[10] it is most probably due to the electric field of the α-particle passing by. The possibility however of an inelastic collision without formation of a real compound nucleus cannot be ruled out. Because of the high internal binding energy, an energy exchange could take place without "dissolving" the α-particle in the nucleus.

II. The Formation of the Compound Nucleus

A nucleus Y in the state α, which is not necessarily the ground state is bombarded by a particle a with energy ϵ. The formation of a compound nucleus depends on two factors: first, the particle must come close enough to the nucleus to be within the range of the nuclear forces, and, second, an energy exchange between the particle and the nucleus must take place since a pure elastic reflection is not considered as involving the formation of a compound state. The short range and high intensity of the nuclear forces make it possible to distinguish between an inside and an outside region of the nucleus. The outer region determines the first factor, the inner region the second.

Let us decompose the incident particle beam into partial beams with orbital angular momenta $\hbar l$, $l=0, 1, 2\cdots$. The cross section $\sigma_{a\alpha}$ for the formation of a compound state can be written as

$$\sigma_{a\alpha}=\sum_l \sigma_{a\alpha}{}^{(l)}=\frac{\pi}{k^2}\sum_l (2l+1)Q_{a\alpha}{}^{(l)}.$$

Here k is the wave number of the incident particle: $k=2\pi/\lambda$. Since $(2l+1)\pi/k^2$ would be the cross section if the entire lth partial wave were absorbed, $Q_{a\alpha}{}^{(l)}$ is always smaller than unity. In order to express the two factors which determine the formation of the compound nucleus we write

$$Q_{a\alpha}{}^{(l)}=P_a{}^{(l)}\xi_{a\alpha}{}^{(l)}. \tag{6}$$

Here $P_a{}^{(l)}$ is the penetration probability of the incoming wave into the surface of the nucleus and $\xi_{a\alpha}{}^{(l)}$ is the sticking probability of the particle. This splitting of $Q_{a\alpha}{}^{(l)}$ into two factors is somewhat artificial since the sticking probability itself depends on the phase and slope of the incoming wave function but it can be used in an attempt to distinguish between the well-known part of the collision process which takes place outside of the nucleus and the unknown part inside. In order to get expressions which are close enough to the classical concepts of penetration and sticking we define $P_a{}^{(l)}$ so that it is unity for an uncharged particle a with zero angular momentum and for any angular momentum for which $l\lambda\ll R$, and that $P_a{}^{(l)}$ goes to zero for $l\lambda\gg R$. Thus in the classical limit of $\lambda\ll R$, $P_a{}^{(l)}$ is unity if a particle with the angular momentum $l\hbar$ would hit the nucleus and is zero if this particle would not hit it. In general $P_a{}^{(l)}\leqq 1$ and is determined by the repulsive effect of the Coulomb field in the case of charged particles and by the centrifugal force if $l\neq 0$.

The penetration probability $P_a{}^{(l)}$ through the field outside the nucleus can be best computed by considering the particle a emitted rather than absorbed by the nucleus, since the wave function of an incident and partly absorbed particle beam depends strongly upon the conditions of elastic reflection, while the wave function of an emitted particle is a simple outgoing radial wave. The expression for $P_a{}^{(l)}$ which is used in this paper is given by

$$P_a{}^{(l)}=1/|F_a{}^{(l)}|^2 \tag{7}$$

where $F_a{}^{(l)}$ is the radial part* of a wave of the particle a emerging from the center of the nucleus with an orbital angular momentum $l\hbar$ taken at the distance R from the center if its value at infinity is given by $e^{ikr+i\delta}$. δ is an arbitrary phase. The nuclear radius R is put equal to the closest distance from the center in which the nuclear forces can be neglected. $1/|F_a{}^{(l)}|^2$ is defined by the forces outside of the nucleus and represents

[10] K. Lark-Horovitz, J. R. Risser, R. N. Smith, Phys. Rev. **55**, 878 (1939).

* We understand by radial part the function $u(r)$ if the total wave function is given by $\psi=r^{-1}u(r)\cdot Y(\vartheta,\varphi)$.

the factor by which the intensity of an outgoing wave is reduced after having penetrated the potential barrier between the surface of the nucleus and infinity.

This magnitude fulfills the requirements of $P_a^{(l)}$ mentioned above and describes as closely as possible the reducing effect of the potential barrier around the nucleus. Furthermore, it is in agreement with the results of Kapur and Peierls.[11] According to these authors the emission probability $\Gamma_{a\alpha}^{(l)}$ of a particle a with angular momentum leaving the nucleus in the state α is given by

$$\Gamma_{a\alpha}^{(l)}=\frac{\hbar^2}{m}\frac{k}{|F_a^{(l)}|^2}|\phi_a^{(l)}|^2. \qquad (8)$$

Here m is the mass of the escaping particle, k is its wave number at an infinite distance from the nucleus, $|\phi_a^{(l)}|^2$ is the square of the wave function ϕ of the compound nucleus taken at a point in the configuration space which corresponds to the particle a being at the distance R and the residual nucleus being in the state α, and integrated over all coordinates except the distance of a from the center. ϕ is normalized according to $\int|\phi|^2d\tau=1$ where the integral is taken over all coördinates within the nuclear sphere. In the expression (8) the factor $|\phi_a^{(l)}|^2$ is mainly determined by the forces inside the nucleus. In the approximation attempted here we consider this factor as completely independent of the field outside. Thus the remaining factors of (8) must contain the penetrability $P_a^{(l)}$. The requirement that $P_a^{(l)}$ should be unity for uncharged particles and $l=0$ for all values of k, leads at once to (7) since $|F_a^{(0)}|^2=1$ for a free particle.

In case of the bombardment of heavy nuclei it is often impossible to distinguish between different angular momenta because many levels with different angular momenta are in resonance simultaneously. It is more useful then to write

$$\sigma_{a\alpha}=S_a\xi_{a\alpha}, \quad S_a=\frac{\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)P_a^{(l)}. \qquad (9)$$

Here $\xi_{a\alpha}$ is an average sticking probability.

The $P_a^{(l)}$ can be readily calculated according to (7) if the particle is uncharged. It is

$$P_a^{(l)}=(2/\pi Rk)\cdot|H_{l+\frac{1}{2}}^{(1)}(Rk)|^{-2}.$$

Here $H_{l+\frac{1}{2}}^{(1)}(x)$ is the Hankel function of the first kind. We obtain in particular

$$P_a^{(l)}=1 \quad \text{for} \quad Rk\gg l$$

$$P_a^{(l)}=\frac{(Rk)^{2l}}{[(2l+1)(2l-1)(2l-3)\cdots]^2} \quad \text{for} \quad Rk\ll l.$$

The classical limit $R\gg 1/k$ is characterized by the formula

$$S_a=\pi R^2.$$

The expressions for the case of charged particles are calculated with the W-K-B method which can be applied with sufficient accuracy for nuclei with $Z>20$. According to our definition (7) of the penetration probability the values are different from those used so far by Bethe[12] and Konopinski and Bethe.[5] From (7) it follows that

$$P_a^{(l)}=\left(\frac{B_l-\epsilon}{\epsilon}\right)^{\frac{1}{2}}e^{-2C_l}, \qquad B_l=\frac{zZe^2}{R}+\frac{\hbar^2}{2mR^2}l(l+1), \qquad (10)$$

in which C_l is a function given by Eq. (631) in reference 12 and which is plotted in reference 5. B_l is the energy barrier height for a particle with angular momentum $\hbar l$. ze is the charge of the particle a. The approximate formula (10) does not hold for energy values near the barrier B_l. It is, however, possible to use (10) for the calculation of S_a (formula (9)) for elements with $Z>20$ since the contribution of the lth partial wave for $\epsilon\sim B_l$ is only a small part (less than 10 percent) of the total S_a. The values of S_a obtained by this method are plotted in Fig. 1(a) and (b) for protons and α-particles respectively.

A direct calculation of the sticking probability $\xi_{a\alpha}$ is impossible because of our lack of knowledge of the interior of the nucleus. A few conclusions, however, can be obtained by considering the general properties of this quantity. We assume in the following that the sticking probability is equal for neutrons and protons due to the fact that the Coulomb forces do not play an important role inside the nucleus. Furthermore we assume that the sticking probability depends only on ϵ and not on l or on the particular state of the nucleus. The energy dependence of ξ can be predicted in two limiting cases.

[11] P. L. Kapur and R. Peierls, Proc. Roy. Soc. **A166**, 277 (1938).

[12] H. A. Bethe, Rev. Mod. Phys. **9**, 161 (1937).

For very small $k(k \ll 1/R)$ only the s wave ($l=0$) and only a small part of it can be absorbed. The corresponding ξ will be much smaller than unity. $\xi^{(0)}$ should be mainly proportional to k:

$$\xi = c \cdot k. \tag{11}$$

This dependence of ξ is identical with the so-called $1/v$-law for slow neutrons since the cross section σ_a is proportional to $1/v$ if $P_a^{(0)}=1$ and $\xi = c\cdot k$ is inserted into (6). For large k (classical limit) it is very probable that the ξ's should be of the order of unity since the particle interacts strongly with all nuclear constituents when it reaches the nuclear surface; it is likely therefore that the particle's energy is transferred to the nucleus and leads to a process other than elastic reflection.

The value of the constant c in Eq. (11) and the energy for which ξ reaches the order of unity can so far only be taken from experiment. Some evidence can be drawn from slow neutron capture experiments. According to Bethe[12] the escape probabilities of slow neutrons (neutron widths) are roughly proportional to k. The order of magnitude of the actual values can be fairly well represented by the assumption that $\xi = \epsilon^{\frac{1}{2}}$ for $\epsilon < 1$ if ϵ is expressed in Mev. From this result it seems reasonable to assume that $\xi = 1$ for $\epsilon > 1$. The experiments with fast particles are in agreement with this assumption. The scattering of fast neutrons is mostly inelastic* and in Section III of this paper evidence is given for taking the sticking probability for protons above 3 Mev to be between 0.3 and 1.

The above considerations can be applied to the emission and absorption of γ-rays. We write the cross section $\sigma_{\gamma\alpha}{}^{(l)}$ for the absorption of a 2^l-pole radiation of the energy ϵ by a nucleus in a state α in the form

$$\sigma_{\gamma\alpha}{}^{(l)} = ((2l+1)\pi/k^2) P_\gamma{}^{(l)}(\epsilon) \xi_{\gamma\alpha}{}^{(l)}. \tag{12}$$

$P_\gamma{}^{(l)}(\epsilon)$ is identical with $P_a{}^{(l)}$ for uncharged particles with the same wave number, and expresses the fact, that radiation of a higher pole is less likely to be absorbed. If the wave-length of the light is large compared to R the assumption (11) about ξ is again justified. This relation is equivalent to an absorption probability proportional to the intensity of the lth spherical harmonic component of the incident wave at the nucleus.

The sticking probability can be expressed in terms of the average matrix element $\langle |M_\alpha{}^{(l)}(\epsilon)|^2\rangle_{\mathrm{Av}}$ of the 2^l-pole moment of the nuclear transition from the state α to a state with an energy higher than that of α by an amount ϵ. One obtains

$$\xi_\gamma = \mathrm{Const}\cdot k \cdot \langle |M_\gamma{}^{(l)}(\epsilon)|^2\rangle_{\mathrm{Av}} / D(\epsilon), \tag{13}$$

where $D(\epsilon)$ is the average distance between the levels reached by this transition. The constant is independent of ϵ. The relation (11) is then identical with the assumption that $\langle |M_\alpha{}^{(l)}(\epsilon)|^2\rangle_{\mathrm{Av}}/D(\epsilon)$ is approximately independent of the energy ϵ.

The experimental knowledge about these quantities is very limited. The cross section for γ-absorption is known to be $\sim 10^{-26}$ cm^2 for γ-rays of 17 Mev.[13] On the other hand, the lifetime of low excited states of some radioactive elements has been found to be of the order of 10^{-12} sec.[14] which corresponds to a Γ_γ of 10^{-3} ev. The excitation energy of the levels investigated is about 1 Mev. In order to check our assumptions, we determine the factor c in (11) from the observed cross section for 17-Mev γ-rays. Formula (12) gives for quadripole radiation

$$\xi_\gamma \sim \epsilon/270 \qquad \epsilon \text{ in Mev.} \tag{14}$$

The relation between emission and absorption probability can be expressed by

$$\Gamma_\gamma{}^{(l)} = \frac{1}{(2l+1)} \frac{\epsilon^2}{\pi \hbar^2 c^2} D(\epsilon) \sigma_{\gamma\alpha}{}^{(l)}(\epsilon). \tag{15}$$

$\Gamma_\gamma{}^{(l)}$ is the emission probability of a 2^l-pole radiation of energy ϵ leaving the nucleus in the state α, averaged over all states from which this transition is possible. For the sake of simplicity the spin of the state α is assumed to be zero. This expression gives an emission probability for a γ-ray of 1 Mev (quadripole radiation) of $\Gamma_\gamma{}^{(2)} \sim 10^{-4}$ ev by using (14), if one assumes a value of 1 Mev for the distance between the lowest levels with spin 2. Although this value is

* New measurements of R. Bacher have shown that at least 95 percent of neutrons having energies from 3 to 5 Mev are scattered inelastically by Al, Cu and Pb. We are indebted to Dr. Bacher for communicating his results to us before publication.

[13] W. Bothe and W. Gentner, Zeits. f. Physik **106**, 236 (1938).

[14] See H. A. Bethe, Rev. Mod. Phys. **9**, 229 (1937).

about ten times smaller than the observed, we believe that the assumption (14) is justified. The expression (13) shows that ξ_γ contains strongly varying functions, since $D(\epsilon)$ changes by a factor $\sim 10^7$ when ϵ goes from 1 Mev to 17 Mev; a discrepancy of 10 should not therefore be considered as serious in this crude approximation.

Furthermore it is seen in Fig. 3 that the values for the radiation width of slow neutron capture levels ($E \cong 8$ Mev) calculated on the basis of the above assumptions, have the order of magnitude $\sim 10^{-7}$ Mev, which fits the present experimental evidence. This width is calculated by means of (17) and by making use of (14) and (15).

III. The Disintegration of the Compound Nucleus

The division of a nuclear process into two stages—the formation and disintegration of the compound nucleus—is based upon the assumption that the disintegration of the compound nucleus is independent of the way in which it has been created. This assumption is certainly not correct. It is known from the dispersion formula for nuclear reactions that the contributions of the different excited compound states add up with definite phase differences depending on the manner of excitation. If, however, many compound states are excited simultaneously, one may reasonably assume that the phase relations between the different states are random.* The compound states would then, to a certain approximation, disintegrate independently of one another; in other words, the disintegration is incoherent. This does not hold for the disintegration which leads to the original state; the elastic scattering of the incident particle takes place coherently no matter how many intermediate states are excited. The elastic scattering, however, is not considered here as a process involving the formation of a compound nucleus.

Under these assumptions the probability η_b for the decay of the compound nucleus of the compound nucleus by the emission of a particle b is given by (3). The particle widths Γ_b are independent of the way the compound state is formed, and depend on its excitation energy E, its spin and other properties of the nucleus. We understand in the following by Γ_b the average value of the emission probability of b taken over all states having an excitation energy near to E above the ground state of the compound nucleus. The value of Γ_b is then a function of E only. We write

$$\Gamma_b = \sum_{\beta,\, l} \Gamma_{b\beta}^{(l)}, \tag{16}$$

where $\Gamma_{b\beta}^{(l)}$ is the probability of escape of b from the compound nucleus with an angular momentum $l\hbar$ and leaving the residual nucleus in a definite state β with energy E_β. The energy of the escaping particle is $\epsilon_b = E - E_b - E_\beta$. Evidently the $\Gamma_{b\beta}^{(l)}$ are also average values over all compound states with excitation energies near E. $\Gamma_{b\beta}^{(l)}$ is the probability of the reverse process to the formation of a compound nucleus by a particle b hitting the residual nucleus in the state α. The cross section for this, $\sigma_{b\beta}^{(l)}$, has been calculated in the previous section. The following relation for the two opposite processes can be derived from simple statistical considerations:

$$\Gamma_{b\beta}^{(l)} = \frac{(2s+1)(2i+1)}{\omega_C(E)} \frac{m\epsilon_b}{\hbar^2\pi^2} \sigma_{b\beta}^{(l)}(\epsilon_b). \tag{17}$$

Here $\omega_C(E)$ is the level density in the compound nucleus of the excitation energy E; $\omega_C(E)dE$ is the number of levels in the interval dE, an n-fold degenerate state is counted as n levels. s and i are the spins of the particle b and the state β of the residual nucleus, respectively. According to the assumptions about ξ made previously, $\sigma_{b\beta}^{(l)}$ does not depend on the state β and we omit this index in what follows. After summation of expression (17) over l and β and after introduction of (9) we get

$$\Gamma_b = \frac{2s+1}{\hbar^2\pi^2} \frac{m}{\omega_C(E)} \sum_\beta (2i+1) S_b(\epsilon_b) \xi_b \epsilon_b. \tag{18}$$

We replace the sum by an integral and get finally:

$$f_b = \omega_C(E)\Gamma_b$$

$$= \frac{m}{\hbar^2\pi^2}(2s+1)\int_0^{E-E_b} \epsilon S_b(\epsilon)\xi_b \omega_R(E-E_b-\epsilon)d\epsilon. \tag{19}$$

* In order to apply the term "compound state" it is furthermore necessary that the states of the compound nucleus which are not in resonance do not contribute essentially to the cross section.

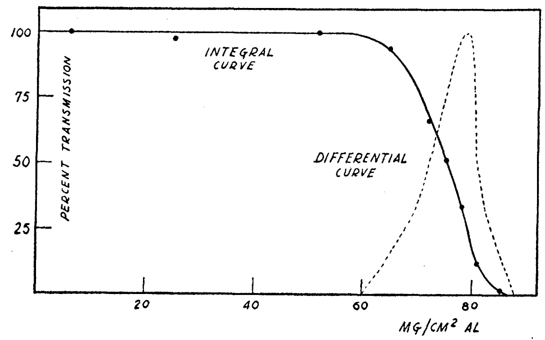

FIG. 8. Transmission of the proton beam by aluminum.

$\omega_R(E)$ is the level density of the residual nucleus and f_b is a function of $E-E_b$ only. The relative energy distribution $I_b(\epsilon)$ of the emitted particles is given by

$$I(\epsilon)=\text{Const}\cdot\epsilon\cdot S_b(\epsilon)\xi_b\omega_R(E-E_b-\epsilon). \quad (20)$$

A large inaccuracy in the computation of f_b is due to the lack of information about level densities. In the present calculations two expressions for $\omega(E)$ are used. The first is

$$\omega^{(1)}(E)=Ce^{(aE)^{\frac{1}{2}}}. \quad (21)$$

The constant a depends on the atomic weight. The choice $a=A/5$ (Mev)$^{-1}$ represents roughly what is known about the level densities and their dependence on atomic weight and energy. The proportionality with A follows from almost all theoretical attempts to derive a formula for $\omega(E)$ and the numerical value is chosen in order to get a level distance of about 10 ev for the region where slow neutrons are absorbed in elements with $A\sim 100$ and about $3-5$ levels below 1 Mev excitation energy. In order to compare nuclei of similar A we assume that the level density is higher for less stable nuclei. We obtain thus the relations:

$$C_{\text{odd-even}}=C_{\text{even-odd}},\ C_{\text{even-even}}<C_{\text{even-odd}}<C_{\text{odd-odd}}.$$

Here $C_{\text{odd-even}}$ refers to a nucleus with odd Z and even $N=A-Z$, etc. $C_{\text{odd-even}}$ is put equal to roughly 0.2 (Mev)$^{-1}$. The absolute value of C drops out of the final result since a common factor in the level density expressions of different nuclei does not alter η_b.

Another expression $-\omega^{(2)}(E)-$ for the level density is taken from the tables computed by Bardeen and Feenberg[15] which should only be applied to nuclei for which $A<60$. The results are similar to those obtained with $\omega^{(1)}$ if one inserts into the formula for $\omega^{(1)}$:

$$C_{\text{even-even}}=2C_{\text{odd, even}}=4C_{\text{odd-odd}}.$$

It should be mentioned, that the expression (3) for η_b is not very sensitive to the choice of the formulae for the level density, since all Γ_b are usually affected in a similar way if a different level density is introduced. The formulae should, however, not be applied to nuclei below $A\sim 50$ because the level density and the sticking probabilities of light nuclei are subject to large individual fluctuations.

IV. Comparison with Experiment

In order to compare the calculated cross sections with experiment several (p,n) reactions have been studied with the 6.5-Mev proton beam of the Rochester cyclotron. To obtain the dependence of the cross section $\sigma(p,n)$ on the energy of the incident particles, thin target excitation functions have been measured by means of the well-known method of stacking thin foils. Special attention has been paid to the homogenity of the beam by measuring the energy distribution before and after bombardment. This has been done by means of a cylindrical evacuated chamber covered with an aluminum window having a thickness of one-thousandth of an inch. The current received by the chamber was measured as a function of the thickness of aluminum absorbers placed in front of it. A typical curve is shown in Fig. 8. From this curve the energy distribution of the beams has been computed taking account of the straggling in the Al foils. Under the conditions of the measurements the inhomogeneity was of the order of 300 kev. As the variation of the theoretical and experimental excitation functions in such a small interval is not very large, this inhomogeneity has been neglected. The radioactivity produced was measured by counters and by ionization chambers and the calibration was done by means of a standard U_3O_8 sample.

The reactions investigated were [16]

[15] J. Bardeen and E. Feenberg, Phys. Rev. **54**, 809 (1938).

[16] Reaction 5 has been measured by T. Enns and reaction 7 by S. Barnes, both of this laboratory. We are indebted to both observers for the communication of their results before publication.

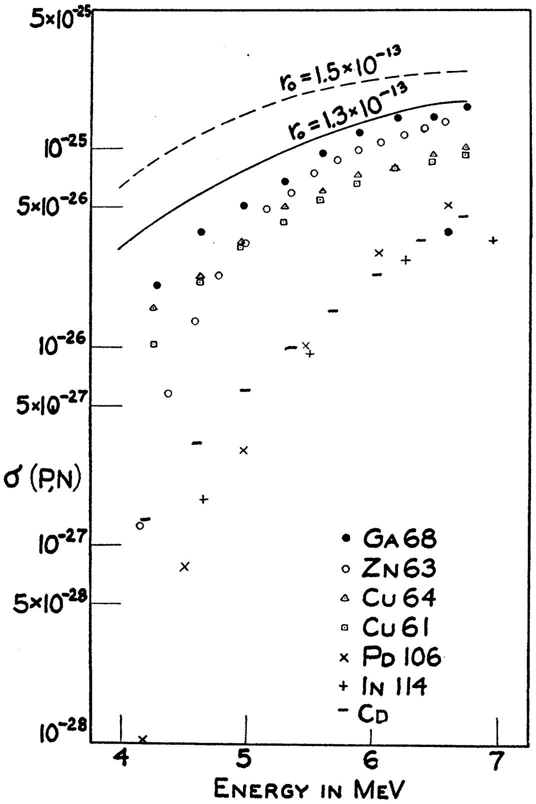

FIG. 9. Observed (p,n) cross sections as a function of the proton energy. The elements listed in the figure are in each case the radioactive elements produced by the reaction. The two curves are the theoretical cross sections S_p for $Z=30$ for two values of r_0.

1. Ni^{61} (p,n) Cu^{61} (half-life 3.4 hours) threshold 3.0 Mev
2. Ni^{64} (p,n) Cu^{64} (half-life 12.2 hours) threshold 2.5 Mev
3. Cu^{63} (p,n) Zn^{63} (half-life 38.2 min.) threshold 4.1 Mev
4. Zn^{68} (p,n) Ga^{68} (half-life 72 min.) threshold 3.7 Mev
5. Pd^{106} (p,n) Ag^{106} (half-life 25 min.)
6. Ag (p,n) Cd (half-life 6.4 hours)
7. Ca^{114} (p,n) In^{114} (half-life 48 hours)

The excitation functions (cross sections as functions of the energy) are plotted in Fig. 9. The accuracy of these values is not better than 20 percent and probably worse for low energies. The reaction 6 is not yet assigned to an isotope of Ag. The abundance, however, of both silver isotopes is nearly equal so that cross sections could be evaluated. Another difficulty arises from the fact that the radioactivity of Cd produced in that reaction consists of an emission of a 92 kev γ-ray following a K capture.[17] Since the internal conversion coefficient of this γ-ray is not known the excitation curve of the reaction 6 is undetermined by a constant factor. We have chosen the factor to fit the theoretical value of the cross section for the highest energy. This would correspond to an internal conversion coefficient of about 1/10 which is roughly the value expected for a dipole radiation of this energy.

The ratio $\sigma_{\text{obs}}/\sigma_{\text{theor}}$ of the observed and the calculated value is plotted as function of the energy in Fig. 10. According to the formulas (1) and (2) we get

$$\sigma_{\text{obs}}/\sigma_{\text{theor}}=\xi_p\cdot\eta_n/(\eta_n)_{\text{theor}},$$

since for our theoretical calculations we have assumed that $\xi_p=1$. By taking the value and energy dependence of ξ_n equal to that of ξ_p, the ratio $\eta_n/(\eta_n)_{\text{theor}}$ is just about unity. It is not exactly unity because of the different energies of the neutrons and protons emerging from the compound state. Within the accuracy of the present calculations and measurements we are allowed to put

$$\sigma_{\text{obs}}/\sigma_{\text{theor}}=\xi_p.$$

The results show that with our choice of $r_0=1.3\times10^{-13}$ cm the sticking probabilities of protons range between 0.3 and 1.3. With the exception of reaction 5 they do not show a definite energy dependence. The low values for the reactions 1 and 2 are perhaps due to a wrong assumption

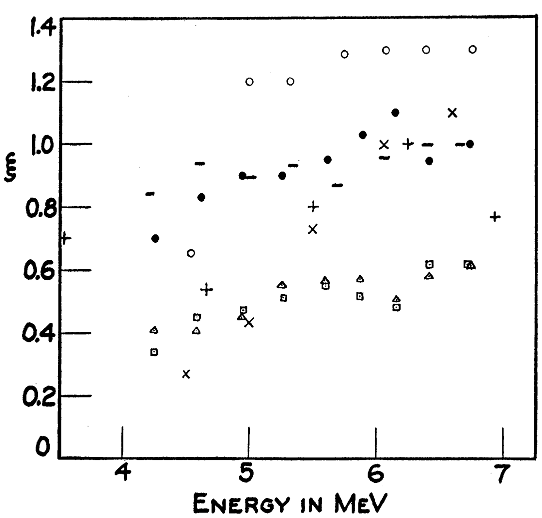

FIG. 10. Proton sticking probabilities for different (p,n)-reactions at different energies. The significance of the signs is explained in Fig. 9.

[17] G. E. Valley and R. L. McCreary, Phys. Rev. **56**, 863 (1939).

about the abundance of the Ni isotopes which are not very well known. The assumption of a larger radius r_0 would produce smaller values of ξ_p and would give rise to a definite tendency to increase with higher energies. This would be in disagreement with our knowledge about sticking probabilities, insofar as we are allowed to apply evidence taken from neutron experiments to proton reactions and to the extent that we may trust the theoretical reasons for choosing a sticking probability near unity for the energies in question.

These assumptions can be tested by determining the nuclear radius with the aid of other independent methods. The most direct method is the measurement of the cross section for an inelastic neutron-collision ((n,n)-reaction), which should be equal to πR^2 for neutron energies of several Mev or more. The existing measurements of this cross section by Graham and Seaborg[18] are not very conclusive. Their observed values give $r_0=1.60$ for C, 1.65 for Al, 1.7 for Zn, 1.5 for Sn and Sb and 1.35 for Pb in 10^{-13} cm. The values for C and Al[19] are in definite disagreement with the very accurate values obtained by Wigner[6] from the maximum energy of the β-decay leading to these elements.

Pollard, Schultz and Brubaker[20] have tried to determine the magnitude r_0 for Cl, Al and A by fitting observed excitation curves for α-induced reactions to the theoretical curves. They find values of 1.94×10^{-13} cm and higher. However, the expression used for the penetration probability is the part for $l=0$ only. The contribution of higher l's is by no means negligible and is even higher than the $l=0$ contribution for energies comparable to the barrier. Taking all l's into account their observed curves can be fitted with a value of $r_0=1.5\times10^{-13}$ cm. It should be added furthermore that the expressions derived by the W-K-B method are very poor approximations for light elements.

Further measurements of the cross section for inelastic neutron scattering together with more and better measurements of the excitation functions for (p,n) and (α,n) reactions would greatly clarify the proper assumptions for a valid theory.

We are much indebted to Dr. L. A. DuBridge and Dr. N. VanVoorhis for their continuous interest and for their invaluable help in our work and to Mr. A. B. White for several numerical computations.

[18] D. Graham and G. Seaborg, Phys. Rev. **53**, 795 (1938).

[19] The experiments on Al have been done by G. Kuerti and S. N. VanVoorhis, Phys. Rev. **56**, 614 (1939)) and suggest a value of not more than $r_0=1.35\times10^{-13}$ cm.

[20] E. Pollard, N. Schultz and G. Brubaker, Phys. Rev. **53**, 351 (1938).

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The Ionization Loss of Energy in Gases and in Condensed Materials*

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It is shown that the loss of energy of a fast charged particle due to the ionization of the material through which it is passing is considerably affected by the density of the material. The effect is due to the alteration of the electric field of the passing particle by the electric polarization of the medium. A theory based on classical electrodynamics shows that by equal mass of material traversed, the loss is larger in a rarefied substance than in a condensed one. The application of these results to cosmic radiation problems is discussed especially in view of the possible explanation on this basis of part of the difference in the absorption of mesotrons in air and in condensed materials that is usually interpreted as evidence for a spontaneous decay of the mesotron.

THE determination of the energy lost by a fast charged particle by ionization and excitation of the atoms through or near which it is passing has been the object of several theoretical investigations. The essential features of the phenomenon are explained as well known

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