$k \neq n \times 3$, cannot be explained by a regular lattice. In both cases the continuous scattering is weak. It follows therefore that most elements of the crystal mosaic have one or more translated layers and that some elements have many displaced layers. This is intermediate between dickite and cronstedite.

Pyrophyllite and one biotite mica sample differed from the other substances examined in showing some continuous scattering along $(h_a k_a l)$, $k_a = n \times 3$, curves. In both cases this was very weak compared with that observed along $(h_a k_a l)$, $k_a \neq n \times 3$ curves, on the same photographs. Continuous scattering on the pyrophyllite photographs further has a tendency to be restricted to the region between two normal reflections with a maximum of intensity midway between them. The intensity maximum is evidence of a tendency for the crystals to form a definite superlattice. No entirely satisfactory explanation has been found for the continuous scattering along $(h_a k_a l)$, $k_a = n \times 3$, curves for pyrophyllite, but the following might possibly hold. The parameter along the b axis (y) defining the relative positions of the layers might differ slightly from $nb_0/12$. Irregular sequence of layers in an element of the crystal mosaic thus would have some effect on the (hkl), $k = n \times 3$ reflections. An alternative explanation would be that the layers in pyrophyllite are somewhat distorted in a manner similar to that of muscovite but not sufficiently to prevent random orientations. Reflections from (hkl), $k = n \times 3$ and l odd, would normally appear though weak for a regular structure and would be broadened to the point of not being observable as irregularity was introduced. The actual photographs might be an intermediate stage in this process. Composition of the specimen examined eliminates any possibility that the phenomenon was caused by the nonhomogeneous distribution of some component.

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Excited States of the O^{16} Nucleus

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IN a recent Letter to the Editor, Oppenheimer and Schwinger[1] have discussed the excited states of the nucleus O^{16} which are formed subsequent to the bombardment of fluorine by protons. Fowler and Lauritsen[2] have shown that this bombardment gives rise, in addition to short and long range groups of α-particles, to monochromatic γ-rays of energy 6.3 ± 0.1 Mev and to the production of pairs with the energy of 5.9 ± 0.3 Mev. According to the interpretation of Oppenheimer and Schwinger, the pairs are produced by internal conversion from an excited state of O^{16} which has zero angular momentum and even parity to the ground state which is likewise a 0^+ state. The γ-rays, on the other hand, result from a transition from an excited 1^+ or possibly 2^- state to the ground state. It is rather essential for this explanation that there exist no intermediate excited levels of such a character that transitions to them from the upper states will be of sufficient intensity to compete actively with the pair production or with the 6.3-Mev γ-ray transitions. The object of the present note is to examine the levels of O^{16} as predicted by the α-particle model and to see to what extent they may satisfy the requirements postulated by Oppenheimer and Schwinger. It is clear that the independent particle model will not readily predict a low excited 0^+ state since the first steps of excitation will probably consist in raising a particle from a P state to a $2S$ state, thus producing levels of the type 2^-, 1^- and 0^-.

The O^{16} nucleus will be thought of as a close packed grouping of four α-particles whose equilibrium configuration is that of a regular

[1] J. R. Oppenheimer and J. S. Schwinger, Phys. Rev. **56**, 1066 (1939).
[2] W. A. Fowler and C. C. Lauritsen, Phys. Rev. **56**, 840 (1939).

tetrahedron. An estimate of the vibration amplitudes indicates that the α-particle model should have some validity as a means of describing O^{16} and that in first approximation the energies and wave functions may be separated into vibrational and rotational parts. As has been pointed out by Wheeler[3] and by Hafstad and Teller,[4] the requirement that the α-particles must satisfy Bose statistics greatly reduces the number of allowed states. The energy of a rotating vibrating tetrahedral model may be written in the following form:

$$W=(j^2+j)\hbar^2/2A+\omega_1\hbar(n_1+\tfrac{1}{2})+\omega_2\hbar(n_2+1)+\omega_3\hbar(n_3+\tfrac{3}{2})+I_{n_{3j}}\pm\epsilon_i.$$

A is here the moment of inertia, and ω_1, ω_2 and ω_3 are three normal frequencies expressed in circular units ($\omega_i=2\pi\nu_i$). As is well known, these frequencies are single, double and triple, respectively. $I_{n_{3j}}$ represents an interaction between vibration and rotation which arises from the fact that the motion associated with the frequency ω_3 possesses an internal angular momentum $\zeta\hbar$. The final term $\pm\epsilon_i$ denotes the two energy states associated with the tunneling process by which a tetrahedron expressed in a right-hand coordinate system passes over into a tetrahedron in a left-hand system.

The states whose wave functions are invariant under an interchange of any two α-particles (Bose statistics) and which correspond to low vibrational and rotational quantum numbers are listed in Table I. The first four columns give the quantum numbers, the fifth column the parity, and the last column the energy above the normal state. The term $9\hbar^2/8A$ in the last column represents the interaction energy, which for this state is $-(j+1)\zeta\hbar^2/A+\zeta^2\hbar^2/2A$. It may be shown that $\zeta=-\frac{1}{2}$ when $n_3=1$.

The normal frequencies ω_1, ω_2 and ω_3 may be found by the usual methods of normal coordinates. There are six internal coordinates, and these will be chosen as the six displacements $q_1\cdots q_6$ along the edges of the tetrahedron. Let q_1, q_2; q_3, q_4; and q_5, q_6 represent displacements along opposite edges. The potential energy is a function of the q_s and can be developed in a power series. Retaining no powers beyond the second, we have

$$V=\tfrac{1}{2}[a(q_1^2+q_2^2+q_3^2+q_4^2+q_5^2+q_6^2)+2b(q_1q_3+q_1q_4+q_1q_5+q_1q_6+q_2q_3+q_2q_4+q_2q_5+q_2q_6+q_3q_5+q_3q_6+q_4q_5+q_4q_6)+2c(q_1q_2+q_3q_4+q_5q_6)].$$

The three frequencies are readily found as a function of the three constants a, b and c together with M, the mass of an α-particle.

$$\omega_1^2=(4a+16b+4c)/M$$
$$\omega_2^2=(a-2b+c)/M \qquad \omega_3^2=(2a-2c)/M.$$

Although it appears that the three normal frequencies may be arbitrarily assigned, the nature of the nuclear forces would lead us to believe that the constant a must be large compared with either b or c and we would reject as physically unlikely any solution for which this condition is not satisfied. The quantity ϵ_i may be estimated from the theory of the two-minima problem and is given in terms of the frequency ω_3 through whose motion the tunneling may be effected. For a reasonable form of potential function we may estimate $2\epsilon_0/\omega_3\hbar\cong 3\times10^{-3}$ and $\epsilon_1\cong 25\epsilon_0$.

It is clear that the α-particle model, in contrast to the independent particle model, predicts in a natural fashion that one of the low excited states of O^{16} is a 0^+ state which therefore upon transition to the ground state may give rise to the production of pairs. The 6.3-Mev γ-rays, according to Oppenheimer and Schwinger, result from a transition from an excited state with a different parity-angular momentum. From the observed high intensity of the γ-rays one would expect that the captured proton is in an S state and produces a 1^+ excited state of Ne^{20} which subsequently emits an S α-particle to form a 1^+ level of O^{16}. Unfortunately the α-particle model

TABLE I. *States whose wave functions are invariant under an interchange of any two α-particles.*

n_1	n_2	n_3	j	p	W
0	0	0	0	+	0
0	0	0	3	−	$6\hbar^2/A$
0	0	0	4	+	$10\hbar^2/A$
1	0	0	0	+	$\omega_1\hbar$
0	1	0	2	+	$3\hbar^2/A+\omega_2\hbar$
0	1	0	2	−	$3\hbar^2/A+\omega_2\hbar+2\epsilon_0$
0	0	1	1	−	$\hbar^2/A+\omega_3\hbar+9\hbar^2/8A-\epsilon_1+\epsilon_0$

[3] J. A. Wheeler, Phys. Rev. **52**, 1083 (1937).
[4] L. R. Hafstad and E. Teller, Phys. Rev. **54**, 681 (1938).

furnishes no low excited 1^+ state. One explanation which may be made is that the emitted α-particle is in a P state and thus produces a 0^- or 2^- level of O^{16}. This process while less probable than the emission of an S state α-particle may still be sufficiently intense to explain the strength of the γ-rays. Our model, as has been seen, predicts the existence of an excited 2^- level.

A more serious difficulty is the position of the rotational level 3^- which depends inversely upon the square of the nuclear radius. We are here concerned with a mass distribution radius and it appears more logical to adopt the estimate of the nuclear dimensions based upon the Coulomb energy differences[5] between such nuclei as $N^{13}-C^{13}$ and $F^{17}-O^{17}$, namely, $R \leqq 3.1\times 10^{-13}$ cm rather than the radius 3.7×10^{-13} which is derived from the α-particle penetrability of the natural radioactive nuclei. This leads to an energy value for the 3^- level of 4.1 Mev. The existence of a level at this position presents certain difficulties which however are perhaps not insurmountable. In the first place transitions might occur to it from the excited 0^+ level. Since these transitions take place by means of electric octopole radiation and since the energy difference is only 1.8 Mev, their probability would be small and hence would probably not complete actively with the pair production. On the other hand, transitions from the excited 2^- level to this 3^- level might occur with the same order of intensity as the main 6.3-Mev transitions to the ground 0^+ state since the former consist of magnetic dipole and electric quadrupole radiation and the latter of magnetic quadrupole radiation only. Thus the fact that no γ-rays of 4.1 Mev have been observed argues against this choice for the position of the 3^- level.

These difficulties would be almost completely obviated if the 3^- level lay somewhat higher, say at 5.1 Mev since in this case the energy differences between it and the 2^- and the excited 0^+ levels would be small enough to preclude frequent γ-ray transitions. Such a value for the 3^- level leads to a mass distribution radius for O^{16} of 2.8×10^{-13} cm which is lower than we should expect.

The remainder of the calculation follows naturally. If the 2^- level lies at 6.3 Mev and the 0^+ level at 5.9 Mev, we have a condition upon the potential constants which is satisfied by letting $b=-a/14$ and $c=0$. The following values for the excited levels are then readily obtained, $3^-=5.1$, $0^+=5.9$, $2^+=6.29$, $2^-=6.3$, $1^-=6.6$ and $4^+=8.5$ Mev. There should also be two excited levels 0^+ lying at about 7.5 and 9.9 Mev due to the first overtones of ω_2 and ω_3, respectively. These are more uncertain however since the α-particle model will be less valid for the overtones of normal frequencies because these imply larger vibrational amplitudes. The numerical values which we have just calculated would not be greatly altered if the 3^- level were considered to lie at 4.1 rather than at 5.1 Mev.

If the 3^- rotational state of O^{16} possesses the energy 5.1 Mev, we may estimate the lowest rotational levels of C^{12} and Be^8. We shall employ the arguments of Wheeler[3] concerning the appropriate relative mass distributions of these nuclei although these cannot be regarded as very certain. The lowest level of C^{12} would be a 2^+ state and would lie at 4.3 Mev. Experimentally the lowest level lies at 4.3 Mev. In the case of Be^8 the lowest level, also a 2^+ state, would be predicted from the O^{16} datum to be a 6.5 Mev. Actually it is observed to lie at 2.9 Mev. This circumstance would argue for a much looser grouping of the α-particles in Be^8 than in O^{16} and might be brought in harmony with the fact that the binding energy of the α-particles is practically zero for this nucleus.*

I would like to extend my most sincere thanks to Professor J. R. Oppenheimer and to Dr. L. I. Schiff for their encouragement and helpful criticism which have made this note possible.

[5] The Coulomb energy for this series is given by $0.6z(z-1)/A^{1/3}$ Mev. For a surface distribution of charge, $A=16$, this gives $R=3\times 10^{-13}$; for a uniform volume distribution, which is surely the better approximation, $R=3.6\times 10^{-13}$. Even taking into account the somewhat tighter binding in O^{16}, $R=3.1\times 10^{-13}$ seems to us a *minimum* permissible value.

* *Note added in proof.*—The assumption of the separability of the rotational and the vibrational energies is considerably less well satisfied for the α-particle model than for molecular models. An estimate of the vibration-rotation interaction terms for the low-lying states of O^{16} shows that they are of the order of 0.5 to 1 Mev; some contributions being positive and some negative and depending upon the details of the potential function. This means that the numerical values which we have obtained must not be taken too literally. In particular, the 3^- level may lie as high as 5.1 Mev without the nuclear radius being as small as 2.8×10^{-13} cm.