A Determination of the Masses and Velocities of Three Radium B Beta-Particles

The Relativistic Mass of the Electron

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An accurate determination of the values of m/e and v of three Ra B β-particles was made by means of an electrostatic spectrograph. It is shown that these particles obey the Lorentz model for electrons rather than the Abraham model, for velocities as high as 0.75 of the velocity of light. The uncertainty in the experiment is less than one-tenth the magnitude of the difference between the two models.

I. INTRODUCTION

THERE have been many experiments performed[1–9] in order to determine the masses and velocities of β-rays. All of these experiments were essentially the same. The particles were deflected in a magnetic field, the deflection being a measure of (mv) of the particle, where m is the mass and v is the velocity. They were also deflected in the electric field between two flat plates, the deflection being a measure of (mv^2) of the particles. Thus since (mv) and (mv^2) were known, m and v could be obtained. Usually the magnetic and electric fields were applied simultaneously. In some experiments, such as Kaufman's,[1] the electric and magnetic fields were parallel; other experiments, such as Bucherer's,[2] consisted in balancing the magnetic and electric deflections against each other. But in all of these experiments, a flat parallel plate condenser was used to give the electric field in which the particles were deflected; hence no focusing of the β-particles was obtained. But in spite of this lack of resolving power these experiments yielded results that indicated that the electronic mass varied in such a way as to conform more nearly to the Lorentz expression[10] than to the Abraham.[10] Since most of the uncertainties in the above experiments were obscured by the large quantity of data taken, the results were accepted as adequate proof of the validity of the Lorentz expression until 1938, when Zahn and Spees[11] reopened the question with a critical survey of all the previous work. They showed that the resolving power of these experiments had been of the same order of magnitude as the effect to be measured, and they suggested that a new and more precise attempt be made to distinguish between the two electronic models. It was this statement that caused the present experiment to be undertaken.

The recent high precision determinations[12–14] of the absolute value of the $H\rho$ of the most intense line of the Ra B β-particle spectrum (which thus allows the highly precise absolute specification of the $H\rho$ values of the remaining lines in the Ra(B+C) β-particle spectrum from the precise relative values determined by Ellis and Skinner[15] and by Ellis[16,17]) give us highly accurate data on the (mv) of these β-particle lines. Hence, in view of the recent advances in the use of radial electric fields for the focusing of

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charged particles,[18–20] it was considered that an experiment to determine the variation of mass with velocity of β-particles could be performed in which the chief source of uncertainty in the previous experiments, i.e., their lack of focusing, could be eliminated. Thus it was hoped that it would be possible to obtain, by a practically new and independent method, precise absolute values for the masses and velocities of three of the discrete spectra β-particles of Ra(B+C) with sufficient accuracy to allow a conclusive decision as to which model the β-particles more nearly conform.

II. Theory

A β-particle of mass m, velocity v, and charge e, moving in a uniform magnetic field H, perpendicularly to H, describes a circular path of radius ρ, satisfying the equation

$$H\rho = mv/e. \tag{1}$$

$H\rho$ is observable and thus yields the product mv/e.

If this particle moves in a radial (in two dimensions), electric field X (see Fig. 1), it can describe a circular path of radius R given by

$$XR = mv^2/e. \tag{2}$$

XR is observable and thus yields the product mv^2/e.

Therefore we get v and m/e in terms of $H\rho$ and XR by the equations

$$v = (XR)/(H\rho); \tag{3}$$

$$m/e = (H\rho)^2/(XR). \tag{4}$$

Hence the determination of XR and $H\rho$ for a particle is sufficient to enable us to determine the v and m/e of the particle.

For future reference in the analysis of the data obtained in this experiment, we record the well-known theoretical formulae for the transverse mass of the β-particle as a function of the velocity of the particle. If the particle follows the Abraham model, its transverse mass, m_A, is

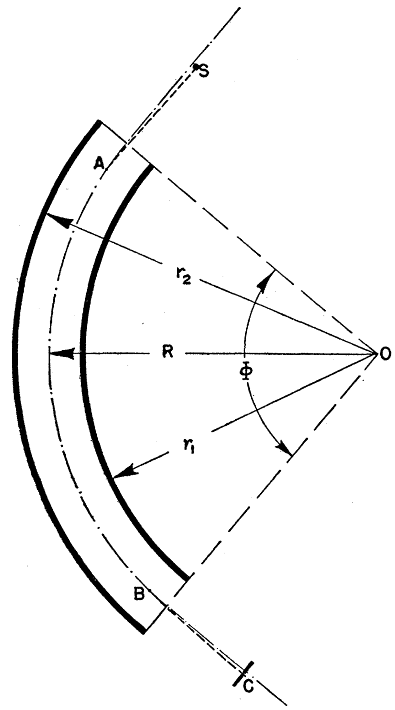

FIG. 1. S and C are source and counter, respectively.

given by

$$\frac{m_A}{e} = \frac{3}{8}\frac{m_0/e}{\beta^3}\left[(1+\beta)^2 \ln\frac{1+\beta}{1-\beta} - 2\beta\right], \tag{5}$$

where m_0 is the rest mass of the electron and $\beta = v/c$. If the particle follows the Lorentz model, its transverse mass, m_L, is given by

$$m_L/e = (m_0/e)/(1-\beta^2)^{\frac{1}{2}}. \tag{6}$$

III. Experiment

In order to determine the XR of the three β-particles, an electrostatic analyzer (Fig. 1) was used. It was constructed of segments of two concentric cylinders so that the mean radius, R, was 16.05 ± 0.007 cm, the height, 7.16 cm, and the angle subtended, $89° 51' \pm 05'$. If P is the potential difference between the plates of the electrostatic spectrograph and if r_1 and r_2 are the radii of the inner and outer surfaces of these plates, then

$$X = P/(R \cdot \ln [r_2/r_1]). \tag{7}$$

In this instrument, $r_2 - r_1 = 0.5989 \pm 0.0032$ cm.

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Thus Eq. (7) becomes simply

$$XR = 26.79P, \tag{8}$$

which (aside from possible errors which might be present in P) is certainly accurate to within 0.5 percent.

The potential difference P was measured by a potentiometer arrangement which measured the potential drop across a 958.3-ohm resistance, caused by the current flowing through a 48.74 ± 0.04-megohm resistance (a bank of Taylor self-shielded x-ray resistances, manufactured by Shallcross) across the output of the high potential applied to the plates of the spectrograph. Thus the only current drain was about 0.25 milliampere through this 48.74-megohm resistance. The high potential was obtained thus: The current from a 500-cycle generator was converted to high potential by a 3-kv-amp. transformer rated at 100,000 volts output at 110 volts input. The output voltage of this transformer was controlled by a finely adjustable resistance network in the primary circuit. Two kenotrons rated at 75,000 volts each were used in the rectifying circuit. The filter network was a pair of 0.2-microfarad condensers rated at 60,000 volts d.c. The electrical centers of the transformer secondary, of the filter condensers, and of the voltage measuring resistor were connected to ground.

The spectrograph plates were mounted by fastening them to porcelain beehive insulators about $2\frac{1}{2}$ inches high. These insulators were screwed to a brass base plate. Insulators of this same sort were also put on top of the plates and brass cross-braces were screwed to the top of these insulators. Hence, the path that any leakage current must take is about seven inches across porcelain. Another advantage of this mounting is that the insulators were not in the strong field between the plates. The mounting was quite strong, no difference in the separation being detectable when pressure was exerted in the same manner and of the same magnitude as that existing when a large potential difference is applied to the plates. This type of mounting was found to be superior to Bakelite spacers, because the Bakelite has a tendency to break down in vacuum even in the absence of strong fields.

The spectrograph was enclosed in an iron box, evacuated by a three-inch oil diffusion pump, backed by a Cenco Megavac pump.

The source of β-particles was a fine platinum wire, 0.75 cm in length, coated with radium active-deposit. It was inserted into the spectrograph container by a device which allowed the wire to be put accurately in place without losing the vacuum in the box. A stopcock was ground to allow a number of turns of fine wire to be wrapped around it. The source was inserted in a groove in a short, thin metal rod. This rod was lowered by means of the wire into an air lock which could be evacuated between two stopcocks. The lower stopcock was then opened and the rod was lowered into the iron box and finally into a hole in a block fastened to the base plate of the spectrograph. This hole held the rod tightly and enabled us to place the source in the same and proper place each time. Two windows in the top of the iron box enabled the source to be observed while it was being put in place.

The particles after being deflected by the plates were incident on a slit one mm wide in front of a Geiger-Müller counter with a glass bubble window 5/10,000 inch thick. The voltage at which the largest number of counts was recorded was taken to be the voltage necessary to deflect the particles along the path of mean radius R.

Two slight corrections had to be made to allow for the lack of ideal conditions. One allowed for the stray fields which the particles are in after leaving the plates. This correction was made by calculating how much the source must be moved toward or away from the center in order to cause the particles emitted from it to come into the spectrograph at A along the path of radius R (Fig. 1). Then the detector likewise must be moved this same amount so that particles coming along the R path at B will fall upon it. This correction assumes a perfectly aligned set of plates. Actually we do not have constant spacing of the plates, the maximum deviation from the mean being 0.0040 cm. Hence another correction must be made to allow for this. In order to make this correction, the separation was measured at seven points along the spectrograph, corresponding to $\phi = 0°$, $15°$,

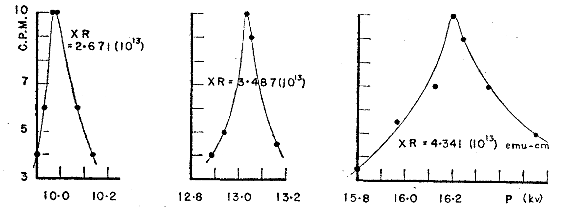

FIG. 2. The ordinates are proportional to the counting rates in particles per minute; the maxima are all normalized to ten.

30°, 45°, 60°, 75°, 90°, by using a Brown and Sharpe dial gauge calibrated in ten-thousandths of an inch. The mean separation was taken and the deviation from the mean plotted as a function of ϕ. A function was found which closely approximated this, and this function was used in the correction to be added to the field expression in the differential equation of motion of the particle between the plates of the spectrograph. The equation was solved subject to the proper boundary conditions, and the spot at which the detector should be placed in order to detect the particles which have taken the R path was found. Both these corrections turned out to be small, of the order of magnitude of one millimeter.

IV. Experimental Results

Five independent runs were made for each β-particle line, and the voltages at which the maximum numbers of counts were found were noted. In practice, very sharp maxima were found. Fig. 2 shows graphs giving for each β-particle line typical data for counting rate *versus* plate voltage. This voltage was almost identical for a given line for all five runs, the deviations found being far below the estimated error. The data are given in Table I, along with the v and m/m_0 values calculated from it by Eqs. (3) and (4), using for e/m_0 the value 1.7591(10⁷) e.m.u./g, given by Dunnington.[21]

[21] F. G. Dunnington, Rev. Mod. Phys. **11**, 65 (1939).

The m/m_0 values are also calculated from Eqs. (5) and (6) and tabulated as m_A/m_0 and m_L/m_0. The values of $H\rho$ are those previously measured by one of us.[13]

The results may be summarized by the graph in Fig. 3. It is obvious that the experimental points follow closely the curve given by Eq. (6), i.e., the Lorentz equation. The difference between the experimental values and the values computed on the Abraham theory is about ten times the maximum experimental uncertainty to be expected, as will be shown in the following paragraph.

The maximum error to be expected may be estimated thus. If ϵ_3 and ϵ_4 are the maximum errors which might be present in $H\rho$ and XR, respectively, then the maximum relative errors, ϵ_1 and ϵ_2, which can be present in v and m/e are obviously

$$\epsilon_1 = \epsilon_3 + \epsilon_4$$
$$\epsilon_2 = 2\epsilon_3 + \epsilon_4.$$

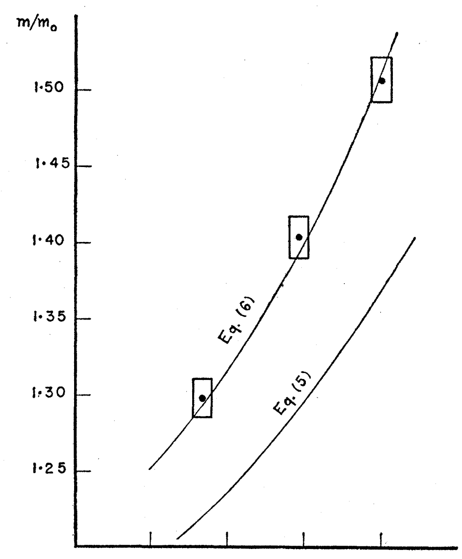

FIG. 3. Curve of m/m_0 *vs.* β for the Lorentz and Abraham equations. Boxes indicate the maximum error to be expected in the measurements.

TABLE I. *Data on β-particles.*

Line	$H\rho$ gauss·cm	P volts	XR e.m.u.·cm (10^{-13})	v cm/sec. (10^{-10})	β	m/m_0 (obs.)	m_L/m_0 (calc.)	m_A/m_0 (calc.)
1	1406.0	9970	2.671	1.8998	0.6337	1.298	1.293	1.220
2	1671.1	13017	3.487	2.0868	0.6961	1.404	1.393	1.290
3	1931.5	16200	4.341	2.2470	0.7496	1.507	1.511	1.369

Now the maximum uncertainties present in the $H\rho$'s are no more than 1/3000, or $\epsilon_3 = 0.03$ percent, and the maximum uncertainty present in XR is no more than 0.8/100, or $\epsilon_4 = 0.8$ percent. Thus we may say that v is good to less than 0.9 percent and m/e is accurate to well within 1.0 percent. These are the *maximum* relative errors which can be present (aside from any undetected systematic errors) in the values of v and m/e got in this experiment.

Thus the evidence seems to point conclusively to the fact that the RaB β-particles conform more nearly to the expression derived on the Lorentz theory than to the expression derived on the Abraham theory.

Finally, the writers wish to express their sincere appreciation for the valuable aid given them by Dr. H. A. Wilson throughout the experiment. They wish to thank Mrs. O. S. Moilliet for her help in taking the data.

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Relativistic Magnetic Moment of a Charged Particle

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Formulas are derived for the magnetic moment of a particle moving rapidly in a central field of force. Possible nuclear applications, particularly to the problem of the deuteron, are discussed. In view of the greatly increased accuracy in the measurement of magnetic moments, the relativity effect appears to be of measurable magnitude.

THE fact that the Zeeman splitting of the hydrogen lines is given correctly by the Landé formula, even when the relativistic wave equation is used in the calculation, was demonstrated by Dirac.[1] Since his interest was confined to the slowly moving electron in the hydrogen atom, he neglected terms of order v^2/c^2. Breit[2] has given a formula for the magnetic moment of an electron in a heavy atom. At present there is renewed interest in the Zeeman effect problem because of its intimate relation with nuclear magnetic moments. The precision which has recently been achieved in the measurements of the latter, chiefly by the ingenious magnetic-resonance method of Rabi, makes it appear worth while to inquire how the magnetic moment of a charged particle depends in detail on its velocity. The basis of the computation will be Dirac's equation.

In nonrelativistic theory there are two equivalent ways of calculating the magnetic moment of a particle, both giving the same result. One is to determine the energy change of the particle in a weak field and subsequently to compute $\partial E/\partial H$. The other is to calculate the mean value of the operator $(e/2mc)(L_z+2S_z)$. Relativistically, the two procedures give different answers, and the second is probably not justified. Nevertheless it will be discussed briefly later. We first calculate the magnetic energy of a particle moving in a central field of force.

The magnetic term in the Dirac Hamiltonian is $-e\boldsymbol{\alpha}\cdot\mathbf{A}$, $\boldsymbol{\alpha}$ being the operator for $\boldsymbol{v}/c$ and $\mathbf{A}$ the vector potential. When the uniform field H is chosen along z this perturbation term takes the form

$$V = \tfrac{1}{2}eH(\alpha_x y - \alpha_y x).$$

Written as a matrix[3] it becomes

$$V = ir\sin\theta \begin{pmatrix} 0 & 0 & 0 & e^{-i\varphi} \\ 0 & 0 & -e^{i\varphi} & 0 \\ 0 & e^{-i\varphi} & 0 & 0 \\ -e^{i\varphi} & 0 & 0 & 0 \end{pmatrix}.$$

We must calculate the diagonal elements of this operator for the two states $j=l+\frac{1}{2}$ and $j=l-\frac{1}{2}$. If the components of a ψ-function are $u_1\cdots u_4$,

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