

# The Meson Theory of Nuclear Forces

## I. General Theory\*

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In this paper, the meson theory of nuclear forces is presented in a simplified way. As in Yukawa's first paper, the forces between two nuclear particles are derived directly from the field equations and the Hamiltonian of the meson field (§2,3), without quantization of the field. The charge dependence of the forces is discussed (§4) and it is found that only two assumptions are in agreement with experimental facts, notably the equality of the forces between two like and two unlike nuclear particles in the singlet state. These assumptions are either (1) that nuclear particles interact only with neutral mesons (neutral theory) or (2) that they interact equally strongly with neutral, positive and negative mesons (symmetrical theory). It is

then shown (§5) that the part of the force which does not depend on the spin of the nuclear particle does not fulfill any useful function in the theory. Accordingly, the hypothesis is made that this part is absent so that there is only a spin-dependent interaction (single force hypothesis). Finally (§6), it is pointed out that the interaction must be cut off at small distances in order to obtain finite eigenvalues for the deuteron. Such a cutting off is to be expected from the general theory, particularly because of the possibility of the simultaneous emission of two or more mesons. The cut-off is to be expected at a distance  $r_0$  of about one-third the range of the nuclear forces.

### §1. INTRODUCTION

YUKAWA<sup>1</sup> first pointed out that nuclear forces can be explained by assuming that particles of mass about 200 times the electron mass (mesons) exist and can be emitted and absorbed by nuclear particles (neutrons and protons). With such an assumption a force between nuclear particles of the right range ( $\sim 2 \times 10^{-13}$  cm) and the right shape (rapid decrease at large distances) was obtained. If the mesons were assumed to be charged (positive or negative) the resulting force between nuclear particles turned out to be of the "exchange" type which had been found successful in the interpretation of empirical facts in nuclear physics. The mesons must obey Bose statistics because they are emitted in the transformation of a neutron into a proton (or vice versa) both of which obey Fermi statistics; their spin could be either zero or one to be reconcilable with their emission in the neutron-proton transformation; zero was chosen by Yukawa for simplicity.

It was suggested by Yukawa that the mesons are intermediate stages in the  $\beta$ -decay. A positive

$\beta$ -decay, e.g., would follow the scheme:

Proton  $\rightarrow$  Neutron + Positive Meson (virtual state)  $\rightarrow$  Neutron + Positive Electron + Neutrino.

In the intermediate state energy is not conserved, in agreement with other dispersion processes. Because of the comparatively high mass of the meson the shape of the  $\beta$ -spectrum is practically the same as in the Fermi theory.

Theories similar to Yukawa's were suggested almost simultaneously by Wentzel<sup>2</sup> and by Stückelberg.<sup>3</sup>

A new impulse was given to these theories by the discovery in cosmic radiation of a particle of mass intermediate between electron and proton. Evidence was obtained by Anderson and Neddermeyer<sup>4</sup> for the occurrence of two kinds of particles in cosmic rays characterized by different losses of energy, *viz.* (1) the shower particles which behaved in every respect as required by theory for electrons, and (2) the particles occurring singly which apparently lost energy only by ionization. These particles of the second type could therefore certainly not be electrons. Anderson and Neddermeyer also showed that

\* A second part, on the theory of the deuteron, is in course of publication in *The Physical Review*. Section numbers from §7 on, equation numbers from (36) on, and references from 35 on refer to that second part.

<sup>1</sup>H. Yukawa, Proc. Phys. Math. Soc. Japan **17**, 48 (1935); H. Yukawa and S. Sakata, *ibid.* **19**, 1084 (1938).

<sup>2</sup>G. Wentzel, Zeits. f. Physik **104**, 34 (1936); **105**, 738 (1937).

<sup>3</sup>E. C. G. Stückelberg, Phys. Rev. **52**, 41 (1937); Nature **137**, 1032 (1936).

<sup>4</sup>S. H. Neddermeyer and C. D. Anderson, Phys. Rev. **51**, 884 (1937).

the particles could not be protons because their ionization was too small. This conclusion was confirmed by Street and Stevenson<sup>5</sup> who showed that single cosmic-ray particles of fairly low momentum ( $H\rho \sim 10^6$ ) could penetrate large thicknesses of lead which could not possibly be penetrated by protons of the given momentum (nor by electrons).

Since its discovery, the medium heavy particle has been observed directly in the cloud chamber by several authors and its mass  $\mu$  has been determined from the curvature and the ionization produced,<sup>6</sup> from curvature and energy loss in solid plates,<sup>7</sup> or from curvature and range.<sup>8</sup> The most reliable measurements give

$$\mu = 150 - 220 \text{ electron masses.} \quad (1)$$

It was natural to identify these cosmic-ray particles with the particles in Yukawa's theory of nuclear forces. Yukawa's theory was therefore more closely investigated by several authors. It was found that the theory in its original form (charged mesons of zero spin) gave the wrong sign for the interaction in the deuteron, i.e., repulsion instead of attraction in the  $^3S$  state.<sup>9</sup> This could be remedied by assuming spin "one" corresponding to a vector wave function of the meson. This vector theory was developed simultaneously by Yukawa, Sakata *et al.*,<sup>10</sup> by Heitler, Froehlich and Kemmer<sup>11,12</sup> and by Bhabha.<sup>13</sup>

With a vector wave function for the mesons two distinct forces between two nuclear particles are obtained of which one ( $U$ ) does not and one ( $V$ ) does depend on the relative spin directions

of the two interacting nuclear particles. If the interaction is due only to charged mesons the forces are of the exchange type. The first force ( $U$ ) is simply a Heisenberg force and is attractive in the  $^3S$  (ground) state of the deuteron, repulsive in the  $^1S$  state. The second force,  $V$ , differs from the first by a factor  $\sigma_1 \cdot \sigma_2$  where  $\sigma_1$  and  $\sigma_2$  are the spin operators of the two interacting nuclear particles; it is attractive in both the  $^1S$  and  $^3S$  state of the deuteron (like a Majorana force) and three times as strong in  $^1S$  as in  $^3S$ . By choosing a suitable linear combination we can obtain the empirical positions of the  $^1S$  and  $^3S$  state on this theory.

Any theory involving charged mesons only, will give no force between like nuclear particles (two protons or two neutrons) in first approximation (i.e., with emission and absorption of one meson). In the second approximation there will be a strong repulsion at very small distances.<sup>11</sup> This is in contradiction with experiments on proton-proton scattering<sup>14</sup> which show<sup>15</sup> that the force between two protons is strongly attractive and very nearly equal to that between neutron and proton in the singlet state. A force between two like particles in the first approximation can only be obtained with neutral mesons. While such particles have not yet been observed in cosmic rays it seems not unreasonable to assume their existence from reasons of symmetry. Kemmer<sup>16</sup> has developed a theory of nuclear forces in which neutral and charged mesons occur in a symmetrical way and which explains in a natural way the equality of the forces between like and unlike nuclear particles. An alternative way of explaining this equality is to assume interaction with neutral mesons only; then the charge of the nuclear particle (i.e., whether it is a neutron or a proton) becomes entirely irrelevant and the equality of forces follows immediately. This alternative will be discussed in the present paper.

In the papers quoted the method of second

<sup>5</sup> J. C. Street and E. C. Stevenson, *Phys. Rev.* **51**, 1005 (1937).

<sup>6</sup> J. C. Street and E. C. Stevenson, *Phys. Rev.* **51**, 1005 (1937); E. J. Williams and E. Pickup, *Nature* **141**, 634 (1938); P. Ehrenfest, *Comptes rendus* **206**, 428 (1938); D. R. Corson and R. B. Brode, *Phys. Rev.* **53**, 773 (1938).

<sup>7</sup> S. H. Neddermeyer and C. D. Anderson, *Rev. Mod. Phys.* **11**, 191 (1939); J. G. Wilson, quoted by P. M. S. Blackett and B. Rossi, *Rev. Mod. Phys.* **11**, 278 (1939).

<sup>8</sup> A. J. Ruhlig and H. R. Crane, *Phys. Rev.* **53**, 266 (1938); H. Maier-Leibnitz, *Zeits. f. Physik* **112**, 569 (1938); Y. Nishina, M. Takeuchi and T. Ichimiya, *Phys. Rev.* **55**, 585 (1939).

<sup>9</sup> W. E. Lamb and L. I. Schiff, *Phys. Rev.* **53**, 651 (1938).

<sup>10</sup> H. Yukawa, S. Sakata and M. Taketani, *Proc. Phys. Math. Soc. Japan* **20**, 319 (1938); H. Yukawa, S. Sakata, M. Kobayasi and M. Taketani, *ibid.* **20**, 720 (1938).

<sup>11</sup> H. Froehlich, W. Heitler and N. Kemmer, *Proc. Roy. Soc.* **166**, 154 (1938).

<sup>12</sup> N. Kemmer, *Proc. Roy. Soc.* **166**, 127 (1938).

<sup>13</sup> H. J. Bhabha, *Nature* **141**, 117 (1938); *Proc. Roy. Soc.* **166**, 501 (1938).

<sup>14</sup> M. A. Tuve, N. P. Heydenburg and L. R. Hafstad, *Phys. Rev.* **50**, 806 (1936); **51**, 1023 (1937); **53**, 239 (1938); **55**, 603 (1939). R. G. Herb, D. W. Kerst, D. B. Parkinson and G. J. Plain, *ibid.* **55**, 998 (1939).

<sup>15</sup> G. Breit, E. U. Condon and R. D. Present, *Phys. Rev.* **50**, 825 (1936); G. Breit, H. M. Thaxton and L. Eisenbud, *Phys. Rev.* **55**, 1018 (1939); G. Breit, L. E. Hoisington, S. S. Share and H. M. Thaxton, *Phys. Rev.* **55**, 1103 (1939).

<sup>16</sup> N. Kemmer, *Proc. Camb. Phil. Soc.* **34**, 354 (1938).

quantization was used to derive the force between nuclear particles. Stückelberg<sup>17</sup> and especially Møller and Rosenfeld<sup>18</sup> have shown that this is not necessary and that the interaction in the first approximation can be obtained from a purely classical treatment of the meson field by use of contact transformations. We shall here follow the same procedure without actually putting the contact transformation in evidence: This will lead to a derivation which is even simpler but not as rigorous and far-reaching as that of Møller and Rosenfeld. A similar derivation was given by Yukawa<sup>1</sup> in his first and fourth paper.

Considerable discussion has been devoted to the question of naming the new particle. Heavy quantum,<sup>1</sup> heavy electron,<sup>11</sup> barytron,<sup>19</sup> or barytron,<sup>20</sup> mesotron<sup>21</sup> and meson<sup>22</sup> have been used in the literature, besides the name yukon (after Yukawa) used in conversation. It seems to us that the only name definitely to be avoided is "heavy electron." If there is any truth in the current theories then the new particle differs from an electron as much as any particle can: It has a different mass, a different spin and different statistics. Even a proton could be more rightfully called "heavy electron" because it has at least the same spin and statistics. If we have to assume a neutral modification of the meson as well as charged ones it becomes even more awkward to use the name "heavy electron" for the charged modification: It would mean that very similar particles are called by different names (heavy electron and "neutretto") and very different particles by similar names (electron and heavy electron; neutrino, neutretto and neutron). The use of the name "heavy electron" is even dangerous because it leads easily to misconceptions.<sup>23</sup> As long as this name is avoided it is not important which name is actually chosen. However, it seems more characteristic for the particle that it has "medium weight" than that it is "heavier" than an electron. Linguistic reasons<sup>22</sup> then decide in favor of meson over mesotron.

## §2. GENERAL THEORY

In this section we shall derive the general expression for the interaction between nuclear

<sup>17</sup> E. C. G. Stückelberg, Phys. Rev. **54**, 889 (1938).

<sup>18</sup> C. Møller and L. Rosenfeld, to be published in Kgl. Dansk. Vidensk. Proc.

<sup>19</sup> L. W. Nordheim and G. Nordheim, Phys. Rev. **54**, 254 (1938).

<sup>20</sup> F. J. Belinfante, Nature **143**, 201 (1939).

<sup>21</sup> C. D. Anderson and S. H. Neddermeyer, Nature **142**, 878 (1938); R. A. Millikan, Phys. Rev. **55**, 105 (1939).

<sup>22</sup> H. J. Bhabha, Nature **143**, 276 (1939).

<sup>23</sup> G. E. M. Jauncey, Phys. Rev. **52**, 1261 (1937); **53**, 106, 197, 265, 669 (1938); S. H. Neddermeyer, Phys. Rev. **53**, 102 (1938).

particles. By "nuclear particle" we shall generally understand a neutron or a proton; a short common name for these two modifications of the nuclear particle would be highly desirable. Our results will in no way go beyond those previously obtained;<sup>1,11-13,16-18</sup> in the derivation we shall only attempt simplicity rather than rigor and completeness. The nuclear particle will be treated nonrelativistically;<sup>24</sup> the meson must, of course, be treated relativistically. The meson field is not quantized; therefore the interaction between nuclear particles is only obtained to the order  $g^2$  (for the significance of  $g$  see below) while terms of order  $g^4$ , etc. are neglected. To show the analogy with the electromagnetic field, all quantities describing the meson field are denoted by the same letters as the corresponding electromagnetic quantities, *viz.*  $\varphi$ ,  $\mathbf{A}$ ,  $\mathbf{E}$ ,  $\mathbf{H}$ . This will not give rise to confusion because we do not consider actual electromagnetic fields in this paper. In this section we shall only consider *neutral* mesons; theories involving charged mesons will be discussed in §4.

The meson field will be described by a four-vector potential  $\varphi$ ,  $\mathbf{A}$  which satisfies the usual condition

$$\operatorname{div} \mathbf{A} + \partial\varphi/c\partial t = 0 \quad (2)$$

and by a six-vector (antisymmetric tensor)  $\mathbf{E}$ ,  $\mathbf{H}$ . In the absence of nuclear particles  $\mathbf{E}$  and  $\mathbf{H}$  can be derived directly from the potentials:

$$\mathbf{E} = -\operatorname{grad} \varphi - \partial\mathbf{A}/c\partial t, \quad (3a)$$

$$\mathbf{H} = \operatorname{curl} \mathbf{A}. \quad (3b)$$

Likewise in the absence of nuclear particles, the "field intensities"  $\mathbf{E}$  and  $\mathbf{H}$  satisfy the relations

$$\operatorname{div} \mathbf{E} + \kappa^2\varphi = 0, \quad (4a)$$

$$\operatorname{curl} \mathbf{H} - \partial\mathbf{E}/c\partial t + \kappa^2\mathbf{A} = 0, \quad (4b)$$

where  $\kappa$  is a constant of the dimensions of a reciprocal length. (4a,b) correspond to one set of Maxwell equations modified by the  $\kappa^2$  term. The other set follows from (3a,b), *viz.*:

$$\operatorname{div} \mathbf{H} = 0, \quad (5a)$$

$$\operatorname{curl} \mathbf{E} + \partial\mathbf{H}/c\partial t = 0. \quad (5b)$$

<sup>24</sup> A short report of a relativistic treatment was given by E. Feenberg, Phys. Rev. **55**, 602 (1939).

If we consider the meson as a particle, the ten quantities  $\varphi$ ,  $\mathbf{A}$ ,  $\mathbf{E}$ ,  $\mathbf{H}$  are the components of its wave function and correspond to the four components of the Dirac wave function of the electron.  $\mathbf{E}$  and  $\mathbf{H}$  may be eliminated from (4), using (3) and (2); then we obtain

$$\nabla^2\varphi - \partial^2\varphi/c^2\partial t^2 - \kappa^2\varphi = 0, \quad (6a)$$

$$\nabla^2\mathbf{A} - \partial^2\mathbf{A}/c^2\partial t^2 - \kappa^2\mathbf{A} = 0, \quad (6b)$$

which are the Klein-Gordon equations of a free particle of mass

$$\mu = \hbar\kappa/c, \quad (7)$$

so that

$$\kappa = \mu c/\hbar \quad (7a)$$

is the reciprocal Compton wave-length of the meson (times  $2\pi$ ). If we identify the meson of our theory with the particle observed in cosmic rays, we have  $\mu \sim 180m$  where  $m$  is the electron mass. For the numerical calculations in the theory of the deuteron it will be convenient (cf. Eq. (39a)) to use

$$\mu = 177m, \quad (8)$$

which is within the limits of error of the experimental determination (1). Then we have

$$\kappa = 4.58 \times 10^{12} \text{ cm}^{-1}, \quad (8a)$$

$$1/\kappa = 2.18_s \times 10^{-13} \text{ cm}, \quad (8b)$$

$1/\kappa$  is the "range" of the nuclear forces (cf. Eq. (19)).

In the presence of a nuclear particle, (4) is replaced by

$$\text{div } \mathbf{E} + \kappa^2\varphi = 4\pi\rho, \quad (9a)$$

$$\text{curl } \mathbf{H} - \partial\mathbf{E}/c\partial t + \kappa^2\mathbf{A} = 4\pi\mathbf{j}/c, \quad (9b)$$

where

$$\rho = g\psi^*\psi, \quad (10a)$$

$$\mathbf{j}/c = g\psi^*\boldsymbol{\alpha}\psi. \quad (10b)$$

$\psi$  is the wave function of the nuclear particle,  $\boldsymbol{\alpha}$  the Dirac operator and  $g$  a constant.  $g$  has the same significance for the interaction of a heavy particle with the meson field as the charge  $e$  for the interaction of a charged particle with the electromagnetic field. Eqs. (9) are relativistically invariant, left and right sides being the components of a four vector.

Equations (3) are modified by the addition of an antisymmetrical tensor involving the nuclear

particle, *viz.*

$$\mathbf{E} + \text{grad } \varphi + \partial\mathbf{A}/c\partial t = 4\pi\mathbf{N}/\kappa, \quad (11a)$$

$$\mathbf{H} - \text{curl } \mathbf{A} = 4\pi\mathbf{M}/\kappa, \quad (11b)$$

$$\mathbf{N} = if\psi^*\boldsymbol{\beta}\boldsymbol{\alpha}\psi, \quad (12a)$$

$$\mathbf{M} = f\psi^*\boldsymbol{\beta}\boldsymbol{\sigma}\psi. \quad (12b)$$

The constant  $g$  is independent of  $f$ . The right-hand side of (11b) represents the spin density.  $\mathbf{M}$  and  $\rho$  are "static" quantities, i.e., they are large for nuclear particles at rest, while  $\mathbf{j}$  and  $\mathbf{N}$  vanish in this case and are generally of the order  $v/c$  compared with  $\rho$  and  $\mathbf{M}$  where  $v$  is the velocity of the nuclear particle. In a theory in which the nuclear particles are treated non-relativistically,  $\mathbf{j}$  and  $\mathbf{N}$  may be neglected.

Combining (9) and (11), we obtain (cf. 2)

$$\nabla^2\varphi - \frac{1}{c^2} \frac{\partial^2\varphi}{\partial t^2} - \kappa^2\varphi = -4\pi\rho + \frac{4\pi}{\kappa} \text{div } \mathbf{N}, \quad (13a)$$

$$\nabla^2\mathbf{A} - \frac{1}{c^2} \frac{\partial^2\mathbf{A}}{\partial t^2} - \kappa^2\mathbf{A} = -\frac{4\pi\mathbf{j}}{c} + \frac{4\pi}{\kappa} \left( \text{curl } \mathbf{M} - \frac{1}{c} \frac{\partial\mathbf{N}}{\partial t} \right). \quad (13b)$$

In a nonrelativistic approximation for the nuclear particles,  $\mathbf{N}$  and  $\mathbf{j}$  should be neglected and  $\rho$ ,  $\mathbf{M}$  considered independent of the time. Then the solution of (13) is

$$\varphi(\mathbf{r}) = \int d\tau' \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} \exp[-\kappa|\mathbf{r}-\mathbf{r}'|], \quad (14a)$$

$$\mathbf{A}(\mathbf{r}) = -\frac{1}{\kappa} \int d\tau' \frac{\text{curl } \mathbf{M}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} \times \exp[-\kappa|\mathbf{r}-\mathbf{r}'|]. \quad (14b)$$

The divergence condition (2) is fulfilled. If the wave function  $\psi$  of the nuclear particle is concentrated in a small space (small compared with  $r$ ) around the origin ("point charge") we may write

$$\varphi(\mathbf{r}) = (g/r)e^{-\kappa r}, \quad (15a)$$

$$\mathbf{A}(\mathbf{r}) = -\frac{f}{\kappa} \text{curl} \left( \mathbf{s} \frac{e^{-\kappa r}}{r} \right), \quad (15b)$$

where

$$\mathbf{s} = \int \psi^* \boldsymbol{\sigma} \psi d\tau' \approx \int \psi^* \boldsymbol{\sigma} \psi d\tau' \quad (16)$$

is the spin of the nuclear particle.

If a nuclear particle is subject to a meson field the potential energy is

$$E_{\text{pot}} = \int d\tau \left( \rho\varphi - \frac{1}{c} \mathbf{j} \cdot \mathbf{A} + \frac{1}{\kappa} \mathbf{M} \cdot \mathbf{H} - \frac{1}{\kappa} \mathbf{N} \cdot \mathbf{E} \right). \quad (17)$$

The general form of this potential energy follows from the field equations (9), (11) and from considerations of relativistic invariance. The sign of the first two terms is the same as in electrodynamics; in general, the sign can only be derived from the Hamiltonian of the field which will be discussed in the next section.

We shall now assume that the meson potential (15) is produced by a nuclear particle 1 located at the origin and acts on a particle 2 located at  $\mathbf{r}$ . Then we have, neglecting relativistic terms and using (11b):

$$E_{\text{pot}} = \frac{g_1 g_2}{r} e^{-\kappa r} - \frac{f_1 f_2}{\kappa^2} \mathbf{s}_2 \cdot \text{curl curl} \left( \frac{e^{-\kappa r}}{r} \mathbf{s}_1 \right) + \frac{4\pi}{\kappa^2} f_1 f_2 \mathbf{s}_1 \cdot \mathbf{s}_2 \delta(\mathbf{r}), \quad (18)$$

$\delta$  being Dirac's singular function. Evaluating the curl curl, we obtain:

$$\begin{aligned} -\text{curl curl} \left( \frac{e^{-\kappa r}}{r} \mathbf{s}_1 \right) &= -\text{grad div} \left( \frac{e^{-\kappa r}}{r} \mathbf{s}_1 \right) \\ &+ \mathbf{s}_1 \nabla^2 \left( \frac{e^{-\kappa r}}{r} \right) = -\text{grad} \left( \mathbf{s}_1 \cdot \text{grad} \frac{e^{-\kappa r}}{r} \right) \\ &+ \kappa^2 \frac{e^{-\kappa r}}{r} \mathbf{s}_1 - 4\pi \mathbf{s}_1 \delta(\mathbf{r}). \end{aligned} \quad (18a)$$

The last term of (18a) cancels the last term of (18). The first term can be transformed in a straightforward way. The final result is

$$E_{\text{pot}} = U + V, \quad (19)$$

$$U = g_1 g_2 e^{-\kappa r} / r, \quad (19a)$$

$$V = V_1 + V_2, \quad (19b)$$

$$V_1 = (2/3) f_1 f_2 \mathbf{s}_1 \cdot \mathbf{s}_2 e^{-\kappa r} / r, \quad (19c)$$

$$\begin{aligned} V_2 &= f_1 f_2 \frac{e^{-\kappa r}}{r} \left( \frac{1}{\kappa^2 r^2} + \frac{1}{\kappa r} + \frac{1}{3} \right) \\ &\times \left( -3 \frac{\mathbf{s}_1 \cdot \mathbf{r} \mathbf{s}_2 \cdot \mathbf{r}}{r^2} + \mathbf{s}_1 \cdot \mathbf{s}_2 \right). \end{aligned} \quad (19d)$$

The interaction  $U$  (19a) arises from the term  $\rho\varphi$  in (17); it does not depend on the spins of the nuclear particles and is repulsive for two nuclear particles of the same kind ( $g_1 = g_2$ ). Thus it behaves in every respect like the Coulomb force in electrodynamics from which it differs only by the exponential factor  $e^{-\kappa r}$ . This factor provides the rapid decrease of the force at large distances which is required by experiment.  $1/\kappa$  is the range of the force (cf. (8b)).

The interaction  $V$  is spin-dependent and comes from the term  $\mathbf{M} \cdot \mathbf{H}$  in (17).  $V$  can conveniently be split into two parts of which the first does not depend on the direction of the vector  $\mathbf{r}$  between the two particles (relative to the spin  $\mathbf{s}$ ) while the second does depend on it and gives zero when averaged over all directions of  $\mathbf{r}$ .  $V_2$  diverges as  $1/r^3$  at the origin while  $U$  and  $V_1$  behave as  $1/r$ .  $V_1$  and  $V_2$  are quantitatively related to each other, both depending on the value of  $f$ ;  $U$  is independent of  $V$  (depending on  $g$ ). For like particles ( $f_1 = f_2$ ),  $V_1$  is repulsive when the spins are parallel (triplet), attractive for opposite spins (singlet).

### §3. SIGN OF THE INTERACTION

In this section we shall discuss the Hamiltonian of the meson field mainly in order to determine the sign of the potential energy of nuclear particles in the field which had been assumed arbitrarily in (17). The Hamiltonian of the *electromagnetic* field interacting with matter is

$$\mathcal{H}_{\text{elec}} = \int d\tau \{ (E^2 + H^2) / 8\pi - \mathbf{A} \cdot \mathbf{j} / c \}. \quad (20)$$

The occurrence of the term  $\mathbf{A} \cdot \mathbf{j}$  without its relativistic partner  $\rho\varphi$  should be noted. With the help of the field equations, the Hamiltonian can be expressed in terms of the potentials rather than the field intensities; it then takes the form

$$\begin{aligned} \mathcal{H}_{\text{elec}} &= \int d\tau \left\{ \frac{1}{8\pi} \left[ \text{grad}^2 \mathbf{A} + \frac{1}{c^2} \left( \frac{\partial \mathbf{A}}{\partial t} \right)^2 \right. \right. \\ &\quad \left. \left. - \text{grad}^2 \varphi - \frac{1}{c^2} \left( \frac{\partial \varphi}{\partial t} \right)^2 \right] + \rho\varphi - \frac{\mathbf{A} \cdot \mathbf{j}}{c} \right\}. \end{aligned} \quad (21)$$

This form is more symmetrical than (20) and suitable for immediate derivation of the field

equations. It has essentially been used by Fermi in his formulation of quantum electrodynamics.<sup>25</sup> It has the disadvantage of being more complicated than (20) and of making it less evident that the Hamiltonian is positive definite (see below).

The Hamiltonian of the meson field expressed in the field intensities has the form

$$\mathcal{H} = \int d\tau \left\{ \frac{1}{8\pi} [\mathbf{E}^2 + \mathbf{H}^2 + \kappa^2(\mathbf{A}^2 + \varphi^2)] - \mathbf{A} \cdot \mathbf{j}/c - \mathbf{N} \cdot \mathbf{E}/\kappa \right\}. \quad (22)$$

With the help of the field equations this can be transformed into a form analogous to (21):

$$\mathcal{H} = \int d\tau \left\{ \frac{1}{8\pi} \left[ \text{grad}^2 \mathbf{A} + \frac{1}{c^2} \left( \frac{\partial \mathbf{A}}{\partial t} \right)^2 + \kappa^2 \mathbf{A}^2 - \text{grad}^2 \varphi - \frac{1}{c^2} \left( \frac{\partial \varphi}{\partial t} \right)^2 - \kappa^2 \varphi^2 \right] + \rho \varphi - \frac{\mathbf{A} \cdot \mathbf{j}}{c} + \frac{1}{\kappa} (\mathbf{A} \cdot \text{curl} \mathbf{M} - \varphi \text{div} \mathbf{N}) + \frac{2\pi}{\kappa^2} (\mathbf{M}^2 - \mathbf{N}^2) \right\}. \quad (23)$$

We introduce the momenta conjugate to  $\mathbf{A}$  and  $\varphi$  as follows:

$$\mathbf{P} = \frac{1}{4\pi c^2} \frac{\partial \mathbf{A}}{\partial t} - \frac{1}{\kappa c} \mathbf{N}, \quad (23a)$$

$$p = \frac{1}{4\pi c^2} \frac{\partial \varphi}{\partial t}. \quad (23b)$$

Then the field equations (13a,b) follow by the usual method.

To deduce the potential entering the wave equation for the nuclear particles it is convenient to transform the second last bracket in (23) by an integration by parts into

$$\int d\tau (\mathbf{M} \cdot \text{curl} \mathbf{A} + \mathbf{N} \cdot \text{grad} \varphi) / \kappa. \quad (23c)$$

The Dirac equation of the nuclear particle is obtained by taking the derivative of the Hamiltonian (including its "material" part) with respect to  $\psi^*$ . Remembering the definition of  $\mathbf{M}$

and  $\mathbf{N}$  we find

$$W\psi = \beta M c \psi + \boldsymbol{\alpha} \cdot \mathbf{p} \psi + \left[ \varphi - \boldsymbol{\alpha} \cdot \mathbf{A} + \frac{1}{\kappa} \beta \boldsymbol{\sigma} \cdot \left( \text{curl} \mathbf{A} + \frac{4\pi}{\kappa} \mathbf{M} \right) + \frac{i\beta \boldsymbol{\alpha}}{\kappa} \cdot \left( \text{grad} \varphi + \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} - \frac{4\pi}{\kappa} \mathbf{N} \right) \right] \psi, \quad (24a)$$

where  $W$  is the energy. With the use of (11) this simplifies to

$$W\psi = [\beta M c + \boldsymbol{\alpha} \cdot \mathbf{p} + \varphi - \boldsymbol{\alpha} \cdot \mathbf{A} + \beta (\boldsymbol{\sigma} \cdot \mathbf{H} - i \boldsymbol{\alpha} \cdot \mathbf{E}) / \kappa] \psi. \quad (24)$$

Multiplying by  $\psi^*$  from the left and integrating over space, this yields the expression (17) for the potential energy of the nuclear particle.

It may be noted that the terms representing the interaction between field and matter in (22) (i.e., the two last terms) are again unsymmetrical relativistically, just as in electrodynamics (cf. (20)). Only the "small" material quantities,  $\mathbf{j}$  and  $\mathbf{N}$ , appear but not the "large" quantities,  $\rho$  and  $\mathbf{M}$ . The alternative form (23) of the interaction which involves the potentials is again symmetrical.

In (22), there is no term containing the material quantities  $\mathbf{M}$  and  $\mathbf{N}$  quadratically. The transformation from (22) to (23) brings such terms into the Hamiltonian but in such a way that they disappear again in the wave equation (24) and therefore in the potential energy (17) and in the final interaction energy (19). In this respect, our formulation agrees with that of Yukawa, Sakata and Taketani, and differs from that of Froehlich, Heitler and Kemmer.

The form (22) shows most clearly that the Hamiltonian is positive definite for fields not containing matter (i.e., no nuclear particles). From (23) this result follows only when the divergence condition (2) is taken into account. *The positive definite character of the Hamiltonian for the field alone determines essentially the sign of the interaction between two particles.* This point will be considered in more detail in another paper.<sup>26</sup>

<sup>25</sup> E. Fermi, Rev. Mod. Phys. 4, 87 (1932).

<sup>26</sup> H. A. Bethe, to appear shortly in *The Physical Review*.

#### §4. CHARGE DEPENDENCE OF THE NUCLEAR FORCES

##### A. Neutral mesons only

Thus far we have tacitly assumed that the mesons constituting the field are electrically neutral. Only then can we speak of the meson field "associated with a proton" (or a neutron) because the nature (charge) of the nuclear particle does not change by emitting or absorbing a neutral meson. The only freedom we have in this "neutral theory" is to choose values for the "mesonic charges"  $f$  and  $g$  of neutron and proton which may be different for the two kinds of particles. We denote the respective values of  $f$ ,  $g$  for proton and neutron by  $f_P g_P$  and  $f_N g_N$ . To determine the relative values of these constants we use general experimental facts of nuclear physics.

(1) Nuclear forces are symmetrical in neutrons and protons,<sup>27</sup> i.e., the forces between two protons are the same as those between two neutrons (except for the electrostatic force). This identity refers to the magnitude as well as the spin dependence of the forces. Therefore we must have (cf. (19))

$$g_N^2 = g_P^2, \quad (25a)$$

$$f_N^2 = f_P^2, \quad (25b)$$

which permits the solutions

$$g_N = \pm g_P, \quad (26a)$$

$$f_N = \pm f_P. \quad (26b)$$

(2) The force between a neutron and a proton in the  $^1S$  state is practically identical<sup>15</sup> with that between two protons in this state, as revealed by the scattering of slow neutrons and slow protons by protons, respectively. This fact shows that the positive sign must be chosen<sup>28</sup> in both (26a) and (26b), *viz.*

$$g_N = g_P, \quad (27a)$$

$$f_N = f_P. \quad (27b)$$

<sup>27</sup> H. A. Bethe and R. F. Bacher, Rev. Mod. Phys. **8**, 133 (1936).

<sup>28</sup> The equality of neutron-proton and proton-proton interaction in the  $^1S$  state by itself shows only that (cf. (19a,c))

$$g_N g_P - 2f_N f_P = g_P^2 - 2f_P^2. \quad (27c)$$

Only in conjunction with (26a,b) the two equations (27a,b) can be deduced separately.

In the neutral theory, therefore, neutron and proton are completely equivalent and indistinguishable as far as the associated meson fields are concerned. The expressions  $g_1 g_2$  and  $f_1 f_2$  in (19) can be replaced by  $g^2$  and  $f^2$  whatever the interacting particles. The interaction thus obtained will be denoted by  $W_n$  in the following. It is an ordinary (nonexchange) interaction which, however, does depend on spin. In the language customary in nuclear theory, it would be a mixture of a Wigner and a Bartlett force.\*

##### B. Charged mesons only

The emission of a charged meson will be accompanied by a change of charge of the emitting nuclear particle. Thus a neutron ( $N$ ) can only emit a negative ( $M^-$ ) or absorb a positive ( $M^+$ ) meson and will thereby be transformed into a proton ( $P$ ). In the second approximation of the Schrödinger perturbation scheme, i.e., when we consider the emission of one meson by a nuclear particle and its reabsorption by another, this will lead to *forces between a neutron and a proton*. The scheme of the interaction between a particle 1 which is originally a neutron and a particle 2 originally a proton is:

$$\text{Either } N_1 \rightarrow P_1 + M^-, \quad P_2 + M^- = N_2,$$

$$\text{or } P_2 \rightarrow N_2 + M^+, \quad N_1 + M^+ = P_1.$$

It is obvious that in this way *no force will be obtained between two nuclear particles of the same kind*, i.e., two neutrons or two protons. These statements are also valid in the approximation of the field theory which is completely equivalent to the second order Schrödinger approximation since both theories give the interaction terms of order  $g^2$  (or  $f^2$ ).

The force between two nuclear particles obtained with charged mesons may be written

$$W_{ch} = W'(Q_1 Q_2^* + Q_1^* Q_2). \quad (28)$$

Here  $Q$  is an operator transforming a neutron into a proton and  $Q^*$  the reverse operator. If particle 1 is originally a neutron, 2 a proton,  $Q_1 Q_2^*$  will interchange the charges of the two particles while  $Q_2 Q_1^*$  is zero. If both nuclear particles have the same charge, the operator in (28) will be zero.  $W'$  will be identical in form with the interaction due to neutral mesons but

\* Cf. reference 27, p. 105.

the  $g$  and  $f$  will now be the quantities which refer to the strength of the *charged* meson field associated with a nuclear particle. It is important that in the formulae (19) for the various parts of  $W'$  we must now put  $|g|^2$  and  $|f|^2$  for  $g_1g_2$  and  $f_1f_2$ . This is because  $g_1$  refers to one transformation of the nuclear particle and  $g_2$  to the reverse transformation; from general principles of quantum theory (reality of the Hamiltonian<sup>29</sup>) we must then have  $g_2=g_1^*$ ,  $f_2=f_1^*$  (the asterisk denotes complex conjugate). Further, if  $g$  represents the interaction between nuclear particles and mesons of *one* charge, there will be a contribution  $W_{ch}$  from mesons of each sign of charge. The total nuclear potential due to charged mesons is therefore  $2W_{ch}$ .

(28) represents an exchange force of the Heisenberg type. It is most convenient to write the wave function of a system of two nuclear particles in terms of five coordinates for each particle, *viz.*

$$\Psi(\mathbf{r}_1, m_{s1}, m_{\tau 1}; \mathbf{r}_2, m_{s2}, m_{\tau 2}).$$

Here  $\mathbf{r}$  represents the three space coordinates,  $m_s$  the spin and  $m_\tau$  the charge coordinate.  $m_s$  and  $m_\tau$  can each assume the two values  $+\frac{1}{2}$  and  $-\frac{1}{2}$ . These are the values of the spin component in a given direction in the case of  $m_s$ , while  $m_\tau = +\frac{1}{2}$  means that the particle is a proton,  $m_\tau = -\frac{1}{2}$  that it is a neutron. Now the operator  $Q_1Q_2^* + Q_1^*Q_2$  will interchange  $m_{\tau 1}$  and  $m_{\tau 2}$  provided one of the two particles is a proton, the other a neutron, but irrespective of which is which. (If both particles are of the same kind, the operator is zero, as mentioned before.) Now it is easy to show that  $\Psi$  will be either symmetrical or antisymmetrical in the charge coordinates  $m_{\tau 1}m_{\tau 2}$ ; therefore the operator  $Q_1Q_2^* + Q_1^*Q_2$  will be equivalent to a factor  $+1$  or  $-1$ , respectively.

The symmetry with respect to the charge is related to that with respect to space coordinates and spin by the Pauli principle. This principle

<sup>29</sup> The Hamiltonian operator of the interaction between charged meson field and two nuclear particles 1, 2 will be 
$$\mathcal{H} = g[\varphi(r_1)Q_1 + \varphi(r_2)Q_2] + g^*[\varphi^*(r_1)Q_1^* + \varphi^*(r_2)Q_2^*]$$
 + terms depending on spin

where  $\varphi^*$  is the operator of emission of a positive meson,  $\varphi$  the operator of absorption. The same formalism as in §2 leads then to the interaction

$$W = gg^*(e^{-kr}/r)(Q_1Q_2^* + Q_1^*Q_2) + \text{spin terms.}$$

requires that  $\Psi$  be antisymmetrical with respect to interchange of *all* coordinates, *viz.*  $\mathbf{r}$ ,  $m_s$  and  $m_\tau$ . Now the symmetry of  $\Psi$  with respect to  $\mathbf{r}$  and  $m_s$  is as follows:\*

- $\Psi$  is symmetrical with respect to  $\mathbf{r}$  if the orbital momentum  $L$  of the system is even, antisymmetrical in  $\mathbf{r}$ , if  $L$  is odd.
- $\Psi$  is symmetrical in  $m_s$  if the total spin  $s$  is one, *i.e.*, if the system is in a triplet state antisymmetrical in  $m_s$  if  $s=0$  (singlet state).

Therefore, according to the Pauli principle,

- $\Psi$  is symmetrical in  $m_\tau$  for singlet states of even  $L$  and for triplet states of odd  $L$ ,
- $\Psi$  is antisymmetrical in  $m_\tau$  for triplet states of even  $L$  and for singlet states of odd  $L$ .

In particular, the ground state ( $^3S$ ) of the deuteron is antisymmetrical, the virtual  $^1S$  state symmetrical in the charge coordinate. For two particles of the *same* kind, we have of course  $m_{\tau 1} = m_{\tau 2}$  so that only the states symmetrical in  $m_\tau$  are permitted.

Summarizing, we find that the interaction caused by the charged meson field is:

- 0, if the two interacting nuclear particles are of the same kind;
- $+2W'$ , if they are of opposite kinds (proton and neutron) and if the wave function is symmetrical in the charge coordinate  $m_\tau$  (e.g. in the  $^1S$  state);
- $-2W'$ , if they are proton and neutron and the wave function is antisymmetrical in the charge (e.g. in the  $^3S$  state).

Considering in particular the spin-independent part  $U$  (cf. (19a)) of the interaction, this means ( $W'$  positive!) a repulsion in the  $^1S$  state (symmetrical in  $m_\tau$ ), an attraction in the  $^3S$  state (antisymmetrical).

### C. Symmetrical theory

The theory involving charged mesons only was found to contradict experiment by giving no force between two like nuclear particles. This can be remedied by using neutral as well as charged mesons. In this case, the argument 1

\* Cf. reference 27, p. 107.



given in Section A will still be valid, i.e., the  $f$  and  $g$  values for the interaction with neutral mesons must be either equal for proton and neutron, or equal and opposite. In contrast to  $A$ , however, the solution  $g_N = g_P, f_N = f_P$  is to be ruled out: This solution would make the interaction caused by neutral mesons *alone*, equal for like and for unlike nuclear particles. Since for unlike particles the charged mesons give an additional contribution while for like particles they do not, the *total* interaction could not be the same for like and unlike particles in the  $^1S$  state, in contrast to evidence 2 quoted in Section A. Therefore we must now choose the negative sign in (26), putting

$$g_N^0 = -g_P^0, \quad (30a)$$

$$f_N^0 = -f_P^0, \quad (30b)$$

the superscript 0 signifying the fact that these quantities refer to the interaction with neutral mesons ( $M^0$ ).

Then we find that the interaction due to neutral mesons alone is

$$\begin{aligned} &+W_n, \text{ if the interacting nuclear particles} \\ &\quad \text{are alike} \\ &-W_n, \text{ if they are unlike.} \end{aligned} \quad (31)$$

The total interaction in the  $^1S$  state (due to neutral and charged mesons) is therefore

$$\begin{aligned} &W_n \text{ for like particles} \\ &-W_n + 2W' \text{ for unlike particles.} \end{aligned} \quad (31a)$$

Since these two interactions are experimentally equal, we find

$$W' = W_n.$$

In other words, the absolute values of  $g$  and  $f$  must be the same for the interactions with neutral, positive and negative mesons:

$$\begin{aligned} |g^+| &= |g^-| = |g^0| = |g|, \\ |f^+| &= |f^-| = |f^0| = |f|. \end{aligned} \quad (32)$$

This theory is therefore symmetrical in all the three kinds of mesons. It was first investigated by Kemmer<sup>16</sup> using a more elegant method.

With this symmetrical theory, we find for the total interaction energy (cf. (29), (31))

$$\begin{aligned} W &= +W_n \text{ for all states symmetrical} \\ &\quad \text{in the charge coordinate,} \\ W &= -3W_n \text{ for all states antisymmetrical} \end{aligned} \quad (33)$$

in the charge coordinate.

TABLE I. Summary of the sign of various interactions.

INTERACTION (CF. (EQ. 19))	NEUTRAL THEORY	SYMMETRICAL THEORY
$U$	Always repulsive	Repulsive for states symmetrical in the charge Attractive for states antisymmetrical in the charge
$V_1$	Repulsive for triplet states Attractive for singlet states	Repulsive for odd $L$ Attractive for even $L$
$V_2$	Zero for singlet states Attractive for triplet states if total spin $S$ and vector $\mathbf{r}$ from one particle to the other are parallel Repulsive when $S$ perpendicular to $\mathbf{r}$	Zero for singlet states Repulsive for triplet states of even $L$ (attractive for odd $L$ ) when $S$ and $\mathbf{r}$ are parallel Attractive for even $L$ (repulsive for odd $L$ ) when $S$ and $\mathbf{r}$ are perpendicular

This result may be written in the form

$$W = W_n \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2, \quad (33a)$$

where  $\boldsymbol{\tau}_1, \boldsymbol{\tau}_2$  are the isotopic spin operators<sup>16</sup> of the two nuclea r particles.  $\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$  has the value  $+1$  for states symmetrical in  $m_r$  and  $-3$  for antisymmetrical states, just as for the ordinary spin  $\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 = +1$  for triplet states and  $-3$  for singlets. As has been pointed out before,  $W_n$  is given by (19) with  $g_1 = g_2 = g$  and  $f_1 = f_2 = f$ .

We see from (33) that the symmetrical theory gives the same interaction as the neutral theory for the  $^1S$  state (and all symmetrical states), but the opposite sign, and three times the magnitude, for the  $^3S$  state (and all antisymmetrical states). The potential in the symmetrical theory is a mixture of exchange and ordinary force which resembles qualitatively an exchange force; the spin-independent part (19a) is similar to a Heisenberg, the spin-dependent part (19c) to a Majorana force.

#### D. Summary of the sign of the various interactions

A summary of the sign of the various interactions is given in Table I. The statements about  $U$  and  $V_1$  follow from the foregoing formulae and discussions considering that  $\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 = +1$  for triplets and  $-3$  for singlets.  $V_2$  will be discussed in detail in §8, 9.

## §5. THE SINGLE FORCE HYPOTHESIS

The spin-independent interaction  $U$  (cf. (19a)) alone cannot explain the known experimental facts about two-particle systems. For this force is repulsive in the neutral theory, and therefore also in the symmetrical theory for the  ${}^1S$  state (cf. §4C) whereas experimentally there is an attractive potential in the  ${}^1S$  state. Moreover, the potentials in  ${}^1S$  and  ${}^3S$  states are experimentally different.

Therefore the spin-dependent interaction  $V$  (19b,c,d) must certainly be present, i.e.,  $f$  must be different from zero. We shall show in this section that it is possible with  $V$  alone, to explain qualitatively all known facts about nuclear two-body systems and also the saturation of nuclear forces. In the quantitative discussion in Part II we shall show that the interaction  $U$  is actually harmful in many cases. Therefore we make the hypothesis:

*The spin-dependent interaction  $V$  is the sole nuclear force* (Single Force Hypothesis, S. F. H.).

It is obvious that this hypothesis means an enormous simplification in concept (though not in mathematics) compared with the older nuclear theory with its four different kinds of forces (Wigner, Bartlett, Majorana and Heisenberg) to which another force, not spherically symmetrical, had to be added by Schwinger in order to explain the quadrupole moment of the deuteron.

With the single-force hypothesis, the interaction between two nuclear particles becomes explicitly (cf. (19))

$$V = V_1 + V_2, \quad (34)$$

where for the neutral theory

$$V_1 = \frac{2}{3} f^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 e^{-\kappa r} / r, \quad (34a)$$

$$V_2 = f^2 \left( -3 \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{r} \boldsymbol{\sigma}_2 \cdot \mathbf{r}}{r^2} + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \right) \frac{e^{-\kappa r}}{r} \\ \times \left( \frac{1}{\kappa^2 r^2} + \frac{1}{\kappa r} + \frac{1}{3} \right), \quad (34b)$$

while in the symmetrical theory these expressions must be multiplied by  $\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$ . (We have used in (34) the spin operators  $\boldsymbol{\sigma}$  rather than the average value  $\mathbf{s}$  of the spin; this is convenient for later use in the wave equation.)

We shall now show that this force is actually capable of explaining all the empirical facts about two-body systems. For any singlet state,  $V_2$  is identically zero. This can be seen either by direct evaluation, or from the fact that the average of the first parenthesis over all orientations of the spin must be zero, and that for a singlet state this average is equal to the actual value. In  $V_1$ , the factor  $\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$  is equal to  $-3$ . In the neutral theory, we therefore get an attractive potential  $V_1$  for any singlet state. The same is the case in the symmetrical theory for the  ${}^1S$  state since in this case  $\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 = 1$ . This result is in agreement with the experimental evidence which shows attraction in the  ${}^1S$  state both for the deuteron and the double proton.

Regarding the ground state of the deuteron which is essentially a  ${}^3S$  state, the two alternative theories differ. In the neutral theory, the central force  $V_1$  will be repulsive since  $\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 = +1$  for triplets. In the symmetrical theory,  $\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 = -3$  for this state (cf. §4C) so that  $\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$  has the same value as for the  ${}^1S$  state, *viz.*  $-3$ . Therefore  $V_1$  will be attractive and equal to its value for  ${}^1S$ .

Whichever theory is used, the potential  $V_2$  ("tensor interaction") will mix the  ${}^3S$  state with a  ${}^3D_1$  state. This will cause the ground state of the deuteron to have a quadrupole moment in agreement with the observations of Kellogg, Rabi, Ramsey and Zacharias. In addition,  $V_2$  will lower the triplet state (second-order perturbation of a lowest state). In the case of the symmetrical theory, this will automatically make the  ${}^3S$  state lower than the  ${}^1S$  because the two states would coincide if  $V_2$  were neglected. This result agrees with experiment according to which the triplet state is the ground state while the  ${}^1S$  state is unstable and close to zero energy. In the neutral theory a greater lowering effect of  $V_2$  is required, first to offset the repulsive action of  $V_1$ , then to make up for the attraction in the  ${}^1S$  state, and finally to depress the triplet state below the singlet. However, owing to its divergence with  $1/r^3$ , the interaction  $V_2$  becomes very large (cf. §6) and can therefore easily make the triplet lower than the singlet even in the neutral theory. That this is actually the case will be shown by the detailed calculations in §12, 13.

We have thus shown that the position of the

deuteron levels can be explained by the potential  $V$ , the singlet level being determined exclusively by the central force  $V_1$  while for the triplet the most important force is the tensor interaction  $V_2$ . It is not necessary to lower the triplet (and raise the singlet) by means of the spin-independent force  $U$  (cf. (19a)) as has been believed;<sup>11, 13, 16</sup> on the contrary, the use of the tensor interaction for this purpose has the advantage of giving a quadrupole moment at the same time.

A very important requirement for any nuclear theory is the saturation of the nuclear forces for heavy nuclei.<sup>30</sup> A necessary (though not sufficient) condition for saturation is that the average of the interaction over all directions of the spins  $\sigma_1 \cdot \sigma_2$  be zero or repulsive. The average of our potential  $V$  is exactly zero which can be seen most easily by averaging over the two possible values,  $+\frac{1}{2}$  and  $-\frac{1}{2}$ , of  $m_{s1}$  and  $m_{s2}$ . This would probably be also *sufficient* for saturation if we could show in addition that our forces favor a small value of the total spin of the nucleus (in agreement with the experimental behavior of nuclear spins). The central force  $V_1$  will in the neutral theory indeed show such a tendency, being attractive for antiparallel spin of the two interacting particles (singlet) repulsive for parallel spins (triplet state). On the other hand, the tensor interaction  $V_2$  will favor a parallel alignment of the spins provided the vectors  $\mathbf{r}$  are also parallel to the resultant spin. This force, if acting alone, would lead to a very oblong rather than a spherical shape of the nucleus, a tendency which is strongly opposed by the marked increase of the kinetic energy of the particles for a non-spherical shape. For lighter nuclei, for which the kinetic energy is most important, the central force  $V_1$  will certainly win out, yielding almost zero spin in agreement with experiment. For very heavy nuclei, however, the kinetic energy will increase more slowly than the potential energy, *viz.* as<sup>31</sup>  $A^{11/16}$  as compared with  $A^2$ . Therefore there will be the danger of a predominance of  $V_2$  with its consequences of large spin, non-spherical shape and, worst of all, nonsaturation.

<sup>30</sup> See, e.g., G. Breit and E. Wigner, *Phys. Rev.* **53**, 998 (1938); H. Volz, *Zeits. f. Physik* **105**, 537 (1937); E. Feenberg, *Phys. Rev.* **52**, 667 (1937).

<sup>31</sup> I am indebted to Professor Wigner for this communication and for pointing out the saturation difficulties mentioned in the text.

Whether this will actually occur, and whether it will occur in the range of existing nuclei ( $A < 240$ ) depends entirely on the behavior of the tensor interaction at very small distances. This is because the effect of nonsaturation is a collapse of the nucleus to a small size which then permits every particle to interact with every other. If  $V_2$  has the same sign at small distances as at large ones, and especially if it becomes very large, there will almost certainly be nonsaturation; if  $V_2$  changes sign, the nonsaturation will not set in except at a much higher nuclear mass  $A$ ; and if  $V_2$  tends to zero, saturation will be preserved. Of course, there is no evidence at the moment on the behavior of  $V_2$  at small distances, but the assumptions leading to saturation are at least not implausible.

In any case, it seems that the addition of a spin-independent force  $U$  will not help matters greatly because such a force would only increase the already existing repulsive central force  $V_1$  in the triplet state. The symmetrical theory gives exactly the same result as the neutral one for nuclei containing only protons or only neutrons so that the same danger of nonsaturation exists here. The nuclei containing both neutrons and protons will not so easily show nonsaturation in the symmetrical theory because the force vanishes when averaged over the charge coordinates  $m_{r1}$  and  $m_{r2}$ ; on the other hand, it is then more difficult to prove that the total spin tends to be small.

Summarizing we may say that the single force hypothesis is not in contradiction with any known qualitative fact, and that nothing is gained by adding a spin-independent force  $U$ .

#### §6. THE NECESSITY OF CUTTING OFF

The tensor interaction  $V_2$  behaves at small distances as  $1/r^3$  and will therefore give rise to infinite negative eigenvalues. This follows most easily from the fact that there are certain states, particularly the  ${}^3P_1$  state (cf. §8, 9), for which the tensor interaction is equivalent to a simple attractive central force depending in the same way on  $r$ . (In other states, e.g.,  ${}^3P_0$ , the equivalent central force is repulsive. If the sign of the whole interaction were changed, the role of attractive and repulsive states would be interchanged so that there would still be states with

an attractive  $1/r^3$  potential.) Now it is well known that in an attractive  $1/r^3$  potential there is no lowest eigenvalue, and this fact is not changed by taking into account the centrifugal force acting in the  ${}^3P_1$  state because this force diverges only as  $1/r^2$  and therefore becomes negligible compared with the tensor interaction at small distances.

It has been suggested that the divergence of the potential might not be so serious because the situation might be similar to that found<sup>32</sup> with respect to some terms in the relativistic interaction of two electrons. As Breit<sup>32</sup> has shown, only the first-order perturbation energy caused by these terms has physical significance while the terms should not be included in the potential used for calculating the wave function. If the same were true of the tensor interaction there would be no divergence difficulty: For then the wave function of the  ${}^3P_1$  state would have to be calculated using only the centrifugal force  $\sim 1/r^2$  and potentials which diverge only as  $1/r$ . Then the wave function would behave as  $r$  at small distances, and the average value of the tensor interaction taken over this wave function (first-order perturbation energy) would be finite (given by an integral which behaves for small  $r$  as  $\int r^2 dr r^2/r^3$ ).

However, this solution is impossible. The tensor interaction is genetically entirely different from the relativistic terms in the electron interaction. The former is a "static" interaction<sup>18</sup> and can be derived from the Hamiltonian of the meson field by a contact transformation just as the Coulomb force is derived from the Hamiltonian of the electromagnetic field. The relativistic terms in the electron interaction are dynamic terms and therefore subject to the same uncertainties as, e.g., the self-energy.

Even if the treatment of the tensor interaction as a true potential could not be justified from its derivation it would still be necessary *a posteriori* from its applications. It is obvious that the quadrupole moment of the deuteron could not be explained if the tensor interaction were to be excluded in the calculation of the wave function. But we can say even more: The tensor interaction and the spin-dependent central force  $V_1$

are genetically related, therefore the latter would have to be excluded as well. This would leave only  $U$  as a true potential for the calculation of the wave function, and  $U$  is repulsive for the singlet state in both the neutral and the symmetrical theory, in contradiction with experiment.

It is therefore necessary to cut off the potential at small distances and to replace it by one which diverges more slowly (less than  $1/r^2$ ) or not at all. Only such a cutting off will make it possible to obtain finite results for the binding energy of a system consisting of two nuclear particles. To my knowledge, *this is the first time that a cutting off is necessary in a purely mechanical problem.*

There are many reasons why the potential (34) will become invalid at small distances. The first, and probably the easiest to take into account, is the relativistic correction, both in the potential ("small" terms  $\mathbf{j}$  and  $\mathbf{N}$ ) and in the wave equation. The first of these effects means the taking into account of the retardation, the second the relativistic change of mass. However, it seems very doubtful whether these corrections will give the desired effect. Relativity can only be expected to be important when the potential energy becomes about  $2Mc^2$  (factor 2 because there are two particles) but actually the explicit calculations show that the potential must be cut off at a value of about  $\frac{1}{4}Mc^2$  in the neutral theory (straight cut-off, cf. §13, Table IV) and at only  $0.015Mc^2$  in the symmetrical theory. Moreover, it is not at all certain in which direction the relativistic corrections will act: The relativistic change of mass will certainly act like an increase in the effective potential, i.e., give an effect in the wrong direction. The retardation will probably give an effect in the right direction but whether this will be sufficient to overbalance the effect of the change of mass is doubtful. In the hydrogen problem, as is well known, the change of mass effect predominates.

A more promising reason for cutting off seems to be the interaction of higher order in  $f^2/\hbar c$ . The unquantized field theory as presented in §2 gives only the terms of order  $f^2$  in the interaction. Terms involving higher powers of  $f$  could be calculated by the Schrödinger perturbation theory in conjunction with a quantization of the meson field. The fourth-order terms have been calculated by Yukawa,<sup>1</sup> by Froehlich, Heitler

<sup>32</sup> G. Breit, Phys. Rev. **39**, 616 (1932).

and Kemmer<sup>11</sup> and by Møller and Rosenfeld;<sup>18</sup> they behave at small distances like

$$W^{(4)} \sim \frac{f^2}{r} \frac{f^2}{\hbar c} \frac{1}{(\kappa r)^4} = \frac{f^4}{\mu c^2 r^2} \frac{1}{(\kappa r)^3}, \quad (35)$$

thus their ratio to the terms of order  $f^2$  is:

$$W^{(4)}/W^{(2)} = (f^2/\hbar c)(\kappa r)^{-2}. \quad (35a)$$

From the quantitative calculations in §13, (Table IV) we obtain  $f^2/\hbar c = 0.08$ . Accordingly, the higher order terms can be expected to become important for  $\kappa r \sim 0.3$ . In view of the fact that numerical factors have been left out, the agreement with the calculated cutting-off distance in the neutral theory, *viz.*,  $\kappa r = 0.32$  or  $0.40$  with two different methods of cutting off, is satisfactory.

The third, and perhaps most important, consideration is that the meson field produced by nuclear particle  $A$  in the neighborhood of  $B$ , will interfere with the proper field of  $B$ . Since the latter is certainly very large, it cannot be expected to obey the linear field equations of §2. Therefore there will not be a linear superposition of the proper field and the external field. This point has been particularly emphasized by Heisenberg.<sup>33</sup> It is like the second point in making use of the nonlinearity of the field equations (higher than quadratic terms in the field energy). However, it differs from both the preceding points in permitting deviations from the simple fields of §2 even for small values of the "external" field, *i.e.*, for large distances of the interacting particles, because the proper field is always large. In other words, it is possible that in this theory we should not speak of a cut-off at all but of a general modification of the interaction potential.

As Heisenberg<sup>33</sup> has shown the interference between external and proper field will be equivalent to an "inertia" of the spin of the nuclear particle. This is exactly what we need in order to reduce our interaction at small distances because the tensor interaction causes a rapid motion of the spin of the nuclear particles.

A last possibility is that all interactions, nuclear as well as electromagnetic, etc., break down at small distances for some reason unknown at present and distinct from the nonlinearity of

the field equations discussed above. There is some indication that such a breakdown is required in order to make the self-energy and the proper field of a particle finite. At what distances (or momentum changes, or field strengths) such a breakdown occurs can, of course, not be estimated at present.

Summarizing we may say that there is ample reason for cutting off the interaction between nuclear particles at small distances. A cutting off distance  $r_0$  of the order of one-third of the range of the nuclear forces,  $1/\kappa$ , would seem plausible. The exact value of  $r_0$ , and the way in which the interaction must be cut off cannot at the moment be deduced from first principles but can only be obtained from a quantitative treatment of the two-body problem in conjunction with empirical data on binding energies, scattering, etc.

In the absence of a rational theory of the cutting off, and in order to obtain an idea of the sensitivity of the results to the unknown behavior of the potential at small distances, we shall investigate two alternative ways of cutting off, *viz.*: (A) The potential is assumed to be zero inside a certain radius  $r_0$  (zero cut-off), (B) The potential is assumed to be constant for  $r < r_0$  and continuous at  $r_0$  (straight cut-off). In formulae:

$$V(r) = 0 \quad \text{for} \quad r < r_0 \quad (A)$$

$$\text{or} \quad V(r) = V(r_0) \quad \text{for} \quad r < r_0. \quad (B)$$

For  $r \geq r_0$ , we assume in either case the validity of the potential (34).

Our theory contains then two<sup>34</sup> unknowns, *viz.*, the strength of the interaction,  $f$ , and the cutting-off radius  $r_0$ . These two constants will be determined from the binding energy of the deuteron (triplet state) and from the scattering cross section of slow neutrons by protons which gives evidence on the singlet interaction of two particles. When  $f$  and  $r_0$  are known, we can calculate further properties of the deuteron, in particular its quadrupole moment, and the agreement or disagreement of such quantities with experiment will provide a test of the theory.

<sup>34</sup> The constant  $\kappa$  (reciprocal range of the forces) is assumed to be known from the mass of the meson (Eq. (8)). Cf. however, the calculations of L. E. Hoisington, S. S. Share, and G. Breit, *Phys. Rev.* **56**: 884 (1939), on the proton-proton scattering, which seem to require a larger value for  $\kappa$ , corresponding to  $\mu = 326m$ .

<sup>33</sup> W. Heisenberg, *Zeits. f. Physik* **113**, 61 (1939).