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The Positive Column of the Nitrogen Arc at Atmospheric Pressure. II

ALBERT M. STONE AND EDWARD S. LAMAR
George Eastman Research Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts

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Further work with the theory of the positive column, previously reported by the authors, has made possible its extension to include convection in the power loss. The convection heat loss function employed in this extended theory was developed empirically subject to certain restrictions imposed by dimensional analysis, and subject to the need for describing accurately existing experimental data on the arc itself. These restrictions have proved to be sufficiently rigid to limit greatly the choice in form for the heat loss function. Hence the authors feel some confidence in the view that the ultimate theoretical solution of the convection heat loss problem must yield a heat loss function very nearly identical with the one developed here. The inclusion of convection makes it possible to satisfy the customary boundary conditions in the differential equation governing the positive column, thus removing the necessity for employing any artifice such as the assumption of extreme power dissipation in the solution of the arc problem. The theory yields values of the power dissipation per unit length, over the range from one to ten amperes, and values of the arc temperature, which are in good agreement with existing experimental data. Values of the visual arc radius, for comparison with those obtained experimentally, are not available from the theory because of lack of detailed knowledge concerning the distribution of radiation in the arc. An average radius is computed, however, which should, and does, show the same general trends as those exhibited by the visual arc radius as determined from experiment. The present theory provides a satisfactory solution of the arc problem so long as the power loss by radiation constitutes a negligible fraction of the whole.

INTRODUCTION

In a paper by the authors,[1] a theory of the positive column of the nitrogen arc at atmospheric pressure was presented in which convection was not included in the differential equation for the positive column, and was considered only insofar as it affected the boundary conditions for the integration. This earlier theory is applicable, therefore, only to cases in which the convection term remains negligible sufficiently far out from the center of the arc to include all volume elements contributing appreciably to the power generation. The present paper concerns itself with an extension of this earlier work designed to include convection as one of the mechanisms by means of which power is lost from the arc. The results of this investigation are applicable, therefore, to all cases except those in which radiation accounts for an appreciable fraction of the total power loss. Furthermore, the inclusion of a specific convection term makes it possible to satisfy the outer boundary condition, i.e., that the temperature gradient must go to zero at ambient temperature. If this were not so, one would continue to have conduction heat loss out to infinity.

A resumé of most of the experimental work on the nitrogen arc at high pressure was presented in the earlier paper,[1] together with arguments supporting the use of the Saha equation for determining the electron concentration. Subsequently two experimental papers by Suits[2, 3] have appeared, and two theoretical papers by Elenbaas[4] and by Suits and Poritsky,[5] respectively. Discussion of these articles will be deferred to a later section. There is some experimental information not included in the earlier resumé, nor in the papers listed above, which is of importance here and which will now be presented.

Although radiation and sound velocity measurements in the positive column of the arc fail to show a variation of arc temperature over the range of arc current from one to ten amperes, there is experimental evidence for expecting a variation with arc current within the accuracy of these measurements. As was mentioned in the earlier paper, it has been found[6] that the product of the visual cross-sectional area, the electric intensity, and the current density per unit potential gradient at the measured temperature equals the measured current. The assump-

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[1] E. S. Lamar, A. M. Stone and K. T. Compton, Phys. Rev. **55**, 1235 (1939); A. M. Stone, E. S. Lamar and K. T. Compton, Phys. Rev. **55**, 1145(A) (1939).

[2] C. G. Suits, Phys. Rev. **55**, 561 (1939).

[3] C. G. Suits, Phys. Rev. **55**, 198 (1939).

[4] W. Elenbaas, Phys. Rev. **55**, 294 (1939).

[5] C. G. Suits and H. Poritsky, Phys. Rev. **55**, 1184 (1939).

[6] L. S. Ornstein and H. Brinkmann, Physica **1**, 797 (1934).

tion that this relationship holds exactly provides a means of determining the arc temperature from measurements of current, gradient, and cross-sectional area. Dr. C. G. Suits, in a private communication, has kindly supplied the authors with values of the arc temperature determined in this way. Without inquiry into the absolute accuracy of these temperatures, the measurements from which they are derived are such as to make them self-consistent to within much narrower limits than the experimental error in the more direct measurements of temperature. These later determinations of Suits' give temperatures varying from 5950°K at one ampere to 6400°K at ten amperes for an arc in nitrogen at atmospheric pressure.

It was kindly pointed out to the authors by Dr. Joseph Slepian that the dimensional arguments employed in the earlier paper practically imposed the necessity for some variation in arc temperature with arc current. It will be seen later in the present paper that this variation in temperature appears as a result of the theory.

THEORY

The differential equation governing the positive column of the arc may be written as

$$dW=-2\pi\frac{d}{dr}\left(r\Theta\frac{dT}{dr}\right)dr+2\pi r\sigma dr=2\pi E^2Frdr. \quad (1)$$

This equation differs from that developed in the earlier paper to which reference has been made only in the inclusion of convection. In this equation W is the power generation per unit length, r the radial distance out from the center of the arc, Θ the generalized thermal conductivity, E the potential gradient, T the temperature, F the current density per unit potential gradient, and σ is the power loss per unit volume by convection. As was pointed out in the earlier paper, the power loss by radiation represents less than five percent of the total for arcs at one atmosphere pressure with currents below 30 amperes. The present theory is designed to cover the range from one to ten amperes, and thus the neglect of radiation is justifiable.

A single integration of Eq. (1) leads to

$$\frac{dT}{dr}=-\frac{1}{r\Theta}\left\{\int_0^r E^2Frdr-\int_0^r \sigma rdr\right\}, \quad (2)$$

where dT/dr has been set equal to zero at $r=0$. The other boundary condition to be satisfied by the second-order differential equation (Eq. (1)) is simply that dT/dr must vanish when $T=T_0$, the ambient temperature.

Since the publication of the previous paper, certain minor revisions have been made in the functions F and Θ, as reported therein. Dr. C. G. Suits, of the General Electric Company Research Laboratory, has very kindly suggested the following possible revisions in the constants entering into the calculation of the functions. He points out that a later and probably more reliable value of the ionization potential of molecular nitrogen is 15.8 volts instead of the 16.5 volts used previously, and the dissociation energy is more likely 7.34 volts than the 7.9 volts recorded by Lozier. Both these later values are to be found in *Gasentladungstabellen* by Knoll, Ollendorf and Rompe. The use of the new value for the dissociation energy together with the value of the free energies of both molecular and atomic nitrogen as given by Giauque and Clayton[7] yield values of the equilibrium constant which differ from those reported by these authors, and which can be expressed with sufficient accuracy by

$$K=1.46(10^7)e^{-8.902(10^4)/T}\text{ atmos.} \quad (3)$$

The change in the ionization potential affects the Saha equation for the molecule, Eq. (8) of the previous paper. The current density per unit voltage gradient becomes with no further change

$$F(T)=\frac{2.17(10^2)T^{3/4}}{P}\{P_a e^{-169300/T}$$
$$+8(P-P_a)e^{-184500/T}\}^{\frac{1}{2}}$$
$$\times\{1+0.183P_a/P\}\text{ amp./volt-cm,} \quad (4)$$

where P_a, the partial pressure of the atoms, is given by

$$P_a=\tfrac{1}{2}K\{(1+4P/K)^{\frac{1}{2}}-1\}\text{ atmos.} \quad (5)$$

and P is the total pressure, both expressed in atmospheres. Values of P_a, F, and n, the electron concentration, are given in Table I. It will be

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[7] W. F. Giauque and J. D. Clayton, J. Am. Chem. Soc. **55**, 4887 (1933).

TABLE I. *Values of the current density per unit potential gradient $F(T)$; electron concentration n; atomic nitrogen pressure P_a; generalized conductivity Θ; and the two functions $\alpha(T)$ and $\beta(T)$ occurring in the convection heat loss function σ.*

T °K	F AMP./CM^2 PER VOLT/CM	n PER CM^3	Pa ATMOSPHERE	Θ JOULE/SEC. CM°	α JOULE/CM^4	β PER CM^3
6500	4.040×10^{-1}	5.907×10^{13}	9.456×10^{-1}	4.372×10^{-3}	885.6	25.38
6300	2.586 "	3.856 "	9.205 "	5.019 "	905.7	27.66
6100	1.603 "	2.443 "	8.835 "	5.902 "	920.0	30.49
5900	9.540×10^{-2}	1.491 "	8.310 "	7.003 "	924.9	34.08
5700	5.434 "	8.737×10^{12}	7.600 "	8.163 "	916.7	38.66
5500	2.940 "	4.883 "	6.705 "	9.119 "	892.9	44.41
5300	1.502 "	2.586 "	5.666 "	9.585 "	853.6	51.41
5100	7.198×10^{-3}	1.289 "	4.565 "	9.398 "	802.4	59.57
4900	3.241 "	6.017×10^{11}	3.497 "	8.607 "	745.0	68.74
4700	1.356 "	2.614 "	2.544 "	7.415 "	687.9	78.74
4500	5.272×10^{-4}	1.053 "	1.774 "	6.078 "	638.3	89.37
4300	1.878 "	3.879×10^{10}	1.149 "	4.796 "	594.2	100.9
4100	6.112×10^{-5}	1.303 "	7.102×10^{-2}	3.762 "	562.2	113.3
3900	1.791 "	3.935×10^{9}	4.135 "	2.771 "	540.4	126.8
3700	4.647×10^{-6}	1.052 "	2.253 "	2.223 "	527.6	141.9
3500	1.047 "	2.440×10^{8}	1.139 "	1.810 "	522.0	159.0
3300	1.998×10^{-7}	4.805×10^{7}	5.288×10^{-3}	1.541 "	521.7	179.0
3100	3.086×10^{-8}	7.661×10^{6}	2.218 "	1.343 "	525.1	202.6
2900				1.226 "	530.5	230.9
2700				1.164 "	537.2	265.7
2500				1.112 "	545.0	308.7
2300				1.058 "	552.4	362.9
2100				1.001 "	559.0	423.6
1900				9.406×10^{-4}	564.4	524.4
1700				8.762 "	567.4	648.5
1500				8.072 "	566.8	821.8
1300				7.327 "	559.7	1073
1100				6.514 "	541.6	1456
900				5.628 "	502.7	2071
700				4.669 "	422.6	3112
500				3.642 "	253.3	4726
300				2.500 "	0	0

noticed that differences between these functions and those recorded previously become more apparent as the temperature decreases. At 4000°K the change introduced in F corresponds to 100° change in temperature; at 7000°K the old and the new functions are practically identical.

The generalized thermal conductivity, Θ, requires alteration because of the changes made in the equilibrium constant and in the value used for the dissociation energy of the molecule. Furthermore, a reexamination of the entire function seems advisable at this point.

Riewe and Rompe[8] include in their calculation of the generalized thermal conductivity the contribution of charged particles to the diffusion term. It is easy to show that this contribution is negligible here, and hence is omitted. The second point which needs clarification is the use of the Sutherland equation rather than that due to Hassé and Cook[9] for computing the viscosity, which enters into the calculation of the ordinary conductivity. With respect to molecular nitrogen, the existing data on the viscosity are hardly adequate as a basis for decision between the use of the Sutherland equation and that of Hassé and Cook. However, Hassé and Cook state in their paper that the experimental points for air (which is not far different from molecular nitrogen) lie distinctly below their theoretical curve. It is undoubtedly true that the actual values lie somewhere between those predicted by the two equations. In view of these uncertainties, it seems hardly justifiable to employ an equation involving any more refinements than those possessed by the Sutherland equation. However, it seems wise, wherever possible, to compensate in other factors for the possibly low values of viscosity obtained from the Sutherland equation.

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[8] K. H. Riewe and R. Rompe, Zeits. f. Physik **105**, 478 (1937).

[9] H. R. Hassé and W. R. Cook, Proc. Roy. Soc. **A125**, 196 (1929).

In introducing the correction to take care of the contribution of the atomic gas to the ordinary conductivity, it is found that, if one uses actual values of the various quantities entering into the kinetic theory ratio of atomic to molecular conductivities, rather than purely kinetic theory values, this ratio is 1.1 instead of the 1.3 reported previously. For an *atomic* gas, the experience of Kenty[10] with mercury vapor indicates that the use of the Hassé and Cook formula is to be preferred. The constants for use in the Hassé and Cook formula for atomic nitrogen are not available, but the ratio of the viscosities obtained by the two formulas for *molecular* nitrogen are 1.31 and 1.55 at 3000°K and 7000°K, respectively. The ratios for atomic nitrogen should not be far different. Since specific information with which to perform the calculations using the formula of Hassé and Cook for atomic nitrogen is lacking, the conductivity of the atomic gas is obtained, certainly within the accuracy justified here, by increasing the ratio of molecular to atomic conductivity to 1.5. This same factor was employed without discussion in the earlier paper.

Concerning the diffusion term entering into the generalized conductivity, there are again two possible formulas for calculating the diffusion coefficient, one a Sutherland-like equation, the other an equation by Hassé and Cook.[11] Here again, the formula of Hassé and Cook predicts higher values than does that of Sutherland, but in view of the uncertainties involved in the long extrapolation from existing experimental data the Sutherland-type equation is employed here. Thermal diffusion is opposite in direction to concentration diffusion. An estimate of the magnitude of the thermal diffusion term was obtained using the formula developed by Furry, Jones, and Onsager.[12] Their formula predicts for the present case a thermal diffusion term amounting to about one percent of the term representing concentration diffusion over the range of temperatures employed here. This contribution is neglected.

The alterations in the diffusion term are confined, therefore, to changes in the concentration gradient introduced by the previously mentioned change in the value of the equilibrium constant, and by the above-mentioned improvement in the value of the dissociation energy of the molecule. The latter value is, of course, that for absolute zero, but the change in going from 0°K to 7000°K is only three percent, and is in a direction such as to reduce the heat loss by diffusion. The neglect of this change and the neglect of thermal diffusion both tend to increase the total heat loss by diffusion and hence to make its value intermediate between that predicted by the Sutherland equation and that predicted by the equation of Hassé and Cook.

Values of the generalized thermal conductivity including the above-mentioned alterations are presented in Table I.

CONVECTION

The fact that there is no adequate theory for free convection, even though the problem has merited the attention of many able investigators in the past,[13] presents unique difficulties in the determination of the free convection function σ. This is in sharp contrast with the preceding discussion where adequate theories were available for the other functions involved in the problem. Even with the artificial assumption of laminar flow of the fluid, the resulting equations for convection heat loss become prohibitively difficult of solution. Hence we were led to develop an empirical equation for σ which satisfies the requirements both of dimensional analysis and those imposed by the electric arc itself. As a guide in developing the heat loss function σ, recourse is had to information concerning total heat loss from solid cylinders. This has been dealt with in the past in two ways, the dimensional analysis originated by Rayleigh and developed further by many others, and the "stationary layer" theory developed by Langmuir. Of the two, the dimensional analysis has proved more useful for our present purposes, since no stationary sheath exists in the arc.

Dimensional analysis applied to the data on total heat loss from solid cylinders leads to the conclusion that

$$H/K=M(g\alpha\theta\rho^2d^3/\eta^2)\cdot N(\eta C_p/K), \qquad (6)$$

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[10] C. Kenty, J. App. Phys. **9**, 53 (1938).

[11] H. R. Hassé and W. R. Cook, Phil. Mag. **12**, 554 (1931).

[12] W. H. Furry, R. C. Jones and L. Onsager, Phys. Rev. **55**, 1083 (1939).

[13] See W. J. King, Mech. Engin. May (1932) for a general list of references.

where H is the total heat loss per unit length per unit temperature excess over surroundings, K is the thermal conductivity, g the acceleration due to gravity, α the temperature coefficient of expansion at constant pressure, θ the temperature excess, ρ and η the density and viscosity, respectively, C_p the specific heat, and d is the cylinder diameter. The function N is certainly independent of the particular gas for diatomic molecules and might be expected to vary but little for monatomic or polyatomic molecules. Dimensional plots of H/K *vs.* $g\alpha\theta d^3\rho^2/\eta^2$ for solid cylinders have been made using measured values of the various quantities involved, for cases in which the temperature excess θ is small in comparison with the absolute temperature of the cylinder. For these cases the various quantities involved have been evaluated at the mean temperature. The points so plotted lie on a surprisingly smooth curve, even though the range of values for the abscissa is extremely large. This dimensional method has been used successfully by Suits and his collaborators[5] for correlating arc data. Here the temperature excess θ is no longer negligible in comparison with the absolute temperature at the visual arc edge. He has found empirically that, instead of evaluating the quantities at the mean temperature, he must make the correlation using α evaluated at ambient temperature and the other quantities at the temperature of the arc edge. Here the quantity d is, of course, taken to be the arc diameter as determined either visually or photographically. It is important for our purpose to notice that, if α is evaluated at ambient temperature,

$$-\rho\alpha\theta = \rho_0 - \rho,$$

where ρ_0 is the density at ambient temperature. Hence the relationship used by Suits can be rewritten in the form

$$H/K = M(g(\rho_0-\rho)\rho d^3/\eta^2). \tag{7}$$

On the basis of Suits' analysis, any form selected for the function σ must be such that, when integrated to find the total heat loss, the integral will satisfy Eq. (7).

A number of different empirical functions, each satisfying Eq. (7), were tried until the function employed in this work was finally determined.

The first empirical function tested was the simplest, namely

$$\sigma = A\alpha(T)r, \tag{8}$$

where

$$\alpha(T) = g(\rho_0-\rho)Q/\eta \tag{9}$$

and Q is the heat content per unit volume (to be discussed later). A is a constant and the other quantities are as defined previously. The rapidity with which the function $\alpha(T)$ approaches zero in the vicinity of ambient temperature prevents the function σ from becoming large at large values of r, and causes it to vanish at ambient temperature. A few trial integrations with this function soon showed that, if A was made sufficiently large to ensure adequate power loss for small values of r, the temperature gradient changed sign long before room temperature was reached, which led to an infinite total power dissipation. It was thus apparent that, to avoid such catastrophes, a factor was needed to reduce the value of the function σ for large values of the radius while preserving its initial value. The dimensional restrictions imposed limit one to terms which are functions of $g(\rho_0-\rho)\rho r^3/\eta^2$. This function must have a form such that it will decrease rapidly with increasing values of the argument. Many such functions were tried, and those of exponential form were the most satisfactory. After careful examination of the various possible exponentials, the one finally adopted was

$$e^{-Bg(\rho_0-\rho)\rho r^3/\eta^2}, \tag{10}$$

where B is a constant; and hence, the next stage in the development of the heat loss function is

$$\sigma = A\alpha(T)re^{-B\beta(T)r^3}, \tag{11}$$

where $\beta(T)$ is written for $g(\rho_0-\rho)\rho/\eta^2$.

If one uses experimental values for the center temperature of the arc as initial conditions for the integration, this heat loss function is perfectly satisfactory out to radii such that beyond there is no further contribution to the electrical power generated. It was not possible, however, with any selection of the constants A and B to account for the observed power and *at the same time* satisfy the boundary conditions at room temperature. It might be expected that, as room temperature is approached, the heat loss function should reduce to a simpler form and thus a constant term was added to the exponential factor. The heat loss function finally arrived at, therefore, is

$$\sigma = A\alpha(T)r[e^{-B\beta(T)r^3}+C] \tag{12}$$

which, of course, reduces to the simple form (Eq. (9)) as the argument is increased. With this final function it is possible to account for the observed power loss per unit length and at the same time satisfy the outer boundary conditions.

It now remains to discuss the various quantities entering into the function σ. The first of these is the density, ρ. Perfect gas laws can, of course, be assumed for both atomic and molecular gases, and, hence, given their partial pressures, the expression for the density becomes

$$\rho = \frac{3.431(10^{-1})}{T}\left[(P-P_a)+\frac{P_a}{2}\right] \text{g/cm}^3. \tag{13}$$

The heat capacity per unit volume, Q, is given by

$$Q=(\rho-\rho_a)\int_{300}^{T} C_{p_2}dT+\rho_a\left[\int_{300}^{T} C_{p_1}dT+q\right], \tag{14}$$

where $2q$ is the dissociation energy of the molecule in joules per gram, 300°K is the ambient temperature employed in the calculations, C_{p_1} and C_{p_2} are the specific heats for atoms and molecules, respectively, and ρ_a is the partial density of atoms.

The calculation of the viscosity η for the molecule has been discussed earlier in this paper. The viscosity of the mixture may be closely approximated by taking the weighted average of the viscosities for the molecular and atomic gases. The viscosity of the atoms should be given more nearly by the equation of Hassé and Cook than by that of Sutherland. However, not only are the constants for use with the Hassé and Cook equation lacking, but the accuracy to be expected in the results hardly justifies the labor involved in the use of this formula. It is possible, however, from a critical examination of the situation, to accomplish the same purpose much more simply to well within the accuracy justified by these calculations. The ratio of the viscosities of the molecules as computed by the two equations has the value of 1.55 at 7000°K and 1.31 at 3000°K. This ratio should not be far different for the atomic gas. The ratio of atomic to molecular viscosities as given by kinetic theory is 0.84. Thus, if it is assumed that the viscosity of the mixture is the viscosity of a molecular gas as computed by the Sutherland equation, the resulting values of the viscosity for the mixture will be intermediate between those predicted by the Sutherland equation and those predicted by the equation of Hassé and Cook, approaching values predicted by the former at low temperatures and those predicted by the latter as the temperature is increased. This is believed to yield viscosity values for the mixture that approximate the true viscosities as closely as present knowledge warrants. Any residual errors remaining as a result of these assumptions will be taken care of, to a large extent, by the empirical selection of the constants in the function σ.

Values of the functions $\alpha(T)$ and $\beta(T)$ computed by the method outlined above are presented in Table I.

The solution of the complete arc equation, Eq. (2), was obtained by the method of numerical integration described in the earlier paper. This method, as was pointed out there, is equivalent to a three-term Taylor expansion about each selected value of r. Linear interpolation was used with the functions presented in Table I. The accuracy of the integration procedure was tested by halving the intervals in r for a particular case and again carrying out the integration. The two solutions gave temperature distributions agreeing at every point to considerably less than one percent and hence the procedure was considered entirely adequate.

The three arbitrary constants appearing in the convection heat loss function σ were determined in the following way. The function σ splits up naturally into two parts. Within radii beyond which there is no appreciable contribution to the power generation, the constant C is negligible in comparison with the exponential. Hence pairs of values of the constants A and B were chosen and the integration of Eq. (2) performed for various values of E, using in each case experimental values for the center temperature and carrying out the integration sufficiently far to ensure complete power generation. From this, a single pair of values for the constants A

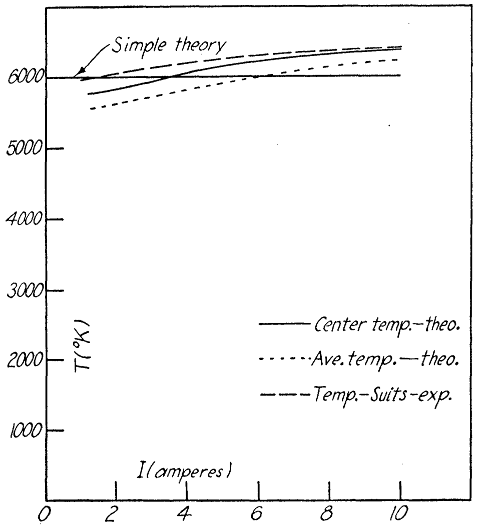

FIG. 1. Arc temperatures as a function of arc current.

and B was chosen which gave the best values for the power loss over the entire range. This condition, together with the restriction that dT/dr must not change sign, was sufficiently stringent to keep all possible pairs of values for A and B within small limits, and to demand that each separately be determined to at least two significant figures. After setting these two constants, the constant C was determined by ensuring that dT/dr vanish at ambient temperature for the ten-ampere point. Both the temperature distribution and power loss were slightly better using a center temperature at the ten-ampere point of 6350°K instead of the 6400°K determined by Suits in the manner described previously. This requirement was sufficiently rigid to fix C to at least two significant figures.

The constants thus determined are: $A=0.21$; $B=0.127$; $C=0.0044$.

Thereafter, without changing the values of A, B, or C, a value of center temperature, for each value of E selected, was sought which would satisfy only the outer boundary conditions. Such values, after considerable laborious computation, were found, and, as formerly, in each case

$$2\pi\int_0^\infty E^2 Frdr,$$

obtained in the course of the calculations, represents the total power generation per unit length.

In our previous paper, it was not possible to use the boundary condition that dT/dr vanish at ambient temperature because of lack of detailed knowledge concerning convection. As a substitute for this boundary condition, recourse was had to a maximum condition imposed on the total power. This, of course, was simply an artifice by means of which a reasonable solution of the arc problem could be obtained applying only to low currents. The authors now believe that the success which resulted from the use of this maximum condition was due to the existence of a sharp maximum in the generalized conductivity tending to tie the arc temperature to a particular value. We do not believe that any general maximum principle, other than that imposed by the peculiar nature of the conductivity, applies to this problem. To anticipate a little, the more extended theory presented in this paper is more satisfactory than was that of the previous paper not only because of its more extensive application, but because of the fact that it predicts decreasing temperature with decreasing current.

RESULTS AND CONCLUSIONS

The arc temperature as a function of arc current is presented in Fig. 1. The solid line represents the center temperature as determined from the theory. The dashed line is a plot of temperature determined by Suits[14] on the assumption that the product of the visual cross-sectional area, the electric intensity, and the current density per unit gradient at *that* temperature equals the measured current. The dotted line represents "average" temperatures obtained from the theory in the following manner. In the previous paper, an "average" cross section was calculated and the temperature appropriate for that cross section was used as the average temperature of the arc. Slightly better agreement is attained, if one calculates from the theory an "average" *radius* rather than an "average" cross section, and defines the temperature appropriate for this radius as the "average" temperature. The "average" radius, so defined, is given by

$$\bar{r}=\int_0^\infty Fr^2dr\Big/\int_0^\infty Frdr \qquad (15)$$

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[14] C. G. Suits, J. App. Phys. **10**, 728 (1939).

and the "average" temperature, $\bar{T}$, is determined by

$$\pi[\bar{r}]^2EF(\bar{T})=I, \qquad (16)$$

where I is the current.

As can be seen from the diagram, the experimental measurements of Suits, the center temperatures and "average" temperatures determined from theory all agree over the current range from one to ten amperes within the uncertainties inherent in the experiments and in the definitions of the various temperatures. The temperatures obtained from the theory decrease in a manner to be expected with decreasing current from ten to one ampere.

Computations below one ampere, however, show that with the constants selected it is impossible to find a center temperature for the arc such that the outer boundary condition is satisfied. Mr. J. J. Hopkins, working in this laboratory, was unable to maintain arcs of sufficient length to ensure a uniform and well-defined positive column of the type postulated in the theory for currents below one ampere. It is to be noticed also that in the published work of Suits no data is presented for currents below one ampere at atmospheric pressure. Hence this theory predicts the impossibility of maintaining well-defined and steady positive columns in arcs running below one ampere. Since this prediction refers to a phenomenon entirely distinct from those which we set out to explain, it provides an independent justification for the theory.

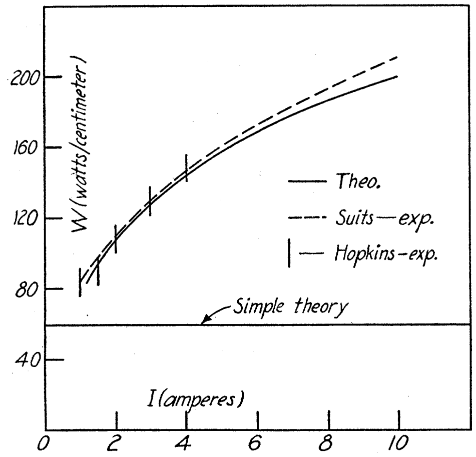

FIG. 2. Power dissipation per unit length as a function of arc current.

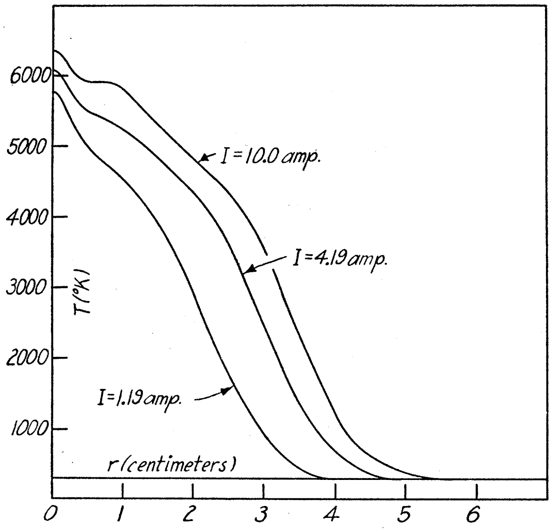

FIG. 3. Temperature distribution as a function of radius for three values of arc current.

There is no ambiguity in terminology in discussing power loss per unit length. It can be measured experimentally and can be determined from theory. The results of both are presented in Fig. 2, where the solid curve is that obtained from theory, and the dotted curve was plotted using the equation given by Suits[2] as the best representation of his experimental data. In his paper, the actual experimental points are presented. These are not reproduced in Fig. 2, but do lie on both sides of the theoretical curve over the entire current range. Values of the power loss obtained by Hopkins in this laboratory are included in this plot as vertical lines, the length of each line giving the uncertainty in the measurement. Both Suits' and Hopkins' arcs were believed to be metal-vapor-free. The agreement among these three sets of determinations is well within experimental limitations. A reexamination of the simple theory presented in the earlier paper in the light of the changes made in the various functions yields a value for the power loss per unit length which is included as a horizontal line in the diagram. The new center temperature determined from the simple theory is 6000°K, a value remarkably close to that observed at the bottom of the experimental current range. This value is shown as a solid line in Fig. 1.

As a matter of interest, the temperature distribution curves for three values of current are presented in Fig. 3. There are, of course, no published reliable measurements of temperature distribution in the free-burning arc for comparison with theory. The two arc radii are presented in Fig. 4. The lower curve represents the visual arc radius as determined by Suits, and the upper curve, the "average" arc radius as determined from theory. As was mentioned earlier, because of lack of knowledge concerning the dependence of radiation on temperature, the visual arc radius could not be determined from theory. The visual arc radius and the "average" arc radius need not necessarily coincide for the reason that they are defined in different ways. However, they should, and do, show the same general trend, so that, although Fig. 4 cannot be used for quantitative comparison between theory and experiment, the agreement in general behavior between the two curves does provide further confirmation of the theory. It is to be noticed in Fig. 3 that the temperature variation over the visual arc radius amounts in most cases to less than 500°, a result in general agreement with the opinion held by Suits from a critical examination of the photographs used in his sound velocity measurements.

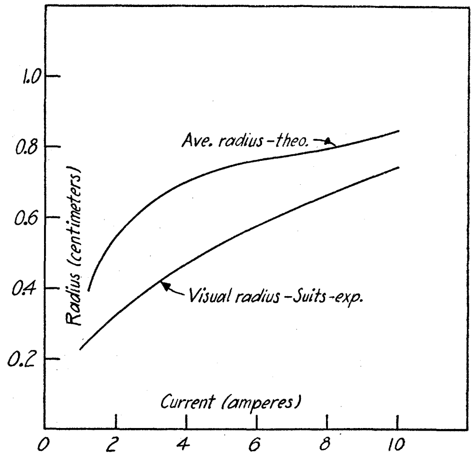

FIG. 4. Arc radii as a function of arc current.

Suits' measurements of luminous efficiencies in arcs[15] indicate that radiation is still negligible at ten atmospheres. Hence some preliminary calculations have been carried out for the ten atmosphere arc using the values of the constants A, B, and C determined at atmospheric pressure in the ten atmosphere convection heat loss function. The results obtained were in surprisingly good agreement with experiment both in temperature and power. The extension of the empirical heat loss function from one to ten atmospheres involves considerable extrapolation, since the gas density occurs essentially squared. The resulting agreement between theory and experiment, therefore, provides additional and almost conclusive evidence for the belief that the heat loss function determined empirically in this work cannot differ greatly from the actual heat loss function.

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[15] C. G. Suits, J. App. Phys. 10, 730 (1939).