A Note on Nuclear Isomerism

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It is suggested that nuclear isomerism may be due to the fact that the two lowest nuclear states have zero angular momentum and opposite parity. The lifetime of the first excited state is estimated under these conditions for the emission of two quanta and for the ejection of two electrons from the K or L shell. The possibility of distinguishing experimentally between this hypothesis and that of Weizsäcker is discussed.

WEIZSÄCKER[1] has suggested that nuclear isomerism can be explained by assuming that the angular momentum of the isomeric nucleus in its first excited state differs considerably from the angular momentum in the ground state. Under this condition, since radiation transitions between the two states are strongly forbidden, the first excited state is metastable. The degree of metastability has been estimated by Bethe[2] for γ-ray transitions involving several units of angular momentum. He found a half-life of 2×10^8 years for an energy difference of 0.025 Mev and for $\Delta l=5$. However, Hebb and Uhlenbeck[3] have pointed out that the possibility of radiationless transitions with the ejection of extranuclear electrons[4] shortens the lives, with the result that the lifetime is of the order of 10^4 years for the same change in energy and angular momentum.

There is, however, another possible explanation of the isomerism. If one assumes that the ground state and the first excited state of the nucleus have zero angular momentum *and* that they have opposite parity, the transition between them is forbidden for γ-radiation, direct ejection of an atomic electron, or pair production.[5] The transition can take place only by means of second-order effects such as two-quanta emission, two-electron ejection, double pair formation, etc. The lifetime for these transitions can be made to be very large by choosing the energy difference between the two nuclear states to be sufficiently small. It turns out that it is necessary to choose this energy difference to be less than 2 mc² in order to obtain sufficiently long lives; therefore it will not be necessary to consider the possibility of pair formation.

Two-Quanta Emission

The theory of the simultaneous emission of two quanta has been carried out by Goeppert-Mayer.[6] She gives for the transition probability

$$w_q=\frac{16}{9}\frac{(2\pi)^4}{\hbar^2c^6}\int_0^{\nu_{n_0n}}\nu'^3\nu^3\left[\frac{P_{n_0n'}P_{n'n}}{\nu_{n'n}-\nu}+\frac{P_{n_0n'}P_{n'n}}{\nu_{n'n_0}+\nu}\right]^2d\nu, \qquad (1)$$

where n_0, n', n refer to the initial, intermediate, and final states, respectively. It is assumed that n' is a state of higher energy than n_0 so that direct transitions from n_0 to n' cannot take place. Actually, there are several states n', but it is necessary to consider only one of them in estimating the order of magnitude of w_q.

In order to have as large a transition probability as possible, the angular momentum in the intermediate state is assumed to be unity; and, for the sake of definiteness, the parity of this state may be taken to be opposite to that of the initial state. Then the first transition ($n_0\rightarrow n'$) is an electric dipole transition, but the second ($n'\rightarrow n$) must be magnetic dipole since it is even→even or odd→odd with $\Delta j=1$. The (n', n) matrix element of the dipole moment, P, in (1) must therefore be replaced by the matrix element of the magnetic dipole moment.

[1] C. F. v. Weizsäcker, Naturwiss. **24**, 813 (1936).
[2] H. A. Bethe, Rev. Mod. Phys. **9**, 69 (1937).
[3] M. H. Hebb and G. E. Uhlenbeck, Physica **5**, 605 (1938). S. M. Dancoff and P. Morrison, Phys. Rev. **55**, 122 (1939).
[4] H. M. Taylor and N. F. Mott, Proc. Roy. Soc. **142**, 215 (1933).
[5] H. Yukawa and S. Sakata, Proc. Phys. Math. Soc. Japan **17**, 397 (1935): They show that the effective potential acting on the pair for a 0→0 transition is spherically symmetrical; thus it is even under inversion through the origin. Such a potential cannot give rise to the necessary odd→even or even→odd transition.
[6] M. Goeppert, Naturwiss. **17**, 932 (1929). M. Goeppert-Mayer, Ann. d. Physik **9**, 273 (1931).

TABLE I. *Mean lives for two-quantum emission and two-electron ejection (K and L electrons) for different separations, E_0-E, of the O_u and O_g states of the nucleus. The last column contains the results of Hebb and Uhlenbeck for a nuclear transition involving 5 units of angular momentum.*

E_0-E VOLTS	τ_q SEC.	τ_{eK} SEC.	τ_{eL} SEC.	$\tau_{\Delta l=5}$ SEC.
10^5	4×10^2	2×10^6	2×10^7	3×10^6
5×10^4	5×10^4	5×10^7	2×10^8	3×10^8
2.5×10^4	6×10^6	10^{10}	2×10^9	3×10^{11}
10^4	4×10^9	—	4×10^{10}	—
5×10^3	5×10^{11}	—	7×10^{11}	—

Taylor[7] has indicated that the most important contribution to the magnetic dipole radiation of a nucleus is due to proton (or neutron) spin transitions. Therefore, as a rough approximation, the magnetic moment of the proton ($\approx 3eh/2$Mc) may be substituted for the matrix element of the magnetic moment. Similarly, the matrix element P_{n_0n} is replaced by eR_r, where e is the charge of the proton and R_r is the "radiation radius" of the nucleus, i.e., that radius which gives the right order of magnitude for ordinary γ-ray transition probabilities. Eq. (1) then becomes

$$w_q\approx 4(2\pi)^4\frac{e^4R_r^2}{M^2c^8}\nu^2{}_{n'n}\nu^3{}_{n_0n}\int_0^1\frac{x^3(1-x)^3dx}{(\alpha-x)^2(\beta+x)^2},$$

with $\alpha=\nu_{n'n}/\nu_{n_0n}$, $\beta=\alpha-1$.

The lifetime is large; that is, the transition probability is small, only if ν_{n_0n} is sufficiently small. For ν_{n_0n} less than $\nu_{n'n}$, w_q is given by

$$w_q\approx 10^{59}(h\nu_{n_0n})^7(h\nu_{n'n})^{-2}R_r^2,\quad \nu_{n_0n}\ll\nu_{n'n}. \quad (2)$$

In order to obtain an estimate of R_r, the transition probability for ordinary (first-order) γ-radiation may be set equal to $2e^2\nu^3R_r^2/3\hbar c^3$. For 1-Mev γ-rays ($\nu=3\times10^{20}$ sec.$^{-1}$), this probability is approximately 10^{13} sec.$^{-1}$, or

$$2e^2R_r^2\hbar^{-1}c^{-3}(3\times10^{20})^3=3\times10^{13}$$

and $R_r\approx 2.5\times10^{-13}$ cm. Substituting this value in Eq. (2), the two-quantum emission probability becomes

$$w_q\approx 6.8\times10^{33}(h\nu_{n_0n})^7(h\nu_{n'n})^{-2}.$$

Approximate values of $\tau_q=w_q^{-1}$ based on this estimate are listed in Table I for $h\nu_{n'n}=1$ Mev.

[7] H. M. Taylor, Proc. Camb. Phil. Soc. **32**, 291 (1936).

TWO-ELECTRON EJECTION

According to the results of Hebb and Uhlenbeck, Dancoff and Morrison,[3] one would expect that the effect of ejection of extranuclear electrons by direct interaction with the nucleus is of the same order of magnitude as the quantum effect since we are dealing with transitions for which $\Delta l=1$. That this is actually the case is shown by the following estimate.

Consider first the simultaneous ejection of two K electrons, and assume that the system of nucleus plus electrons is contained in a "box" of volume V. The transition probability can then be calculated in a manner analogous to the two-quantum case, and, as is to be expected, it turns out to be independent of V. The result is[8]

$$w_{eK}=\frac{2\pi}{\hbar}\int_0^{E_0-E-2\epsilon_K}\rho(\epsilon')\rho(\epsilon)\left|\frac{H_{n_0n'}H_{n'n}}{E_0-E'-\epsilon_K-\epsilon'}\right|^2 d\epsilon, \quad (3)$$

where E_0, E', E are the energies in the initial, intermediate, and final nuclear states; ϵ_K is the ionization potential of the K shell; ϵ, ϵ' are the kinetic energies of the ejected electrons, $\epsilon+\epsilon'=E_0-E-2\epsilon_K$; and $\rho(\epsilon)$ is the density of electronic states of kinetic energy ϵ,

$$\rho(\epsilon)=2(2\epsilon)^{\frac{1}{2}}m^{\frac{3}{2}}V/(2\pi)^2\hbar^3$$

for free electrons in a box of volume V.

The electronic transition corresponding to $n_0\rightarrow n'$ may be assumed to be even$\rightarrow$odd, $\Delta l=1$. Then the potential acting on the electron is due to the nuclear dipole moment induced by the nuclear $S\rightarrow P$ transition. The matrix element $H_{n_0n'}$ therefore has the form

$$\int\psi_{n_0}\frac{eP_0}{r^2}\psi_{n'}r^2dr=eP_0\int\psi_{n_0}\psi_{n'}dr$$

where P_0 is the effective dipole moment of the nucleus and ψ_{n_0}, $\psi_{n'}$ are the electronic radial wave functions. Since $\psi_{n'}$ is a P function, it has a node of order r/λ', $\lambda'=h/(2m\epsilon')^{\frac{1}{2}}$, at the nucleus, and we may set

$$H_{n_0n'}=(eP_0/\lambda')\int\psi_{n_0}\varphi rdr$$

[8] There is an additional term in w_{eK} corresponding to the two transitions taking place in the opposite order. However, this simply introduces a factor of order two in the estimated transition probabilities.

where $(r/\lambda')\varphi=\psi_{n'}$. For $r>a_K$, where a_K is the radius of the K shell, $\psi_{n_0}\approx 0$, so the range of integration is approximately[9] $r=0$ to $r=a_K$. In this range the functions may be taken to be constant since they are slowly varying functions of r; thus we obtain $H_{n_0n'}\approx eP_0a_K{}^2/\lambda' a_K{}^{\frac{3}{2}}V^{\frac{1}{2}}$, where $2a_K{}^{-\frac{3}{2}}$, $V^{-\frac{1}{2}}$ are the normalization factors for ψ_{n_0}, $\psi_{n'}$, respectively.

The effective dipole moment of the nucleus is again given by $P_0=eR_r$, $R_r\approx 2.5\times 10^{-13}$ cm, since the nuclear transition here is identical with the transition that gives rise to the electric dipole radiation. The matrix element $H_{n_0n'}$ becomes

$$H_{n_0n'}\approx e^2R_ra_K{}^{\frac{1}{2}}V^{-\frac{1}{2}}(2m\epsilon')^{\frac{1}{2}}h^{-1}. \quad (4)$$

The second electronic transition is necessarily odd→odd, which means that the electronic orbital angular momentum remains unchanged (since the parity remains unchanged) but the spin of the electron is turned over. The interaction in this case is therefore due to spin-spin coupling between the nucleus and the electron, and the calculation of the matrix element is similar to that used to obtain the hyperfine structure separation. The order of magnitude of the hyperfine structure separation in an S state is given by[10] $(8\pi/3)\mu_n\mu_e\psi^2(0)$ where μ_n, μ_e are the nuclear and electronic magnetic moments, and $\psi(0)$ is the value of the electronic wave function at the origin. Since we are interested in the nondiagonal matrix element rather than the diagonal one, $\psi^2(0)$ must be replaced by $\psi_{n'}(0)\psi_n(0)$, and μ_n by the effective nuclear magnetic moment, which is again assumed to be due to a single proton. We now have

$$H_{n'n}\approx 2(e\hbar/2c)^2(a_K{}^{\frac{3}{2}}V^{\frac{1}{2}}mM)^{-1}. \quad (5)$$

Making use of Eqs. (4) and (5), Eq. (3) becomes

$$w_{eK}\approx\frac{4R_r{}^2e^4m^2(e\hbar/c)^4}{(2\pi)^5a_K{}^2\hbar^9M^2}\times\int_0^{E_0-E-2\epsilon_K}\frac{\epsilon^{\frac{1}{2}}\epsilon'^{\frac{3}{2}}d\epsilon}{(E_0-E'-\epsilon_K-\epsilon')^2} \quad (6)$$

or

$$w_{eK}\approx 4\times 10^8(R_r/a_K)^2(E_0-E-2\epsilon_K)\int_0^1\frac{x^{\frac{1}{2}}(1-x)^{\frac{3}{2}}dx}{(\alpha-x)^2},$$

where $\alpha=(E'-E-\epsilon_K)(E_0-E-2\epsilon_K)^{-1}$. The result is independent of the volume as expected.

The interesting case again occurs for $E'-E\gg E_0-E$, or $\alpha\gg 1$. w_{eK} then becomes

$$w_{eK}\approx 4\times 10^8(R_r/a_K)^2(E_0-E-2\epsilon_K)^3\times(E'-E-\epsilon_K)^{-2}\int_0^1 x^{\frac{1}{2}}(1-x)^{\frac{3}{2}}dx.$$

The integration can be carried out by elementary methods and the result is

$$w_{eK}\approx(\pi/4)\times 10^8(R_r/a_K)^2(E_0-E-2\epsilon_K)^3\times(E'-E-\epsilon_K)^{-2}. \quad (7)$$

The radius of the K shell is $a_K=\hbar^2/Zme^2$, and for $Z\approx 50$, $a_K\approx 10^{-10}$ cm, $R_r/a_K\approx 2.5\times 10^{-3}$, so the transition probability is

$$w_{eK}\approx 4.9\times 10^2(E_0-E-2\epsilon_K)^3(E'-E-\epsilon_K)^{-2}.$$

When the available energy, E_0-E, is less than twice the ionization potential of the K shell, the ejection of two K electrons cannot take place. Electrons can, however, be ejected from the L_1 shell and the probability for this process is not much smaller than for the K ejection.[3] In order to estimate the probability when both electrons are ejected from the L_1 shell, it is only necessary to replace a_K and ϵ_K in Eq. (7) by the radius and ionization potential of the L_1 shell. Taking $R_0/a_L=0.0625\times 10^{-2}$, Eq. (7) now becomes

$$w_{eL}\approx 3.1\times 10(E_0-E-2\epsilon_L)^3(E'-E-\epsilon_L)^{-2}.$$

Values of $\tau_{eK}=w_{eK}{}^{-1}$ and $\tau_{eL}=w_{eL}{}^{-1}$ are given in Table I for $E'-E=1$ Mev, $\epsilon_K=10^4$ volts, and $\epsilon_L=10^3$ volts. It is apparent that the ejection of K electrons is not important because of the high ionization potential of the K shell. The L_1 ejection is of the same order of magnitude as the quantum effect for E_0-E between 10^3 and 10^4 volts, but for E_0-E less than 10^3 volts, it cannot take place since the available energy is less than the ionization potential of the L shell.

The results of Hebb and Uhlenbeck[3] for nuclear transitions involving a change of angular momentum of 5 units are reproduced in the last

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[9] In general the range of integration is given by the smaller of a_K, λ'. The electronic energies, ϵ', that enter here will be no greater than 10^5 volts, so $\lambda'\gtrsim h(2m\cdot 10^5\cdot 1.59\times 10^{-12})^{-\frac{1}{2}}\approx 4\times 10^{-10}$ cm. On the other hand, $a_K=\hbar^2/Zme^2$, and for $Z\approx 50$, $a_K\approx 10^{-10}$ cm.

[10] E. Fermi, Zeits. f. Physik **60**, 320 (1930). G. Breit, Phys. Rev. **37**, 51 (1931).

TABLE II. *Energy distribution of ejected K electrons for a $O_u \rightarrow O_g$ transition; $k = 8\times10^4$ volts. w_{eK} is symmetrical with respect to $\epsilon = 4\times10^4$ volts.*

ϵ VOLTS	$w_{eK}(\epsilon)$ (ARBITRARY UNITS)
0	0
0.5×10^4	15.5
1×10^4	21.1
2×10^4	27.3
3×10^4	31
4×10^4	32

column of the table. These values of the lifetime depend very strongly on the special assumptions that have been made about the structure of the nucleus,[3] because they are determined by the multipole moments of high order which depend only on the detailed structure of the nucleus. The values given here are based on the model used by Bethe[2] in calculating the multipole moments of nuclei. The table indicates that the lifetime for the $O_u \rightarrow O_g$ transition is as large as the lifetime for the $\Delta l = 5$ transition only if the energy difference $E_0 - E$ for the former case is several times smaller than for the latter.

The transition $n_0 \rightarrow n$ can also take place by means of combination processes such as the ejection of one quantum and one electron. The probabilities for these transitions may be estimated by taking the geometric mean of the probabilities for the "pure" transitions. However, this will not introduce anything new since these mixed probabilities will necessarily lie between the "pure" probabilities.

EXPERIMENTAL POSSIBILITIES

In recent experiments,[11] internal conversion electrons have been found that can be accounted for only by assuming that they are ejected in a transition from one isomeric form of the nucleus to another. The question immediately arises as to whether these experiments are in agreement with the Weizsäcker hypothesis (I) the one suggested here (II), or both.

The difference between the two hypotheses should show up in two respects. In the first place, it has been shown[3] that the internal conversion coefficient increases with an increase in the angular momentum change associated with the nuclear transition. Therefore one would expect a somewhat larger internal conversion coefficient for I than for II since II involves angular momentum transitions of one unit, at most, and *I* involves changes of three or four units.

The other, and more important, difference is found in the energy distribution of the ejected electrons. According to *I*, a single electron is ejected with an energy equal to the available energy minus the ionization potential of the electron: therefore the distribution curve should show a sharp peak at this energy. On the other hand, in any double process such as two electron ejection, the energy is distributed between the two particles; so, according to the mechanism II, the distribution curve should be broad. The shape of the curve is given by Eq. (6) if we write

$$w_{eK} = \int_0^{E_0 - E - 2\epsilon_K} w_{eK}(\epsilon, \epsilon')d\epsilon.$$

Then

$$w_{eK}(\epsilon, \epsilon') \sim \epsilon^{\frac{1}{2}}\epsilon'^{\frac{3}{2}}(E' - E - \epsilon_K - \epsilon)^{-2},$$

with $\epsilon + \epsilon' = E_0 - E - 2\epsilon_K$. Neglecting ϵ and ϵ_K as compared to $E' - E(=1 \text{ Mev})$, $w_{eK}(\epsilon, \epsilon') \sim \epsilon^{\frac{1}{2}}(k-\epsilon)^{\frac{3}{2}}$, $k = E_0 - E - 2\epsilon_K$. The quantity that is actually measured is

$$w_{eK}(\epsilon) = w_{eK}(\epsilon, \epsilon') + w_{eK}(\epsilon', \epsilon)$$

since the two electrons cannot be distinguished from one another. This has been tabulated in Table II on an arbitrary scale for $E_0 - E = 10^5$ volts, $\epsilon_K = 10^4$ volts ($k = 8\times10^4$ volts). Since $w_{eK}(\epsilon)$ is symmetrical about $\epsilon = 4\times10^4$ volts, its values for ϵ greater than this have not been given. The distribution apparently is very flat: it has a half-value width of about 7×10^4 volts. II should therefore be easily distinguishable from I.

Those distribution curves that have been measured[11] show the discrete electron spectrum that is characteristic of *I* so that, at present, there appears to be no experimental support for the hypothesis II. However, the experiments have been carried out for very few nuclei, and, since the two hypotheses are not mutually exclusive, it is possible that nuclei satisfying the conditions specified by II may be found.

The author wishes to thank Professor Teller for suggesting this mechanism for nuclear isomerism to him and for discussing it with him.

[11] G. E. Valley and R. L. McCreary, Phys. Rev. **56**, 863 (1939). A summary of the other experimental material is given by B. Pontecorvo, Nature **144**, 212 (1939).