the components of $t_\mu{}^\nu$ all vanish for $r>2m$. If one wishes to avoid this, one must give up (29) and (30).

§6

Enough has been given to show that there are advantages from the formal point of view in introducing the Euclidean $\gamma_{\mu\nu}$ into the general relativity theory. It imparts tensor character to quantities which otherwise do not have it, and allows additional conditions to be imposed on the field so as to restrict the form of the solution for a given physical situation.

In conclusion, it is necessary to point out that, having once introduced $\gamma_{\mu\nu}$ into the theory, one has a great number of new tensors and scalars at one's disposal. One can, therefore, set up other field equations than (2). It is possible that some of these may be more satisfactory for the description of nature. Further investigation is here required.

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General Relativity and Flat Space. II

N. Rosen
Massachusetts Institute of Technology, Cambridge, Massachusetts

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The possibility is considered of interpreting the formalism of the general theory of relativity in terms of flat space, the fundamental tensor $g_{\mu\nu}$ being regarded as describing the gravitational field but having no direct connection with geometry. The resulting theory in general leads to the same predictions as the Einstein theory, but there are cases where the predictions differ. The present theory may explain the principal results obtained by D. C. Miller in his "ether-drift" experiments. The implications of the theory for cosmology are briefly touched upon.

§1

In a previous paper[1] (hereafter referred to as I) it was shown that it is useful to introduce into the general theory of relativity the concept of the existence at each point of space-time of a Euclidean metric tensor $\gamma_{\mu\nu}$ in addition to the usual Riemannian metric tensor $g_{\mu\nu}$. From the standpoint of the general theory of relativity, one must look upon $\gamma_{\mu\nu}$ as a fiction introduced for mathematical convenience. However, the question arises whether it may not be possible to adopt a different point of view, one in which the metric tensor $\gamma_{\mu\nu}$ is given a real physical significance as describing the geometrical properties of space, which is therefore taken to be flat, whereas the tensor $g_{\mu\nu}$ is to be regarded as describing the gravitational field.[2]

It has been pointed out in I that the introduction of $\gamma_{\mu\nu}$ leads to the possibility of other laws than those adopted in general relativity. In the present paper, however, no attempt will be made to change the laws of the latter, since they form a self-consistent system and have proved to be quite satisfactory for the description of large scale phenomena, at any rate.

§2

As far as the field equations are concerned, it is immaterial what interpretation one gives to the variables involved. This is not the case with the equations of motion. Let us therefore consider the law of motion for a particle in the field. The latter is given in the general theory of relativity by the equation of the geodesic[3]

$$\frac{d^2x^\mu}{ds^2}+\left\{\begin{matrix}\mu\\ \alpha\ \beta\end{matrix}\right\}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}=0, \tag{1}$$

where ds is the line element, defined in terms of the tensor $g_{\mu\nu}$, (I (1)). Let us now introduce as independent variable the Euclidean line element

[1] N. Rosen, Phys. Rev. **57**, 146 (1940).

[2] In some respects this resembles the theory of gravitation proposed by Nordström (cf. report by M. v. Laue, Jahrbuch f. Rad. u. El. **14**, 263 (1917)). It will be seen that there are important differences, however.

[3] A. Einstein, Ann. d. Physik **49**, 769 (1916).

$d\sigma$, defined in terms of the tensor $\gamma_{\mu\nu}$, (I (6)), instead of ds. After canceling out a factor $d\sigma/ds$, the equation can be written

$$\frac{d}{d\sigma}\left(\frac{d\sigma}{ds}\frac{dx^\mu}{d\sigma}\right)+\left\{\begin{matrix}\mu\\ \alpha\ \beta\end{matrix}\right\}\frac{dx^\alpha}{d\sigma}\frac{dx^\beta}{d\sigma}\frac{d\sigma}{ds}=0. \tag{2}$$

This can be rewritten

$$\frac{d^2x^\mu}{d\sigma}+\Gamma^\mu{}_{\alpha\beta}\frac{dx^\alpha}{d\sigma}\frac{dx^\beta}{d\sigma}=\lambda\frac{dx^\mu}{d\sigma}-\Delta^\mu{}_{\alpha\beta}\frac{dx^\alpha}{d\sigma}\frac{dx^\beta}{d\sigma}, \tag{3}$$

where $$\lambda\equiv\frac{d^2s}{d\sigma^2}\Big/\frac{ds}{d\sigma}\equiv-\frac{d^2\sigma}{ds^2}\Big/\left(\frac{d\sigma}{ds}\right)^2, \tag{4}$$

and $\Gamma^\mu{}_{\alpha\beta}$ and $\Delta^\mu{}_{\alpha\beta}$ are given by I, (7) and (8).

If we multiply (3) by $\gamma_{\mu\nu}dx^\nu/d\sigma$, the left-hand member vanishes as a consequence of the definition of $d\sigma$. Hence it is necessary that

$$\lambda=\gamma_{\tau\nu}\Delta^\tau{}_{\alpha\beta}\frac{dx^\alpha}{d\sigma}\frac{dx^\beta}{d\sigma}\frac{dx^\nu}{d\sigma}, \tag{5}$$

which gives the relation between s and σ. If this expression for λ is substituted into (3), one gets an equation from which s has been completely eliminated.

We can get the equation in a more familiar form if we go back to (2), multiply it through by a constant m_0, and write

$$m\equiv m_0 d\sigma/ds, \tag{6}$$

where m will be considered as the "proper mass" of the particle. Then the equation can be written as

$$\frac{d}{d\sigma}\left(m\frac{dx^\mu}{d\sigma}\right)+m\Gamma^\mu{}_{\alpha\beta}\frac{dx^\alpha}{d\sigma}\frac{dx^\beta}{d\sigma}=-m\Delta^\mu{}_{\alpha\beta}\frac{dx^\alpha}{d\sigma}\frac{dx^\beta}{d\sigma}. \tag{7}$$

On the left we have the absolute derivative with respect to σ of

$$p^\mu\equiv m\frac{dx^\mu}{d\sigma}\left(\equiv m_0\frac{dx^\mu}{ds}\right), \tag{8}$$

which is the energy-momentum vector. Hence the right-hand member is to be interpreted as the gravitational force. We see from (7) that, although inertial and gravitational mass are equal, they are not necessarily constant.

If we define the scalar

$$w^2=g_{\mu\nu}\frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\sigma}, \tag{9}$$

then (6) can be written

$$m=m_0/w \tag{10}$$

as the law of dependence of proper mass on the gravitational field.

Conversely, if we start from (7), multiply by $g_{\mu\nu}dx^\nu/d\sigma$ and make use of I (7), we get

$$\tfrac{1}{2}m dw^2/d\sigma=-w^2dm/d\sigma, \tag{11}$$

which is equivalent to (10).

Thus we see that, from the standpoint of flat space, one can put the equations of motion of a particle into the Newtonian form (generalized to four dimensions) if we assign to the particle a variable mass. In a coordinate system in which the particle is at rest the dependence of mass on the field is given by

$$m=m_0(\gamma_{44}/g_{44})^{\frac{1}{2}}. \tag{12}$$

In a weak gravitational field this can be written in first approximation[4]

$$m=m_0(1-\varphi), \tag{13}$$

where φ is the gravitational potential.

In the case of a light ray we have the additional condition (in the presence of a gravitational field)

$$w=0. \tag{14}$$

We therefore let m_0 vanish also, but take m, as given by (10), to be finite and constant. This allows one to cancel out the m's in (7). That (14) is consistent with (7) is to be seen from (11).

§3

To see the connection between Eq. (7) and the gravitational field equations, (I (3)), let us suppose we have a stream of particles each of which moves according to (7). Let n_0 be the proper particle density and let us assume that all quantities that we may need are smooth functions of the coordinates. We write (7) in the form

$$\left(m\frac{dx^\mu}{d\sigma}\right)_{,\alpha}\frac{dx^\alpha}{d\sigma}=-m\Delta^\mu{}_{\alpha\beta}\frac{dx^\alpha}{d\sigma}\frac{dx^\beta}{d\sigma} \tag{15}$$

and multiply through by n_0. On the left we have

$$\left(mn_0\frac{dx^\mu}{d\sigma}\frac{dx^\alpha}{d\sigma}\right)_{,\alpha}-m\frac{dx^\mu}{d\sigma}\left(n_0\frac{dx^\alpha}{d\sigma}\right)_{,\alpha}.$$

[4] Since it is shown, reference 3, that for a weak field $g_{44}\doteq 1+2\varphi$ in Galilean coordinates.

The scalar $(n_0(dx^\alpha/d\sigma))_{,\alpha}$ can be evaluated in a Galilean coordinate system. Thus

$$\left(n_0\frac{dx^\alpha}{d\sigma}\right)_{,\alpha}=\left(n_0\frac{dt}{d\sigma}\frac{dx^\alpha}{dt}\right)_{,\alpha}=\left(n\frac{dx^\alpha}{dt}\right)_{,\alpha}, \quad (16)$$

where t is the time coordinate and

$$n\equiv n_0dt/d\sigma=n_0/(1-v^2)^{\frac{1}{2}} \quad (17)$$

is the particle density in a reference frame with respect to which the particles at the point in question have a velocity v, the velocity of light being taken as unity. Hence

$$\left(n_0\frac{dx^\alpha}{d\sigma}\right)_{,\alpha}=\frac{\partial n}{\partial t}+\frac{\partial(nv_x)}{\partial x}+\frac{\partial(nv_y)}{\partial y}+\frac{\partial(nv_z)}{\partial z} \quad (18)$$

and this vanishes (equation of continuity) if the number of particles is conserved, which we shall assume. If we set as the proper mass density

$$\rho_0=mn_0, \quad (19)$$

(14) becomes

$$\left(\rho_0\frac{dx^\mu}{d\sigma}\frac{dx^\alpha}{d\sigma}\right)_{,\alpha}=-\rho_0\Delta^\mu{}_{\alpha\beta}\frac{dx^\alpha}{d\sigma}\frac{dx^\beta}{d\sigma}. \quad (20)$$

Taking as the energy-momentum density tensor

$$T'^{\mu\nu}=\rho_0\frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\sigma}, \quad (21)$$

where the prime has the same significance as in I (20), one readily verifies that I (5) and hence I (21) are satisfied. In this way we see that the equations of motion and the field equations are consistent with one another.

§4

The preceding calculations show that the standpoint of flat space at any rate does not lead to internal contradictions. Let us now compare the theory based on this standpoint with the Einstein general relativity theory.

First of all, insofar as the three crucial tests[5] are concerned the two theories lead to the same predictions. Since the equations of motion have been taken the same, except for the independent variables, it is clear that relations among the coordinates, giving the motion of a particle or the path of a light ray, will also be the same in both theories. But more than that, the gravitational red shift will be the same in the two theories, although this is not quite as obvious.

The gravitational red shift in the general relativity theory is usually derived from the assumption that the interval ds connected with the period of oscillation of an atom dt remains unchanged if the atom is put into a gravitational field, together with the relation

$$ds^2=g_{44}dt^2$$

to which I (1) reduces if the space coordinates of the atom are fixed. It is clear, however, that this change in the period of oscillation must also be a consequence of the equations of motion of the particles constituting the atom. These equations being the same in the two theories, the predicted red shift will also be the same. (It should be mentioned that the equations of motion referred to will involve the electromagnetic field and may be of a quantum-theoretical nature.)

One way of describing the situation is to say that all the formal relations involving ds are taken over into the present theory but ds is not regarded as the geometrical interval. Putting the matter in this way makes the comparison of the two theories almost trivial. Thus one sees, for example, that the principle of equivalence is valid here as long as we deal with coordinates and not with intervals.

Indeed it appears at first sight as if the differences between the theories were only questions of conventions.

For example, if a small measuring rod is moved from one place to another in a gravitational field, the interval ds between its end points will remain constant, but the corresponding $d\sigma$ may change.[6] One may say that the length of the rod does or does not change, depending on the point of view. As far as the changes in the coordinate-differences of the end points are concerned, however, no disagreement exists. Incidentally, there is some analogy here to the change in length of a measuring rod with temperature. If all substances had the same temperature coefficients of expansion, one could assume that the length of the measuring rod is independent of the temperature and no contradictions would arise. This corresponds to the point of view of the Einstein theory.

[5] A. S. Eddington, *The Mathematical Theory of Relativity* (Cambridge, 1924).

[6] Reference 3, p. 818.

A similar example involving time is to be found in the case of the gravitational red shift.

Actually it turns out that there are other, more important, differences between the two theories. They are based upon the fact that, after all, the fundamental ideas of the two theories are different. *The present theory involves less relativity*. This may be seen from the following example:

Let us take for the moment the flat-space point of view. Suppose that in a given frame of reference, we have a static gravitational field described by $g_{\mu\nu}$. Then the velocity of light will, in general, be different from that in field-free space. The space containing the field can therefore be looked upon as a medium with an index of refraction different from unity. From the standpoint of special relativity one can expect that such a medium will exert a "drag" upon light, as in the case of the well-known experiment of Fizeau. If we go over to a second frame of reference moving uniformly with respect to the first (Lorentz transformation), it will be possible, by measurements carried out on light in this system, to determine its motion relative to the other system. Thus the static gravitational field determines an "absolute" frame of reference. This will be true whenever the velocity of light differs from its special-relativity value, even in the limiting case when the components $g_{\mu\nu}$ are constant in a part of space, so that there is no gravitational force (but the gravitational potential differs from zero).

Such a situation may account for the principal results of the "ether-drift" experiments of D. C. Miller.[7] A discussion of this question will be presented in a separate paper, since it involves some considerations which would lead us too far afield here.

If we consider the same situation from the point of view of the Einstein theory, we arrive at entirely different conclusions.[8] We cannot carry out a Lorentz transformation in the presence of a gravitational field but must first get rid of the latter by going over to a suitably accelerated frame of reference. Having obtained a coordinate system in which the components $g_{\mu\nu}$ are constant (at least for a small portion of space), we can always make a further coordinate transformation so as to give them their special relativity values. But then no light-drag or "ether-drift" can be observed.

[7] D. C. Miller, Rev. Mod. Phys. **5**, 203 (1933).

[8] I am indebted to Professor Einstein for a helpful discussion of this question.

§5

The question of viewpoint being discussed here has a bearing on the application of relativity theory to cosmology. Thus, if one takes seriously the interpretation in terms of flat space, one must give up the λ-term in the cosmological field equations,[9] since this prevents the $g_{\mu\nu}$ from going over into the $\gamma_{\mu\nu}$ at large distances from matter. Of course, any solution of the equations corresponding to an "expanding universe" must be interpreted as representing the expansion of a distribution of matter and radiation in a flat space. There appears to be no necessity for imposing the condition of homogeneity, as is usually done.

§6

In the Einstein general relativity theory gravitation is explained in terms of geometry. In the theory suggested here based on flat space, this geometrization of gravitation has been given up. Perhaps this may be regarded by some as a step backward. It should be noted, however, that the geometrization referred to has never been extended satisfactorily to other branches of physics, so that gravitation is treated differently from other phenomena. It is therefore not unreasonable to wonder whether it may not be better to give up the geometrical approach to gravitation for the sake of obtaining a more uniform treatment for all the various fields of force that are to be found in nature.

It might be remarked that the theory, as presented here, was obtained by a reinterpretation of the Einstein general relativity theory. This amounts to using the formalism of the latter as a means for obtaining a consistent set of equations. Such a procedure is not entirely satisfactory. It ought to be possible to build up the theory independently from suitable postulates. This will be attempted in the near future.

[9] A. Einstein, Berl. Ber. p. 142 (1917). As a matter of fact the existence of the nebular red-shift weakens the chief argument advanced there for introducing the λ-term.