The eigenvalue problem of the Dirac spin electron has to be a generalization of the former scalar problem which had led to the integral equation:

$$(\mu/2\pi)^3 \int\int S_P(R|R')\psi(R')dV_0'dW_0 = \psi(R). \qquad (2)$$

Here dV_0' and dW_0 are invariant elements in space and in momentum space, and

$$\begin{array}{c} S_P(R|R') = \exp\{i\mu(P\cdot R - R' - E\cdot T - T')\} \\ E = \pm(P^2+1)^{\frac{1}{2}},\ \ T = \pm(R^2+1)^{\frac{1}{2}},\ \ P = p/m_0c,\ \ R = r/a. \end{array} \qquad (3)$$

The only change for the electron with spin is that S is now a matrix, and that there are four ψ-functions. Using the spinor $\psi_+\psi_-$ and its spin-conjugate $\chi_+{}^+\chi_-{}^+$ and calling them $\psi_1\psi_2\psi_3\psi_4$ we arrive at the following set of integral equations as a generalization of (2):

$$(\mu/2\pi)^3 \int\int dV_0'dW_0 S_P(R|R') \times \left\{ \begin{array}{l} (E+P_z)\psi_3(R\,) + (P_x - iP_y)\psi_4(R') \\ (P_x+iP_y)\psi_3(R') + (E-P_z)\psi_4(R') \\ (E-P_z)\psi_1(R') - (P_x - iP_y)\psi_2(R') \\ -(P_x+iP_y)\psi_1(R') + (E+P_z)\psi_2(R') \end{array} \right\} \begin{array}{l} =\psi_1(R) \\ =\psi_2(R) \\ =\psi_3(R) \\ =\psi_4(R). \end{array} \qquad (4)$$

For small P these equations reduce to (2). The bracket has exactly the same structure as the Dirac's differential equations except that in the latter $|E|$ stands for the operator $(\hbar/im_0c^2)\partial/\partial t$, etc. In contrast to the case of the point electron, we are far from knowing the exact eigenfunctions and eigenvalues. Nevertheless it seems interesting that the spin leads to a new eigenvalue problem (although there is no external electromagnatic field) simply because of the presence of large momenta in the probability distribution.

The physical meaning of our signal equation $(ct)^2 - r^2 = a^2$ is shown by its application (Born): Whereas in a *stationary* state of momentum p the probability amplitude of "transition" from the point r at the time t, to the point r' at the same time t, is $S_P(r|r') = \psi(r)\psi^*(r') = \exp\{2i\pi(p\cdot r - r')\}$, Born assumed that the transition probability in a *stationary* state from r at the time t to r' at the advanced or retarded time t' is given by (3).

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The Spin Angular Momenta of Elementary Particles

In the present state of the field theories it is not clear to what extent a consistent formulation of the "particle" as distinguished from the "field" aspects of the formalism can be made. The most characteristic condition in the electron theory is that the square of any component of the spin is an absolute invariant, but this is possible only for spin 0 and $\frac{1}{2}$. I have investigated the looser condition that the squared magnitude of the space part of the spin is a Lorentz invariant. This condition insures that the "magnitude" of the spin is the same for all Lorentz systems, and so has an invariant significance. This gives the three relations

$$S_{42}S_{12} + S_{43}S_{13} - i\hbar S_{41} = 0 \qquad \text{(cyclic)}, \qquad (1)$$

which lead at once to

$$S \equiv S_{12}{}^2 + S_{23}{}^2 + S_{31}{}^2 = S_{14}{}^2 + S_{24}{}^2 + S_{34}{}^2. \qquad (2)$$

If we assume the spin expressible in terms of an operator 4-vector by

$$S_{kl} = \lambda[\beta_k, \beta_l],$$

the general commutation relations become

$$(\beta_k\beta_l\beta_m + \beta_m\beta_l\beta_k) - (\beta_l\beta_k\beta_m + \beta_m\beta_k\beta_l) = (i\hbar/\lambda)(\delta_{km}\beta_l - \delta_{lm}\beta_k), \qquad (3)$$

which are satisfied by both the Dirac ($\lambda = -i\hbar/4$) and the Duffin[1] ($\lambda = -i\hbar$) operators.

With

$$\beta^2 = \beta_1{}^2 + \beta_2{}^2 + \beta_3{}^2$$
$$S^2 = \lambda^2\{\beta^2(\tfrac{1}{2}\beta^2 + i\hbar/\lambda) - \tfrac{1}{2}(\beta_1\beta^2\beta_1 + \beta_2\beta^2\beta_2 + \beta_3\beta^2\beta_3)\}. \qquad (4)$$

For the Duffin operators satisfying

$$\beta_k\beta_l\beta_m + \beta_m\beta_l\beta_k = \beta_k\delta_{lm} + \beta_m\delta_{lk} \qquad (5)$$

we get

$$S^2 = (i\hbar)^2\beta^2(\beta^2 - 3).$$

Introducing condition (1) there results

$$(i\hbar)^2(\beta_1\beta_4 + \beta_4\beta_1)(\beta_2{}^2 + \beta_3{}^2 - 1) = 0. \qquad \text{(cyclic)}$$

It is not apparent that these conditions are in conflict with Eqs. (5), but from the explicit matrices given by Kemmer[2] for the irreducible representations it is found that they are satisfied only for the trivial case of zero spin.

From the general representation $D_{k,l}$ of the Lorentz group[3] obtained by separation of the spin tensor into two self-dual tensors $\mathbf{\Gamma}$ and $\mathbf{\Delta}$ as in Dirac's method,[4] where

$$\mathbf{\Gamma}\times\mathbf{\Gamma} = i\hbar\mathbf{\Gamma}, \quad \mathbf{\Delta}\times\mathbf{\Delta} = i\hbar\mathbf{\Delta}, \quad [\Gamma_k, \Delta_l] = 0$$

conditions (1) become

$$\mathbf{\Gamma}\times\mathbf{\Delta} = 0,$$

yielding

$$\mathbf{S}^2 = \mathbf{\Gamma}^2 + \mathbf{\Delta}^2.$$

By simultaneous quantization of these three operators we get

$$s(s+1) = k(k+1) + l(l+1),$$

where s, k, and l are all to be integral or half-integral. This is possible only for $l=0$, $k=s$ or $k=0$, $l=s$. The wave function in terms of spinors has components of the form

$$\varphi^{\lambda\mu\cdots} \qquad \chi_{\sigma'\epsilon'\cdots},$$

each with $2s$ indices. Since the momentum operator spinor has one primed and one unprimed index, it is not feasible to connect these spinors by a linear wave equation except for $s=\frac{1}{2}$. The possibility of using families of secondary functions along the lines discussed by Fierz[5] has not yet been investigated.

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