Heavy Electron Pair Theory of Nuclear Forces

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(Received April 1, 1940)

The interpretation of the proton-proton scattering experiments on the basis of Bethe's neutral meson theory leads to a range of nuclear forces $\approx \frac{1}{2}\hbar/\mu c$ (μ is meson mass). This implies a mass for the field particle equal to twice the mass of the cosmic-ray meson. It has also been found that a meson theory of nuclear forces, involving the emission and absorption of single charged mesons obeying Bose statistics, does not give the right sign of the quadrupole moment of the deuteron. In view of these difficulties and the fact that the single meson theory cannot correctly explain the β-decay anyway, we have investigated the consequences of a heavy electron pair theory of nuclear forces. The heavy electrons are assumed to be identical with electrons in every respect ("hole" theory, Fermi statistics, etc.) except that their rest mass is taken equal to the cosmic-ray meson mass. A tensor interaction between the nuclear particles and the heavy electron pair field can alone account for the spin dependence of the nuclear forces and the positive quadrupole moment of the deuteron. It is interesting that a pseudo-vector interaction gives both the wrong sign of the quadrupole moment and too much repulsion. The potential function between two nuclear particles behaves at large distances, r, as $e^{-2kr}/r^{2.5}$, $k\approx \mu c/\hbar$ so that the range is effectively one-half the single meson range, and is directly connected with the rest mass of the heavy electron pair field (in contrast to the Gamow-Teller pair theory). At small r, the potential goes as $1/r^5$ so that one has to cut off in the same way as in the original electron-neutrino theory. The advantage of the heavy electron pair theory over a neutral meson theory is that it deals with particles which can be identified with the cosmic-ray mesons.

§1. INTRODUCTION

IN a recent paper[1] Bethe has examined quantitatively the "symmetrical" and "neutral" formulations of the single meson field theory of nuclear forces. He finds that Kemmer's "symmetrical" theory[2] (according to which both charged and neutral mesons obeying Bose statistics are emitted by nuclear particles) leads to several serious difficulties. First, the "cut-off" distance must be considerably larger than the natural range of nuclear forces following from the finite mass of the field particle. This implies that the nuclear force exists only at distances greater than 3×10^{-13} cm and is in conflict with all the strong evidence for short range forces such as the large binding energies of the triton and α-particle as compared with the deuteron, the almost total absence of p wave scattering of protons by protons at low energies, etc. Secondly, the quadrupole moment of the deuteron turns out to have the wrong sign and to be much too large. Finally any attempt to account for the β-decay by means of the "symmetrical" theory predicts a lifetime of the meson itself which is much shorter than that permitted by the cosmic-ray data. In addition to the shortcomings of the "symmetrical" theory revealed by Bethe's calculations, attention has also been called by Weisskopf[3] to the "peculiar" character of the internal charge distribution of an elementary particle with Bose statistics (scalar or vector meson) in contrast to the "well-behaved" charge distribution of an elementary particle obeying Fermi statistics and with spin one-half (electron).

On the other hand, Bethe finds that the assumption of an interaction of the nuclear particles with a "neutral" meson field alone leads to both a reasonable "cut-off" distance and the right sign and magnitude of the quadrupole moment of the deuteron. The neutral mesons are assigned Bose statistics, spin one and the mass experimentally found (150–200 electron masses) for the penetrating component of the cosmic rays. However, investigations by Breit and his collaborators[4–6] on the theoretical interpretation of the experimental data on proton-proton scattering and neutron-proton scattering, using Bethe's form of "neutral" meson theory, point

[1] H. A. Bethe, Part I, Phys. Rev. **57**, 260 (1940); Part II, **57**, 390 (1940).
[2] N. Kemmer, Proc. Camb. Phil. Soc. **34**, 354 (1938).
[3] V. Weisskopf, Phys. Rev. **57**, 1066A (1940).
[4] L. E. Hoisington, S. S. Share and G. Breit, Phys. Rev. **56**, 884 (1939).
[5] G. Breit, C. Kittel and H. M. Thaxton, Phys. Rev. **57**, 255 (1940).
[6] C. Kittel and G. Breit, Phys. Rev. **56**, 744 (1939).

to a range of nuclear forces of about $\frac{1}{2}\hbar/\mu c$ (μ is meson mass). This implies a mass of the field particle equal to about twice the mass of the cosmic-ray meson. Thus far no real evidence exists for neutral mesons of either the same mass as the charged mesons or twice their mass; moreover it would seem desirable to have recourse only as a last resort to a theory which depends on the postulation of unobserved particles.

In view of the above objections to Bethe's "neutral" theory, we have explored the possibility of a theory which only makes use of particles which can be identified with the observed cosmic-ray mesons, leads to a range of nuclear forces of $\frac{1}{2}\hbar/\mu c$, and at the same time retains the following desirable features of Bethe's "neutral" theory:

(a) Single force hypothesis; only one type of interaction between the nuclear particles and the field is needed to give the relative positions of the singlet and triplet levels of the deuteron and the sign of the quadrupole moment.

(b) Like and unlike particle forces arise in the same order of perturbation theory.

(c) Only a small admixture of D function is required in the deuteron ground state to give the magnitude of the quadrupole moment and therefore does not contradict the "almost" additivity of the magnetic moments of the neutron and proton.[7]

(d) A steep increase of the potential at small distances with the consequent possibility of accounting for the larger binding energies of the triton and α-particle.

§2. Interaction of Nuclear Particles with Heavy Electron Pair Field

We assume that the field consists of charged particles which obey Fermi statistics and possess a rest mass equal to the cosmic-ray meson mass. It is also assumed that the Dirac equation, implying a spin one-half, holds for these particles and that "hole" theory can be used to explain the presence of both signs of charge. In other words, the field particles are really *heavy electrons*—identical with electrons in every respect except for their mass which is taken to be about 200 times the electron mass. In this paper we reserve the term "meson" for the particles observed in cosmic-ray experiments and refer to the field particles of negative charge as heavy electrons and to those of positive charge as heavy positrons.

As the Hamiltonian which describes the interaction between the nuclear particles (neutron or proton) and the heavy electron-positron field, we first consider the most general type: a linear combination of the five possible relativistic invariants[8] which can be used when no derivatives of the wave functions are involved. Later we shall adopt Bethe's "single-force hypothesis" and set all constants except one equal to zero. Since we work in the non-relativistic approximation for the nuclear particles, all terms involving the Dirac α's of the nuclear particles can be neglected and the Dirac β's of the nuclear particles can be set equal to one. Thus the pseudo-scalar interaction disappears entirely and we get:[9]

$$H=G\{C_1\Psi^*\Psi\psi\beta\psi+C_2\Psi^*\Psi\psi^*\psi+C_3\Psi^*\boldsymbol{\sigma}_n\Psi\psi^*\beta\boldsymbol{\sigma}_e\psi+C_4\Psi^*\boldsymbol{\sigma}_n\Psi\psi^*\boldsymbol{\sigma}_e\psi\}, \quad (1)$$

where Ψ is the quantized Pauli two-component wave function, $\boldsymbol{\sigma}_n$ the two-component spin operator for the nuclear particle; ψ is the quantized Dirac wave function, $\boldsymbol{\sigma}_e$ the four-component spin operator for the heavy electron. The asterisk signifies creation and its absence destruction; the destruction of a negative-energy heavy electron is equivalent to the creation of a heavy positron in accordance with the Dirac "hole" theory. The second-order perturbation energy which describes the transition of a proton in a specified state π and a neutron in a specified state ν to two other specified states π', ν' is given by:

$$W=-\sum_b\frac{H_{bd}^*H_{ab}}{E_b-E_a}-\sum_c\frac{H_{cd}H_{ac}^*}{E_c-E_a}, \quad (2)$$

where a and d refer to the initial and final composite states, respectively, and b and c to the

[7] L. W. Alvarez and F. Bloch, Phys. Rev. **57**, 111 (1940).

[8] H. A. Bethe and R. F. Bacher, Rev. Mod. Phys. **8**, 82 (1936).

[9] The same coupling constants are taken for both neutron and proton because of the equality of like and unlike particle forces; consequently a calculation performed for proton-neutron is just as valid for proton-proton or neutron-neutron (cf. below).

two intermediate states with a heavy electron pair present in the field—the pair being emitted in the first case by a proton and in the second case by a neutron. In the transition to the final state the pair is reabsorbed by the neutron or the proton. Since these two possibilities are completely equivalent we can just as well consider one and multiply the result by two; we consider the first. We get for the perturbation energy in the transition $\pi, \nu \rightarrow \pi', \nu'$

$$W=-2G^2\sum_{ik,=1}^{4} C_iC_k\sum_b\left[\frac{1}{(p^2+1)^{\frac{1}{2}}+(q^2+1)^{\frac{1}{2}}}\int\int(\Psi_{\nu'}{}^*O^i\Psi_\nu)(\psi_q{}^*O^i\psi_p)(\psi_p{}^*O^k\psi_q)(\Psi_{\pi'}{}^*O^k\Psi_\pi)d\tau_1d\tau_2\right], \quad (3)$$

where O^i, O^k are for the nuclear particles I (identity matrix) or $\boldsymbol{\sigma}_n$, and for the field particles β, I, $\beta\boldsymbol{\sigma}_e$ or $\boldsymbol{\sigma}_e$; p always refers to the momentum of a positive energy heavy electron and q to that of a negative energy heavy electron. Since our unit of momentum is μc, of energy μc^2, of length $\hbar/\mu c$ (μ is heavy electron mass), $(p^2+1)^{\frac{1}{2}}$ and $(q^2+1)^{\frac{1}{2}}$ represent energies of the heavy electrons in the intermediate state. Moreover the recoil energy of the nuclear particle has been neglected since its rest mass is large. Hence, if plane waves are taken for the heavy electrons, the interaction potential in configuration space becomes:

$$J_{NP}(r)=\sum_{i,k=1}^{4} C_iC_kO^iO^kJ_{ik}$$

with

$$J_{ik}=-2G^2\int d\mathbf{p}\int d\mathbf{q}\frac{\exp\left[i(\mathbf{q}-\mathbf{p})\cdot\mathbf{r}\right]}{(p^2+1)^{\frac{1}{2}}+(q^2+1)^{\frac{1}{2}}}\sum_{\text{spins}}{}'\,(\bar{\psi}_q{}^*O^i\bar{\psi}_p\bar{\psi}_p{}^*O^k\bar{\psi}_q), \quad (4)$$

where $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$, $\mathbf{r}_1$ specifies the position of one nuclear particle and $\mathbf{r}_2$ that of the other, and the $\bar{\psi}$'s are the amplitudes of the plane waves. The integration is over all possible values and directions of the momentum and the summation over both orientations of spin corresponding to the positive energies of one heavy electron and the negative energies of the other. To evaluate the spin summation we insert operators before the $\bar{\psi}$'s so that we may sum over both signs of the energy for both heavy electrons. Thus:

$$\frac{1}{2}\left(1+\frac{\boldsymbol{\alpha}\cdot\mathbf{p}+\beta}{(p^2+1)^{\frac{1}{2}}}\right)\bar{\psi}_p=\bar{\psi}_p \quad \text{for positive energies}$$
$$=O \quad \text{for negative energies,}$$

$$\frac{1}{2}\left(1-\frac{\boldsymbol{\alpha}\cdot\mathbf{q}+\beta}{(q^2+1)^{\frac{1}{2}}}\right)\bar{\psi}_q=\bar{\psi}_q \quad \text{for negative energies}$$
$$=O \quad \text{for positive energies}$$

and we get:

$$P^{ik}=\sum_{\text{spins}}{}'\,\bar{\psi}_q{}^*O^i\bar{\psi}_p\bar{\psi}_q{}^*O^k\bar{\psi}_q$$

$$=\sum\tfrac{1}{4}\bar{\psi}_q{}^*O^i\left(1+\frac{\boldsymbol{\alpha}\cdot\mathbf{p}+\beta}{(p^2+1)^{\frac{1}{2}}}\right)\bar{\psi}_p\bar{\psi}_p{}^*O^k\left(1-\frac{\boldsymbol{\alpha}\cdot\mathbf{q}+\beta}{(q^2+1)^{\frac{1}{2}}}\right)\bar{\psi}_q$$

(over spins for both signs of energy)

$$=\sum\tfrac{1}{4}\bar{\psi}_q{}^*\left[O^i\left(1+\frac{\boldsymbol{\alpha}\cdot\mathbf{p}+\beta}{(p^2+1)^{\frac{1}{2}}}\right)O^k\left(1-\frac{\boldsymbol{\alpha}\cdot\mathbf{q}+\beta}{(q^2+1)^{\frac{1}{2}}}\right)\right]\bar{\psi}_q$$

or

$$P^{ik}=\tfrac{1}{4}\,\text{Spur}\left[O^i\left(1+\frac{\boldsymbol{\alpha}\cdot\mathbf{p}+\beta}{(p^2+1)^{\frac{1}{2}}}\right)O^k\left(1-\frac{\boldsymbol{\alpha}\cdot\mathbf{q}+\beta}{(q^2+1)^{\frac{1}{2}}}\right)\right]. \quad (5)$$

P^{ik} can be found for different choices of O^i and O^k by means of the usual rules for evaluating the spurs of products of Dirac operators. The cross terms $(i \neq k)$ do not give any contribution when integrated over momentum space and so we get for the interaction energy between a neutron and a proton:

$$J_{NP}(r) = -2G^2\int d\mathbf{p}\int d\mathbf{q}\exp[i(\mathbf{q}-\mathbf{p})\cdot\mathbf{r}]\cdot\frac{1}{(p^2+1)^{\frac{1}{2}}+(q^2+1)^{\frac{1}{2}}}\left\{(C_1^2+C_2^2)\left(1-\frac{1}{(p^2+1)^{\frac{1}{2}}(q^2+1)^{\frac{1}{2}}}\right)\right.$$
$$+(C_1^2-C_2^2)\frac{\mathbf{p}\cdot\mathbf{q}}{(p^2+1)^{\frac{1}{2}}(q^2+1)^{\frac{1}{2}}}+(C_3^2+C_4^2)\left[\sum_{r,s=1}^{3}\sigma_r^N\sigma_s^P\delta_{rs}\left(1-\frac{1}{(p^2+1)^{\frac{1}{2}}(q^2+1)^{\frac{1}{2}}}\right)\right]$$
$$\left.+(C_3^2-C_4^2)\left[\sum_{r,s=1}^{3}\sigma_r^N\sigma_s^P\left(\frac{p_rq_s+p_sq_r}{(p^2+1)^{\frac{1}{2}}(q^2+1)^{\frac{1}{2}}}-\frac{\delta_{rs}\mathbf{p}\cdot\mathbf{q}}{(p^2+1)^{\frac{1}{2}}(q^2+1)^{\frac{1}{2}}}\right)\right]\right\}. \quad (6)$$

After the angular integrations are performed we get the following two characteristic integrals:

$$\text{(A)} \qquad \int_0^\infty pdp\int_0^\infty qdq\frac{\sin pr\sin qr}{(p^2+1)^{\frac{1}{2}}+(q^2+1)^{\frac{1}{2}}}=\frac{\pi}{2}\left(-\frac{\partial}{\partial z}\right)\int_1^\infty dq(q^2-1)^{\frac{1}{2}}e^{-qz}=-\frac{\pi}{2}\frac{\partial}{\partial z}\left[\frac{K_1(z)}{z}\right],$$

$$\text{(B)} \qquad \int_0^\infty pdp\int_0^\infty qdq\frac{\sin pr\sin qr}{(p^2+1)^{\frac{1}{2}}+(q^2+1)^{\frac{1}{2}}}\cdot\frac{1}{(p^2+1)^{\frac{1}{2}}(q^2+1)^{\frac{1}{2}}}=\frac{\pi}{2}\left(-\frac{\partial}{\partial z}\right)\int_1^\infty\frac{dqe^{-qz}}{(q^2-1)^{\frac{1}{2}}}=\frac{\pi}{2}K_1(z).$$

In (A) and (B) $z=2r$ and $K_\nu(z)$ is a Bessel function of order ν and is related to the well-known Hankel function of the first kind:

$$K_\nu(z)=\tfrac{1}{2}\pi ie^{\nu\pi i/2}H_\nu^{(1)}(z).$$

The asymptotic forms of $K_\nu(z)$ are

for large z: $K_\nu(z)\sim(\pi/2z)^{\frac{1}{2}}e^{-z}$ for all ν,

for small z: $K_0(z)\sim-\log(\tfrac{1}{2}z)-\gamma$ (γ is Euler constant),

$K_1(z)\sim 1/z$.

All the other integrals which arise may be evaluated by performing suitable differentiations on (A) and (B). Making use of the recurrence relations for the K's, we finally obtain for the interaction potential of any two nuclear particles with spin operators $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$:

$$J_{12}(r)=-3f_1^2\left[\frac{K_0(z)}{z^3}+\frac{2K_1(z)}{z^4}\right]+f_2^2\left[\frac{2K_0(z)}{z^3}+\frac{K_1(z)}{z^2}+\frac{4K_1(z)}{z^4}\right]$$
$$+\frac{f_3^2}{3}\left\{(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\left[\frac{K_0(z)}{z^3}+\frac{2K_1(z)}{z^2}+\frac{2K_1(z)}{z^4}\right]\right.$$
$$\left.+\left[-\frac{3(\boldsymbol{\sigma}_1\cdot\mathbf{z})(\boldsymbol{\sigma}_2\cdot\mathbf{z})}{z^2}+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right]\left[\frac{5K_0(z)}{z^3}+\frac{K_1(z)}{z^2}+\frac{10K_1(z)}{z^4}\right]\right\}$$
$$-\frac{f_4^2}{3}\left\{(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\left[\frac{4K_0(z)}{z^3}-\frac{K_1(z)}{z^2}+\frac{8K_1(z)}{z^4}\right]\right.$$
$$\left.+\left[-\frac{3(\boldsymbol{\sigma}_1\cdot\mathbf{z})(\boldsymbol{\sigma}_2\cdot\mathbf{z})}{z^2}+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right]\left[\frac{5K_0(z)}{z^3}+\frac{K_1(z)}{z^2}+\frac{K_1(z)}{z^4}\right]\right\}, \quad (7)$$

where $f_i^2=128\pi^3G^2C_i^2$ $(i=1, 2, 3, 4)$, $\mathbf{z}=2\mathbf{r}$.

We now demand of our theory that one type of coupling alone should be capable of explaining all known facts about nuclear forces ("single force hypothesis"; the arguments for this hypothesis are given in Bethe[1]). Then it is clear that the spin-dependence of nuclear forces at once excludes the attractive f_1^2 (scalar) and repulsive f_2^2 (vector) interactions. The f_4^2 (pseudo-vector) interaction can also be eliminated for two reasons: (a) in the singlet state the spin-orbit-spin term vanishes, $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2=-3$ and we get for the potential function:

$$J_{12}(r)=f_4^2\left[\frac{4K_0(z)}{z^3}-\frac{K_1(z)}{z^2}+\frac{8K_1(z)}{z^4}\right].$$

For large z, the second term dominates, the potential is attractive and goes as $e^{-z}/z^{2.5}$; for small z the third term is the most important, the potential is repulsive and increases as $1/z^5$. The equilibrium value z_m can easily be found from the minimum condition:

$$K_0(z_m)/K_1(z_m)=(z_m^2+40)/(z_m^3-20z_m);$$

the result is that $z_m=5.8$ which corresponds to $2r_m=5.8\hbar/\mu c$ or a distance $r_m=6.3\times10^{-13}$ cm (for $\mu=175$ electron masses). Such a large lower bound for the region of attractive potential is incompatible with the known short range character of nuclear forces (cf. §1).

(b) The pseudo-vector interaction moreover gives rise to the wrong sign for the quadrupole moment of the deuteron. This can be seen from the fact that the spin-orbit-spin term is repulsive when the vector $\mathbf{z}=2\mathbf{r}$ is parallel to the total spin $\mathbf{S}$. The ground state of the deuteron will consist mostly of a $^3D_1$ wave function with an admixture of $^3S_1$ and with $\mathbf{z}$ and $\mathbf{S}$ tending to be at right angles. The quadrupole moment will therefore be negative in disagreement with experiment.[10]

There remains only the tensor interaction as a possible theory fulfilling the conditions of the single force hypothesis. It turns out that this interaction is indeed capable of satisfying the criteria for a correct theory enumerated in §1. We write the f_3^2-interaction potential between any two nuclear particles in the form: (cf. Bethe[1] Eqs. (34), (34a), (34b))

$$V=V_1+V_2, \tag{8}$$

where

$$V_1=f^2(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)F(r), \tag{8a}$$

$$V_2=f^2\left[-\frac{3(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{r})}{r^2}+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right]G(r). \tag{8b}$$

We have used the abbreviations:

$$f^2=f_3^2/24,$$

$$F(r)=\frac{K_0(2r)}{r^3}+\frac{4K_1(2r)}{r^2}+\frac{K_1(2r)}{r^4},$$

$$G(r)=\frac{5K_0(2r)}{r^3}+\frac{2K_1(2r)}{r^2}+\frac{5K_1(2r)}{r^4}.$$

Both $F(r)$ and $G(r)$ are positive monotonically decreasing functions of r—for large r decreasing as $e^{-2r}/r^{2.5}$ while for small r the dominant term becomes $\sim 1/r^5$. The exponential decrease of the potential leads to an effective range of the nuclear forces equal to $(\frac{1}{2})\hbar/\mu c\approx 1.1\times10^{-13}$ cm.

Comparison of the expression for V as given by (8), (8a), and (8b) with the potential function found by Bethe on the basis of the neutral meson theory shows a striking similarity. Both V_1 and V_2 are identical in the two theories as regards signs, spin factors, etc. except for a different dependence on the separation of the nuclear particles, r. Hence all of Bethe's conclusions about two-body systems, which are independent of the explicit form of $F(r)$ and $G(r)$, hold equally well in the heavy electron pair theory. These conclusions with suitable modification are as follows:

(a) The total angular momentum J, the parity, and the total spin $\mathbf{S}$ are good quantum numbers.

(b) The potential $V_1=-3f^2F(r)$ is attractive at all distances while V_2 vanishes identically for any singlet state of the deuteron or double proton.

(c) For triplet states with $L=J$ the central force V_1 is repulsive; since

$$t_{JL}=\left[-\frac{3(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{r})}{2r^2}+\frac{\sigma_1\cdot\sigma_2}{2}\right]_{JL}=-1$$

for all L, the part V_2 is attractive and independent of L. The total interaction, which is attractive at all distances, is:

$$V_{L=J}=-9f^2\left[\frac{K_0(2r)}{r^3}+\frac{K_1(2r)}{r^4}\right].$$

[10] J. M. B. Kellogg, I. I. Rabi, N. F. Ramsey and J. R. Zacharias, Phys. Rev. **56**, 728 (1939).

(d) The other two triplet states $L=J\pm1$, are always mixed together. For $J=1$ the coupling between the two states $L=2$ and $L=0$ is rather large and will tend to lower appreciably the lower of the two states and to raise the upper. Since $t_{10}=0$ while $t_{12}=1$, the diagonal value of V_2 is more repulsive for the $^3D_1$ wave function than for the $^3S_1$; hence the ground state of the deuteron if it is a mixture of $^3D_1$ and $^3S_1$, will be chiefly $^3S_1$ rather than $^3D_1$ (as in the case of the pseudo-vector interaction). Furthermore it is to be expected that the $^3S_1+{}^3D_1$ state will lie below the $^3P_1$ state (cf. Bethe[1] §10) and can be made to lie below the ^1S state by "cutting off" at sufficiently small distances. The ground state of the deuteron will therefore possess a quadrupole moment and in virtue of the attractive character of V_2 for parallel alignment of spin S and vector separation r, the moment will be positive; this is in agreement with experiment.

It is to be noted that in Bethe's theory V_1 has a $1/r$ dependence at small distances so that the $1/r^3$ dependence of the nondiagonal matrix element of V_2 operates to overcome the repulsive action of V_1. In our theory V_1 and V_2 have the same r dependence and the possibility of depressing the triplet state below the singlet arises from the presence of larger numerical coefficients in the nondiagonal matrix element of V_2 than in V_1. Moreover because of the large coupling potential the effect on the energy of the triplet ground state of the deuteron can be considerable without requiring more than a small percentage of D function mixed together with the S function. As far as qualitative considerations go, it would therefore appear that the heavy electron pair theory will give the "almost" additivity of the magnetic moments of neutron and proton.

As has already been implied, the necessity of "cutting off" the potential at small distances exists in the heavy electron pair theory just as it does in Bethe's neutral meson theory. Bethe's arguments for "cutting off" the potential function V (e.g. the interference of the proper fields of the nuclear particles, relativistic corrections, etc.) and his reasons for believing that the saturation requirements for nuclear forces are not in contradiction with the form of V when due account is taken of the "cut-off" (cf. Bethe[1] §6) are equally applicable to the heavy electron pair theory. However these arguments have an admittedly uncertain status at the present time so that we have not attempted any quantitative determination of the "cut-off" distance.

§3. Conclusions

We have seen that the tensor interaction between the nuclear particles and the heavy electron pair field leads to a theory of nuclear forces which is formally identical with the neutral meson theory and is characterized by all the desirable features of the latter theory (cf. §1). In addition, if the single force hypothesis is adopted, the heavy electron pair theory is unique since all the other types of coupling of the nuclear particles to the heavy electron pair field give rise to forces which can be rejected on purely qualitative grounds (cf. §2). If the single force hypothesis is relinquished, many more theories can be constructed by the choice of suitable linear combinations. One can even develop a theory which will lead automatically to repulsion at very small distances and still give the right sign of the quadrupole moment (by a linear combination of the tensor and pseudovector interactions). But this procedure involves the introduction of as many (or more) arbitrary constants as there are experimental data to fit.

In the heavy electron pair theory the effective range of nuclear forces is $\frac{1}{2}\hbar/\mu c$ in contrast to a range of $\hbar/\mu c$ in Bethe's neutral meson theory. Insofar as all the results of Breit *et al.* on fitting the experimental data on proton-proton scattering to a meson type of potential function indicate a range of $\frac{1}{2}\hbar/\mu c$, it would seem that this is a definite advantage for the heavy electron pair theory. Of course it is possible that this conclusion will not be borne out by a detailed investigation of the "cut-off" and the strength of the interaction. Whatever be the outcome of such an investigation this paper has established that the existence of unobserved neutral mesons is not indispensable to a theory of nuclear forces.

In conclusion the author should like to thank Professor V. Weisskopf and Dr. A. Nordsieck for helpful discussion.