Analysis of Proton-Proton Scattering Data

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THE analysis by Breit, Thaxton and Eisenbud[1] of the proton-proton scattering data of Heydenberg, Hafstad and Tuve[2] indicated a value of K_0, the phase shift for the S wave, which appeared too large at 670 kev to fit the theoretical curve deduced from a square and a Gauss error potential well which gave good agreement with the experiments of Herb, Kerst, Parkinson and Plain[3] at higher energies. In the same analysis the values of K_0 at 776 and 867 kev were somewhat doubtful because (1) the values of $\mathcal{R}$, the ratio of the observed scattering to that given by Mott's formula, were read off the curve drawn by Heydenberg, Hafstad and Tuve, rather than being computed from the actual numbers of counts; (2) the coefficients of $\sin^2 K_0$ and $\sin K_0 \cos K_0$ in the theoretical expression for the scattering had been computed only for every 100 kev from 500 to 1000 kev and interpolations made to the experimental energies; and (3) those coefficients had been computed only for every 5° in the value of Θ, the scattering angle, while the experiments were made in most cases at every $2\frac{1}{2}$°.

The present work consisted first in extending the table of coefficients to include every 50 kev and every $2\frac{1}{2}$° from 15° to 45°, and second in fitting the data of Heydenberg, Hafstad and Tuve in the sense of least squares, allowing for the possible effect of scattering of the P wave, i.e., considering terms in K_1 in the theoretical expansion of the scattering.

Since the value of K_1 that might fit the data was expected to be small, of the order of a few tenths of a degree at most, the term in $\sin^2 K_1$ was dropped and the term in $\sin K_1 \cos K_1$ assumed proportional to K_1. The theoretical value of $\mathcal{R}$ was then $\mathcal{R}_0+aK_1$ where $\mathcal{R}_0$ is the value of $\mathcal{R}$ computed for some K_0 chosen near that arrived at by Breit, Thaxton and Eisenbud and a is the coefficient of $\sin K_1 \cos K_1$, computed from the formulas given by Breit, Condon and Present.[4] The percentage difference between $\mathcal{R}_0+aK_1$ and the experimental $\mathcal{R}$ was squared and summed over all scattering angles. This sum was then minimized with respect to K_1 and the corresponding "best" K_1 was found for the particular K_0 chosen. Another K_0 was then used, and the same procedure repeated, giving a new best K_1 and a new sum of squared differences between experiment and theory.

The values of these sums of squares were then plotted against their K_0 and the abscissa of the minimum point of the parabola-shaped curve was taken as the K_0 which best fits the data, along with the corresponding K_1.

An important source of error in scattering experiments is the wobbling of the beam, and since this affects $\mathcal{R}$ more at small than at large scattering angles it was thought advisable to weight the data in evaluating K_0 and K_1. This was done by graphically evaluating the logarithmic derivative of the yield as function of Θ and using the reciprocal of this quantity, ie., $Yd\Theta/dY$, as the weighting factor, to multiply $(\mathcal{R}_{\text{experiment}}-\mathcal{R}_0-aK_1)$. The K_0 as found from the unweighted data varied from that found in this manner by about $-0.03°$, $-0.03°$, and $+0.02°$ at 867, 776, and 670 kev, respectively. By both methods the K_1 found was accurately a linear function of the K_0 chosen.

In Fig. 1 is plotted as a solid line the percentage variation of experiment, from $\mathcal{R}$ calculated with the "best" K_0 and K_1 determined as explained above; and as a dotted line a typical fit in which a value of K_0 alone and no K_1 is used. A better fit is obtained at practically all points by the use of a small amount of K_1 than can be gotten by the use of K_0 alone, as is expected, since the number of parameters is thereby increased. The fact that the K_1 which gives the

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[1] G. Breit, H. M. Thaxton and L. Eisenbud, Phys. Rev. **55**, 1018 (1939).
[2] L. R. Hafstad, N. P. Heydenberg and M. A. Tuve Phys. Rev. **51**, 1023 (1937); **53**, 239 (1938).
[3] R. G. Herb, D. W. Kerst, D. B. Parkinson and G. J. Plain, Phys. Rev. **55**, 998 (1939).
[4] G. Breit, E. U. Condon and R. D. Present, Phys. Rev. **50**, 825 (1936).

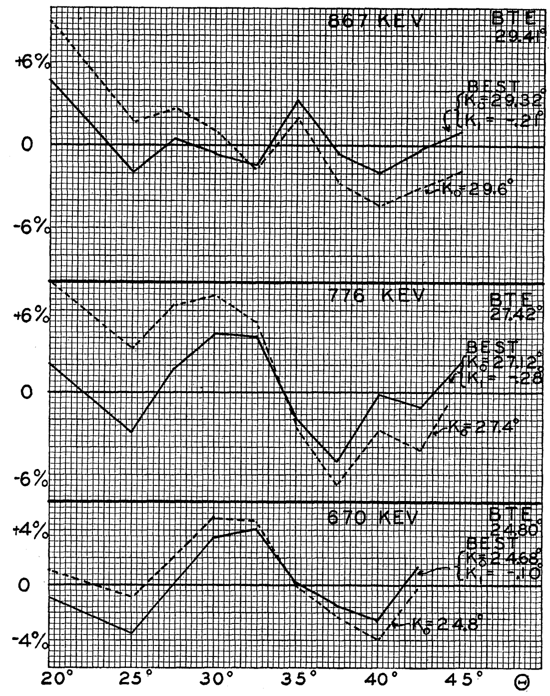

FIG. 1. Solid line: Percentage deviation of experimental values of $\mathcal{R}$ from value obtained with the K_0 and K_1 which give the best fit in the sense of least squares. Dotted line: a typical fit in which a value of K_0 alone and no K_1 is used. Values of K_0 given by Breit, Thaxton and Eisenbud are indicated by BTE.

best fit is not a smooth function of energy but is greater at 776 kev than at 670 or 867 kev probably indicates that no meaning should be attached to it as a real scattering anomaly, but only that the experimental errors are such as to increase the apparent $\mathcal{R}$ at small angles more than at large. For the "best" K_1 the difference between experiment and theory is of the same order of magnitude as for $K_1=0$, and the dips in the full drawn curves in the figure occur at about the same scattering angle at the three energies. If these dips are due to systematic errors, the apparent K_1 may be due to them as well. It appears somewhat inconsistent, therefore, to introduce K_1 into the analysis without bringing in K_2 as well. The latter seems to be improbable theoretically. It should also be mentioned that[1] the data of Herb, Kerst, Parkinson and Plain[3] do not indicate the necessity of K_1. The data are insufficient to render the suggestion of a possible resonance in K_1 in the neighborhood of 776 kev due to the formation of an excited He^2 any more than a mere speculation. It is of interest, however, that this more thorough analysis of the data provides values for K_0 in good agreement with the theoretical values of Breit, Thaxton and Eisenbud, lowering the low energy part of the experimental K_0, E curve.

Table I gives the expansions in powers of $1/E$,

TABLE I. *Series expansions in powers of $1/E$, E being the energy in Mev, for* $\mathbf{X}$, $-2\mathbf{Y}/\eta$, *and* $\mathfrak{M}$.

	$22\frac{1}{2}°$	$27\frac{1}{2}°$	$32\frac{1}{2}°$	$37\frac{1}{2}°$
$\mathbf{X}$	8.000 $-0.3154/E$ $+0.00242/E^2$	5.961 $-0.1410/E$ $+0.0007/E^2$	4.870 $-0.0689/E$ $+0.0002/E^2$	4.287 $-0.0375/E$ $+0.00007/E^2$
$\frac{-2\mathbf{Y}}{\eta}$	26.61 $-0.4034/E$ $+0.00186/E^2$	15.107 $-0.1444/E$ $+0.00043/E^2$	9.565 $-0.0558/E$ $+0.000107/E^2$	6.828 $-0.0233/E$ $+0.000027/E^2$
$\mathfrak{M}$	40.00 $+0.3108/E$ $-0.002011/E^2$	17.65 $+0.1270/E$ $-0.000451/E^2$	9.106 $+0.0495/E$ $-0.000084/E^2$	5.518 $+0.01504/E$ $-0.000009/E^2$

E being the energy in Mev, for $\mathbf{X}$, $-2\mathbf{Y}/\eta$, and $\mathfrak{M}$ for values of Θ not given in the tables of Breit, Thaxton and Eisenbud.[5] The expansions were checked for several values by direct trigonometric substitution in formulas (1) and (2.2) of the above paper. The author wishes to express his appreciation to Professor Breit for suggesting this work and to Mr. R. Davies who checked some of the calculations.

[5] Reference 1, pages 1024–1025.