TABLE I. *Values of x_0 and Δx_0. The maxima are set in bold face type. The probable errors of the values in the first row range from ± 0.3 at the left to ± 3 A at the right.*

x_0 (A)	**25.5**		38		**54**		72.5		**95.5**		117.5		**139**		164
Δx_0		12.5		16		18.5		23		22		21.5		25	
$x_0^{3/4}$	11.5		15.2		19.9		24.7		30.5		35.6		40.4		45.8
$\Delta(x_0^{3/4})$		3.7		4.7		4.8		5.8		5.1		4.8		5.4	

To formulate the theory quantitatively it is necessary to obtain an accurate solution of the wave equation. The W.B.K. first approximation will not serve, since it is just the one which starts by neglecting diffraction effects, and so always gives a nonfluctuating D. To begin with we may replace $V(x)$ by a parabolic potential function. The exact solution can then be obtained in terms of the parabolic-cylinder function but it turns out that in this case D has no fluctuations even for positive electron kinetic energies. It is interesting that here the W.B.K. method gives exactly the same result. This is the counterpart of the fact that for a harmonic oscillator the W.B.K. approximation gives the correct energy levels. As a better approximation we may try using a combination of a parabola with a straight line of slope $-eE$. D is then expressible in a series of which one small term is periodic in $2x_0/\lambda$, indicating that the above x_0 differences should be $\lambda/2$, which is seen to be roughly verified by the table. However, the neglected terms may cancel this result. To settle the question as to whether D really does have fluctuations it would probably be necessary to resort to a numerical or mechanical solution of the wave equation.

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Zeeman Effect in the Hyperfine Structure of Iodine. II

ARTHUR S. FRY* AND RUSSELL A. FISHER
Northwestern University, Evanston, Illinois

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Observations of the Zeeman effect in the hyperfine structure of iodine II lines at fields of 4000 to 16,000 gauss provide an independent confirmation of the value $2\frac{1}{2}$ units as the spin of the iodine nucleus. The patterns observed in intermediate fields have been interpreted with the help of the theory of Goudsmit and Bacher. Evidence of a nuclear quadrupole interaction is detected in some Zeeman patterns.

THE hyperfine structure of iodine I and II has been investigated exhaustively by Tolansky,[1, 4] Tolansky and Forrester,[2] and Murakawa.[3] However, these extensive measurements do not yield directly a conclusive value for the nuclear spin because the interval rule is not obeyed. The departures from the interval rule are only in part attributable to ordinary perturbations. By introducing a nuclear quadrupole and finally octopole moment, Tolansky and Forrester[2] and Tolansky[4] have been able to interpret observed intervals as arising from a spin of $2\frac{1}{2}$ units.

As an alternative approach to determination of the nuclear spin we have studied the Zeeman effect in the hyperfine structure of certain lines of iodine II. Since the magnetic fields attainable in our work were insufficient to produce the Back-Goudsmit effect in the wider hypermultiplets of iodine II we have had to deal with the more complex situation of hyperfine Zeeman effect in intermediate fields. The theory of hyperfine Zeeman effect in intermediate fields as developed by Goudsmit and Bacher[5] has been applied in

* Now with Research Laboratories Division, General Motors Corporation, Detroit, Michigan.

[1] S. Tolansky, Nature **127**, 855 (1931); Proc. Roy. Soc. **136**, 585 (1932); **A149**, 269 (1935); **A152**, 663 (1935); Proc. Phys. Soc. London **48**, 49 (1936).

[2] S. Tolansky and G. O. Forrester, Proc. Roy. Soc. **A168**, 78 (1938).

[3] K. Murakawa, Inst. Phys. and Chem. Res., Tokyo **420**, 285 (1933); Zeits. f. Physik **109**, 162 (1938).

[4] S. Tolansky, Proc. Roy. Soc. **A170**, 205 (1939).

[5] S. Goudsmit and R. F. Bacher, Zeits. f. Physik **66**, 13 (1930).

analysis of the patterns observed and incidentally subjected to a rather extensive experimental test.

The light source employed, which is a modified form of Schüler's metal hollow cathode discharge tube is described in detail elsewhere.[6] In this work on iodine it was necessary to use a cathode of tungsten and anode of molybdenum in order to avoid the formation of troublesome iodine compounds. Helium at approximately 6 mm pressure was used as the exciting gas. The iodine was introduced simply by placing a few crystals inside the anode cylinder. No measures for purifying the gas were necessary in this work because the iodine itself when excited in the discharge served as a most effective getter, the discharge becoming perfectly clean after one minute's operation. Discharge currents of less than 100 milliamperes were ordinarily employed.

A water-cooled Weiss electromagnet having 576 turns and fitted with truncated conical pole pieces produced fields up to 16,100 gauss with a maximum exciting current of 65 amperes and pole separation of 17 mm. Field strengths were measured from the simple Zeeman triplet of the iodine line λ4986.9 whose hyperfine structure is so narrow as to be practically negligible.

The patterns were resolved by means of a ruled Fabry-Perot interferometer[7] having a resolving power of approximately 25 times the order crossed with a glass prism spectrograph. A Rochon prism was used to separate the π from

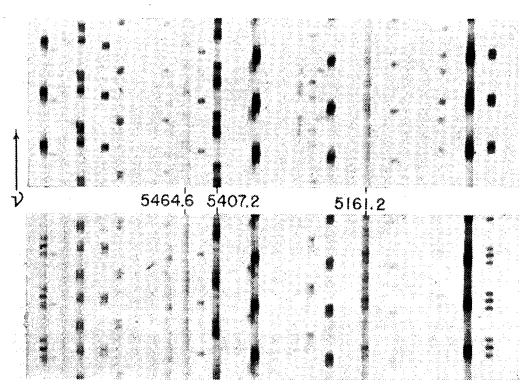

FIG. 1. Enlarged reproductions of spectrograms taken with 5.3 mm interferometer spacer showing π components in hyperfine Zeeman patterns of iodine; above at 4900 gauss, below at 9200 gauss.

[6] R. A. Fisher and A. S. Fry, following paper.
[7] R. C. Machler and R. A. Fisher, J. Opt. Soc. Am. 25, 315 (1935).

the σ components in the patterns. Since the pattern of π components is always simpler and more readily resolved in these structures, only the π components were carefully studied and measured.

Spectrograms were taken at a number of fields ranging from about 4000 to 16,000 gauss. Interferometer spacers of from 3.7 to 10 mm were used depending upon the field and the width of the pattern to be resolved. Fig. 1 is a reproduction of typical spectrograms taken at 4900 and 9200 gauss with a spacer of 5.3 mm.

Lines in a spectrum exhibiting high multiplicities, such as that of iodine, must have certain favorable properties if the hyperfine Zeeman patterns in intermediate fields are to be other than a hopeless confusion. Our study has been confined primarily to the behavior in magnetic fields of three lines of iodine II. These lines are suitable, first because the differences between interval factors of the states involved in each transition are large, thus producing widely spread hyperfine components in the Back-Goudsmit type of pattern. Second, for each of these lines the g values of the states are large, allowing more rapid approach to the Back-Goudsmit condition. Third, the differences between g values are small so that the complete Zeeman patterns are narrow enough to be studied with a Fabry-Perot interferometer. Two lines, λ5464.6 and λ5407.25, have the further simplifying property that the value of J does not change in the transition. These patterns were measured at several field strengths. The third line, λ5161.2, was observed in an interesting and simple pattern only at one particular field.

In seeking an interpretation of the Zeeman patterns observed at the various fields it was necessary for us to make at the outset assumptions concerning the value of I for the iodine nucleus and then to determine which of these values is qualitatively in accord with the observations. By proceeding in this manner we find that the only value for I which is consistent with the observed patterns is $2\frac{1}{2}$, the value assigned by Murakawa and by Tolansky. Adopting this value for I we have available the values determined by Tolansky and Forrester for the hyperfine interval factors. The g values for the states of interest to us are available either

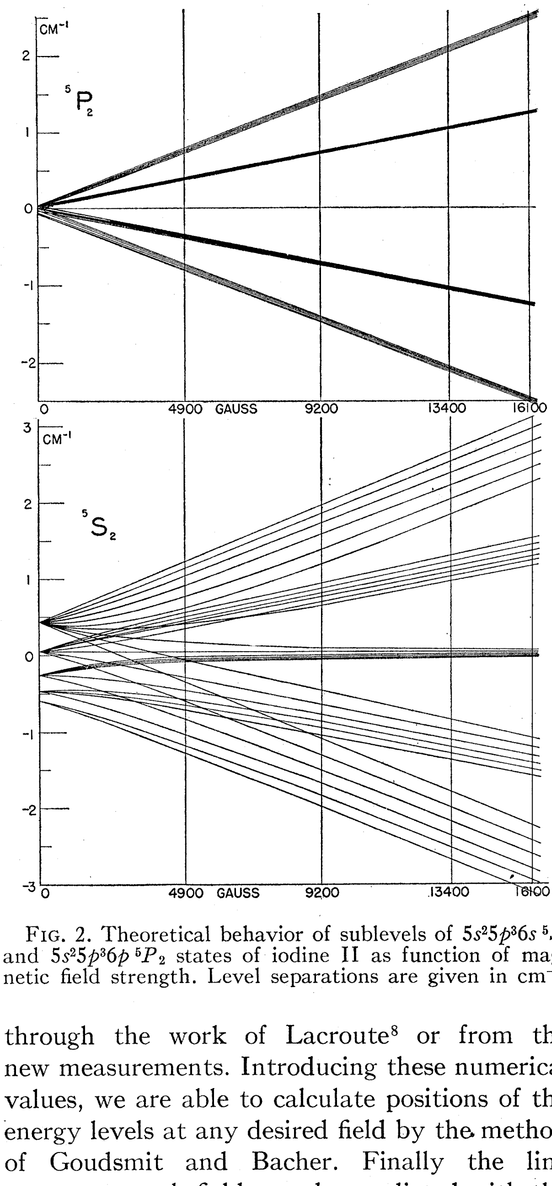

FIG. 2. Theoretical behavior of sublevels of $5s^25p^36s\ ^5S_2$ and $5s^25p^36p\ ^5P_2$ states of iodine II as function of magnetic field strength. Level separations are given in cm^{-1}.

through the work of Lacroute[8] or from the new measurements. Introducing these numerical values, we are able to calculate positions of the energy levels at any desired field by the method of Goudsmit and Bacher. Finally the line pattern at each field may be predicted with the aid of the intensity equations and compared with that observed. Detailed discussion of this comparison with reference to each line follows.

λ5464.6

The states and constants of the states from which the line λ5464.6 originates are:

$$\begin{array}{ccc} 5s^25p^36s\ ^5S_2 & - & 5s^25p^36p\ ^5P_2 \\ g=1.78 & & g=1.67 \\ A=0.0874\ \mathrm{cm}^{-1} & & A=0.0075\ \mathrm{cm}^{-1}. \end{array}$$

[8] P. Lacroute, Ann. d. Physik 3, 5 (1935).

The numerical values of the interval factors A are from Tolansky and Forrester's analysis. The value $g=1.78$ is from Lacroute's observations, while the second g was determined in the present analysis through an assumption of the correctness of the first g. The $^5P_2$ state is already approaching the Back-Goudsmit condition at 2000 gauss. Thus Goudsmit and Bacher's first approximation equations are adequate for treating the behavior of the sublevels of this state at the fields with which we are concerned. The $^5S_2$ state, having wide hyperfine structure, only begins to approach the Back-Goudsmit condition at 16,000 gauss. The general equations of Goudsmit and Bacher must here be used in computing the positions of the energy levels. These computations, which involve the numerical solution of a set of ten algebraic equations ranging from the first to the fifth degree, have been carried out for fields of

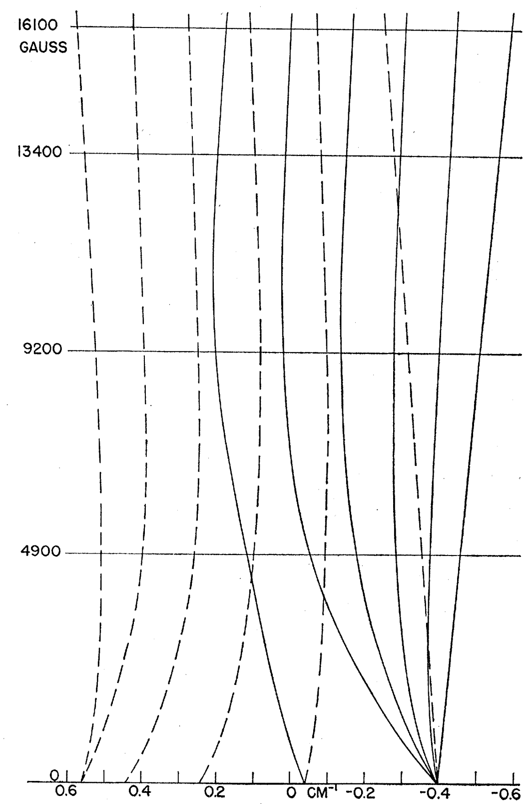

FIG. 3. Theoretical behavior of pattern of strong π components of λ5464.6, iodine II with varying magnetic field. Line distribution is plotted horizontally against field strength in gauss. Dotted components originate in the transition $M_J=-2\rightarrow M_J=-2$, solid components in the transition $M_J=2\rightarrow M_J=2$. These components have comparable intensities at fields above 4000 gauss.

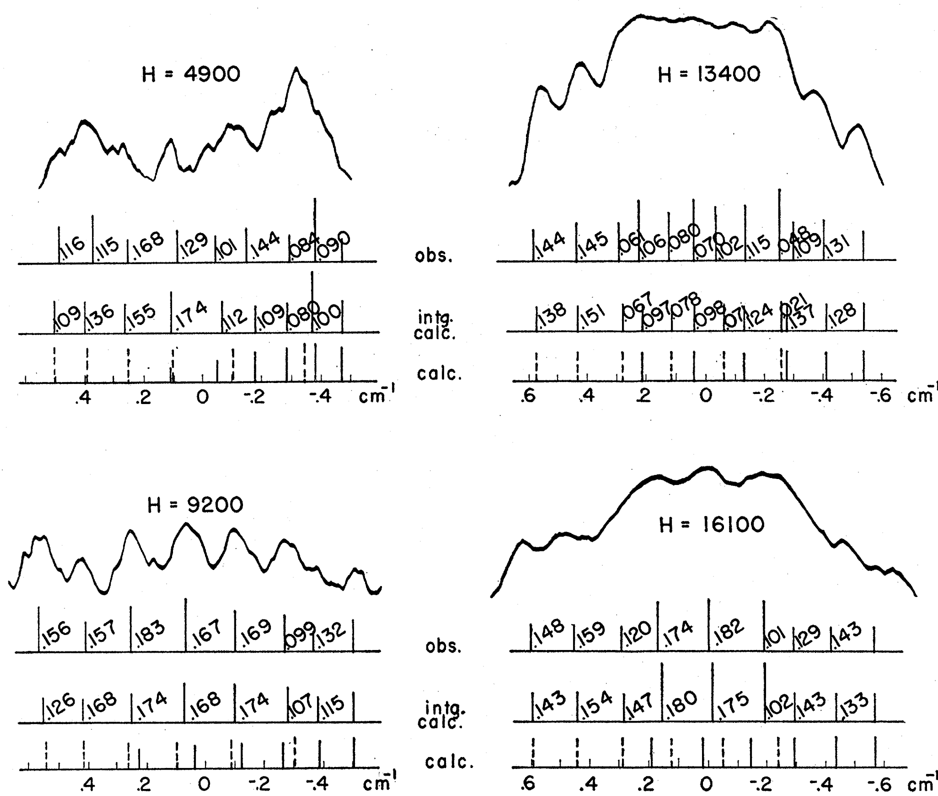

FIG. 4. Graphical comparison of calculated and observed π Zeeman patterns of λ5464.57 at four different fields. Integrated calculated patterns are obtained by representing as a single component close groups of components not expected to be resolved. Lengths of lines are proportional to intensities. Above each set of graphs is a microphotometer trace of a corresponding observed interferometer pattern.

4900, 9200, 13,400 and 16,100 gauss. These fields were chosen because experimental observations there proved particularly satisfactory. The theoretical behavior of the sublevels of the $^5P_2$ and $^5S_2$ states as functions of the magnetic field up to 16,100 gauss is shown graphically in Fig. 2.

The character of the line pattern resulting from the transitions between these two sets of levels may be determined for each field when the relative intensities of the numerous individual components have been found from Goudsmit and Bacher's intensity equations. Confining ourselves to the π components, we find that only 28 have intensities different from zero. Of these components 4 have intensities of less than one percent of that of the strongest at fields above 4900 gauss and 12 more have intensities of less than 20 percent of that of the strongest. The remaining 12, originating in the transitions $M_J=2\rightarrow M_J=2$ and $M_J=-2\rightarrow M_J=-2$, are strong and have comparable intensities which approach equality at the higher fields. This relative simplicity is a consequence of the fact that J does not change in the transition. Fig. 3 shows graphically the behavior of the line pattern made up of these 12 strong π components with varying field. The 6 components originating in the transition between levels having $M_J=-2$ are distinguished from the other set by dashed lines in the figure.

Figure 4 shows a graphical comparison of the observed and theoretically calculated patterns of π components of λ5464.6 at four different fields. The heights of the vertical lines representing components are proportional to their intensities. Intervals between components in cm^{-1} are indicated. The lower pattern for each field shows the calculated positions and intensities of the 12 strong π components. The two groups of components are distinguished in this pattern as in

Fig. 3. The middle pattern marked "intg. calc." is the integrated calculated pattern obtained when neighboring components not expected to be resolved are shown as a single component. The upper pattern shows the measured positions of observed components. Above each group of graphical patterns is an unretouched microphotometer trace of the corresponding observed interferometer pattern reduced to approximately the scale of the graphs. In comparing the microphotometer trace with the graphs it is to be remembered that the correspondence cannot be exact since the interferometer does not give a normal dispersion. However, to improve the apparent correspondence we have chosen orders rather far from the center of the interference pattern but have sacrificed something in resolution thereby. The reported positions of components in the observed pattern are the result of comparator measurements upon several orders of the interferometer pattern.

The comparison represented in Fig. 4 shows, we believe, a satisfactory qualitative agreement in that the general structure and number of components observed at each field corresponds with that expected. Our neglect of the weak components in the predicted patterns will introduce some distortion, but the quantitative agreement also seems satisfactory upon the whole.

λ5407.25

The line λ5407.25 is very similar in character to λ5464.6. It originates in the transition between two states described as follows:

$$\begin{array}{lcl} 5s^25p^36s\ ^3D_2 & - & 5s^25p^36p\ ^3D_2 \\ g=1.26 & & g=1.19 \\ A=0.0584\ \text{cm}^{-1} & & A=0.0068\ \text{cm}^{-1}. \end{array}$$

The value $g=1.19$ was determined in the present analysis by accepting the other g value of 1.26 as found by Lacroute. The A values are again from Tolansky and Forrester. Calculation of the pattern for this line at the four different fields has also been carried out. Fig. 5 shows the comparison between the calculated and observed patterns. The qualitative agreement seems again satisfactory. The quantitative agreement is, however, less complete than for λ5464.6. It is to be noted that the middle portion of the observed pattern is shifted toward the right relative to the predicted pattern in each case.

λ5161.2

A comparatively simple pattern was observed for the line λ5161.2 only at fields near 4900 gauss. The states involved are:

$$\begin{array}{lcl} 5s^25p^36s\ ^5S_2 & - & 5s^25p^36p\ ^5P_3 \\ g=1.78 & & g=1.55 \\ A=0.0874\ \text{cm}^{-1} & & A=0.0009\ \text{cm}^{-1}. \end{array}$$

The value $g=1.55$ was determined in the present analysis. Because this is a transition of the type $J\rightarrow J+1$ there are 30 components in the π pattern of comparable intensity and all must be included in the calculated pattern. The comparison between calculated and observed patterns for this line is shown in Fig. 6.

Discussion

Nuclear spin

Any change in the value for I introduced into the computations alters radically the predicted pattern both in regard to the number and the arrangement of components. The agreement between the theoretically calculated and the observed Zeeman patterns discussed above constitutes, we believe, a demonstration of the correctness of the value $I=2\frac{1}{2}$ for the iodine nucleus.

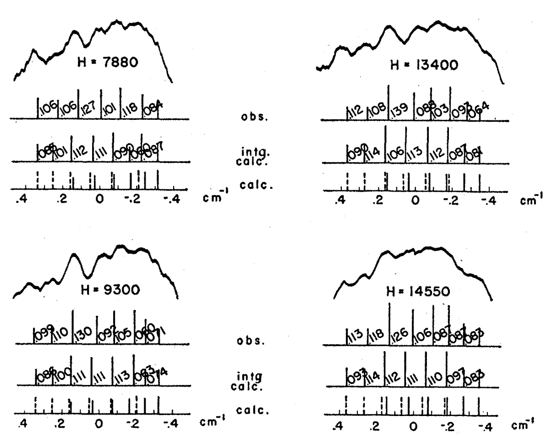

Fig. 5. Graphical comparison of calculated and observed π Zeeman patterns of λ5407.25 at four different fields.

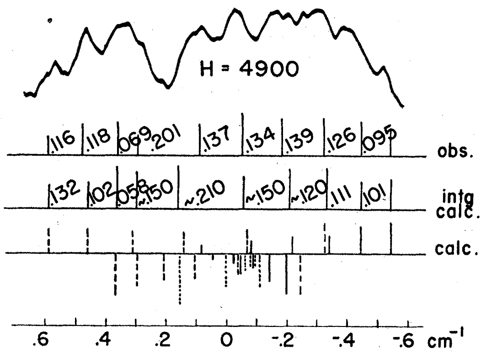

FIG. 6. Graphical comparison of calculated and observed π Zeeman patterns of λ5161.18 at 4900 gauss.

Theory of hyperfine Zeeman effect

In applying the theory of Goudsmit and Bacher to prediction of the Zeeman patterns of iodine II in a wide range of fields we have subjected the theory to a severe test. The agreement between prediction and experiment indicates that the theory is adequate in such highly complex cases.

Nuclear quadrupole effects

It has been pointed out that the experimental patterns of λ5407.2 were apparently distorted as compared with the predicted patterns. This affect we believe is to be attributed to the influence of a nuclear quadrupole moment. The appearance of this interaction in the Zeeman effect has been predicted by Schmidt.[9] He obtains the following approximate expression for the quadrupole interaction energy in the Back-Goudsmit condition:

$$Eq=b[\tfrac{2}{3}I(I+1)J(J+1)+6M_I^2M_J^2 -2M_J^2I(I+1)-2M_I^2J(J+1)],$$

where b is the quadrupole coupling constant. This term introduces an unsymmetrical shift of the hyperfine Zeeman levels although the total width of a group associated with a particular value of M_J is unchanged, as is that of the entire Zeeman pattern. A numerical value for b of the $5s^25p^36s\ ^3D_2$ state has been determined by Tolansky and Forrester from their hyperfine structure analysis. Their value is -0.00028 cm^{-1}.[10] Using this value we obtain the following corrections to be applied to the positions of the respective components of each M_J group in the line pattern, beginning at the high frequency side of the group: $+0.013$, $+0.007$, 0, 0, -0.007, -0.013 cm^{-1}. The result is a shift toward lower frequencies of the central components of the combined pattern. This is precisely the type of distortion observed in λ5407.2 and the quantitative agreement is satisfactory. We conclude, therefore, that the quadrupole interaction is observed in these Zeeman patterns.

[9] T. Schmidt, Zeits. f. Physik **111**, 332 (1938).

[10] We employ the coefficient obtained by Tolansky and Forrester when only a quadrupole interaction was assumed. The coefficient obtained later by Tolansky when an octopole interaction was introduced seems less satisfactory.

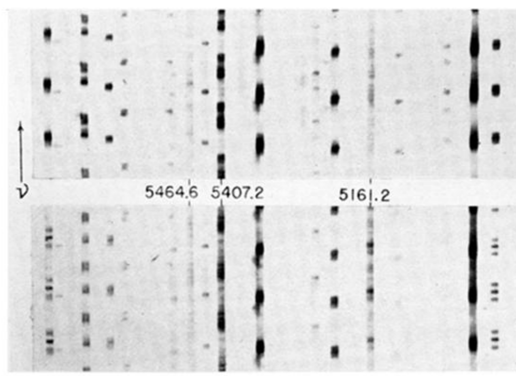

FIG. 1. Enlarged reproductions of spectrograms taken with 5.3 mm interferometer spacer showing π components in hyperfine Zeeman patterns of iodine; above at 4900 gauss, below at 9200 gauss.