The Periodic Deviation from the Schottky Line

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The width of the potential hump traversed by thermions moving between parallel plates depends upon the strength E of the external field at the cathode. Partial reflection at this hump, analogous to the partial reflection of light passing through a thin plate, may account for the small deviations from the Schottky line, depending in a periodic manner on $\sqrt{E}$, observed by Phipps, Seifert and Turnbull. A calculation based on this analogy shows rough agreement between the theory and the data. An approximate solution of the wave equation yields a small term of suitable period, but to start the theory, one needs a really accurate solution probably best obtained by mechanical integration.

THE explanation which suggests itself for the results described in the preceding papers is partial reflection of the emitted electrons at the potential barrier formed by the combination of the Schottky image force with the external field. This potential $V(x) = -e^2/4x - eEx$ (E = field strength at cathode surface, x = distance from this surface) has a maximum at $x = x_0 = (e/4E)^{\frac{1}{2}}$, and the parameter x_0 also fixes the scale of length for the potential distribution; so that, e.g., the distance between two points where the potential is V_0 less than the maximum varies as $x_0^{\frac{3}{2}}$ when V_0 is small. Considering the optical analogy we see that for electrons tunneling through the barrier the transmission coefficient D varies monotonically with electron energy and some arbitrarily defined width of the barrier, which will depend on E. But for electrons passing over the peak, D may be a fluctuating function of the energy and of E, because of interference between the partial waves scattered from the two sides of the barrier. For instance, if we compare the barrier with a uniform plate of thickness, t, D will pass from a maximum to a minimum or vice versa when t changes by a quarter wave-length. The thickness t must depend on the parameter x_0 because x_0 determines the scale of lengths, and for simplicity we may begin by assuming $t = x_0$. On this picture the maxima and minima in the curve of deviations from the Schottky line should correspond to values of x_0 equally spaced by the amount $\lambda/4$ where λ is some average wave-length for electrons with a Maxwellian velocity distribution with the temperature T of the cathode. From column 4, Table I of the preceding paper[1] one can see that the differences Δx_0 actually are roughly constant, though there is a decided trend to lower values as x_0 itself decreases. There is some indication that the equivalent plate thickness t varies approximately as $x_0^{3/4}$. This is shown by columns 6 and 7 of the same table.

Dr. D. Turnbull has kindly prepared the following table for me based on the combined results of his own experimental work and that of Seifert and Phipps.[2] It is seen to be substantially in agreement with the table mentioned above. Taking λ to correspond to the electron energy kT, the range of λ was from 42 to 47A for the different temperatures used. In agreement with the above theory, Seifert and Phipps[2] found that the positions of the maxima and minima were the same for a tantalum cathode as for tungsten. There was no observable dependence upon cathode temperature, but this may be due to the relatively small temperature range of the experiments.

When x_0 is several times λ there will be several maxima of D in that part of the Maxwellian distribution that makes an appreciable contribution to the current, and consequently the fluctuations of the average of D over the Maxwellian distribution will be less than those of D itself. We should get the greatest fluctuations when x_0 is of the order of λ, so that the amplitude of the fluctuations should increase from right to left in Table I. This is the case, the increase in amplitude being very marked.

[1] D. Turnbull and T. E. Phipps, Phys. Rev., this issue.
[2] R. L. E. Seifert and T. E. Phipps, Phys. Rev., this issue.

TABLE I. *Values of x_0 and Δx_0. The maxima are set in bold face type. The probable errors of the values in the first row range from ±0.3 at the left to ±3 A at the right.*

x_0 (A)	**25.5**		38		**54**		72.5		**95.5**		117.5		**139**		164
Δx_0		12.5		16		18.5		23		22		21.5		25	
$x_0^{3/4}$	11.5		15.2		19.9		24.7		30.5		35.6		40.4		45.8
$\Delta(x_0^{3/4})$		3.7		4.7		4.8		5.8		5.1		4.8		5.4	

To formulate the theory quantitatively it is necessary to obtain an accurate solution of the wave equation. The W.B.K. first approximation will not serve, since it is just the one which starts by neglecting diffraction effects, and so always gives a nonfluctuating D. To begin with we may replace $V(x)$ by a parabolic potential function. The exact solution can then be obtained in terms of the parabolic-cylinder function but it turns out that in this case D has no fluctuations even for positive electron kinetic energies. It is interesting that here the W.B.K. method gives exactly the same result. This is the counterpart of the fact that for a harmonic oscillator the W.B.K. approximation gives the correct energy levels. As a better approximation we may try using a combination of a parabola with a straight line of slope $-eE$. D is then expressible in a series of which one small term is periodic in $2x_0/\lambda$, indicating that the above x_0 differences should be $\lambda/2$, which is seen to be roughly verified by the table. However, the neglected terms may cancel this result. To settle the question as to whether D really does have fluctuations it would probably be necessary to resort to a numerical or mechanical solution of the wave equation.

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Zeeman Effect in the Hyperfine Structure of Iodine. II

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Observations of the Zeeman effect in the hyperfine structure of iodine II lines at fields of 4000 to 16,000 gauss provide an independent confirmation of the value $2\frac{1}{2}$ units as the spin of the iodine nucleus. The patterns observed in intermediate fields have been interpreted with the help of the theory of Goudsmit and Bacher. Evidence of a nuclear quadrupole interaction is detected in some Zeeman patterns.

THE hyperfine structure of iodine I and II has been investigated exhaustively by Tolansky,[1, 4] Tolansky and Forrester,[2] and Murakawa.[3] However, these extensive measurements do not yield directly a conclusive value for the nuclear spin because the interval rule is not obeyed. The departures from the interval rule are only in part attributable to ordinary perturbations. By introducing a nuclear quadrupole and finally octopole moment, Tolansky and Forrester[2] and Tolansky[4] have been able to interpret observed intervals as arising from a spin of $2\frac{1}{2}$ units.

As an alternative approach to determination of the nuclear spin we have studied the Zeeman effect in the hyperfine structure of certain lines of iodine II. Since the magnetic fields attainable in our work were insufficient to produce the Back-Goudsmit effect in the wider hypermultiplets of iodine II we have had to deal with the more complex situation of hyperfine Zeeman effect in intermediate fields. The theory of hyperfine Zeeman effect in intermediate fields as developed by Goudsmit and Bacher[5] has been applied in

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[1] S. Tolansky, Nature **127**, 855 (1931); Proc. Roy. Soc. **136**, 585 (1932); **A149**, 269 (1935); **A152**, 663 (1935); Proc. Phys. Soc. London **48**, 49 (1936).
[2] S. Tolansky and G. O. Forrester, Proc. Roy. Soc. **A168**, 78 (1938).
[3] K. Murakawa, Inst. Phys. and Chem. Res., Tokyo **420**, 285 (1933); Zeits. f. Physik **109**, 162 (1938).
[4] S. Tolansky, Proc. Roy. Soc. **A170**, 205 (1939).
[5] S. Goudsmit and R. F. Bacher, Zeits. f. Physik **66**, 13 (1930).