existence of maxima and minima in the A curve is not as pronounced as in the ϕ curve. The ϕ values show definite maxima and minima at values of $E^{1/2}$ corresponding, respectively, to minima and maxima in the emission curves (compare Figs. 8 and 3). It appears then that the Richardson work function, ϕ, is a periodic function of $E^{1/2}$, which exhibits an increasing amplitude with increasing field. The deviation from a least-squares line is small; the maximum value up to applied fields of the order 250,000 volts cm^{-1} is about 0.1 percent of the value of ϕ. The fact that the Richardson A as well as the Richardson ϕ may be a periodic function of $E^{1/2}$ could be due to the form of the Richardson equation; for, if the work function is not constant, but varies linearly with the temperature, the coefficient of T appears in the experimental constant A.

The periodic deviations from the Schottky line reported above are given theoretical consideration by Dr. H. M. Mott-Smith in an accompanying paper.

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Evidence of a Periodic Deviation from the Schottky Line. II

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The periodic deviation from the Schottky line observed by Seifert and Phipps was studied in greater detail for tungsten up to fields of 6.5×10^5 volts cm^{-1}. The deviation continued with increasing period and amplitude up to the highest fields investigated. The characteristic Schottky distance from the surface, x_0, was calculated for all values of the field at which maximal and minimal values of the deviation occurred. The difference between successive x_0 values was of the order 24A between $E^{1/2}=160$ and $E^{1/2}=266$; of the order 17A between $E^{1/2}=266$ and $E^{1/2}=505$; and of the order 12A between $E^{1/2}=505$ and $E^{1/2}=752$. The temperature coefficient of the position of these maximal and minimal values along the $E^{1/2}$ axis, $dE_m^{1/2}/(E_m^{1/2}dT)$, if it exists at all, is less than 5.9×10^{-5} deg.$^{-1}$.

Introduction

Recently Phipps and Seifert[1] measured the electron emission from clean tungsten as a function of the applied electrical field and found that the Schottky equation,

$$i=i_0 \exp\left[(e^{3/2}E^{1/2})/(kT)\right],$$

in which i_0 is the specific electron current at zero field and E is the electrical field, is valid only as a first approximation. By plotting $\Delta \log_{10} i$, which is the deviation from a reference line calculated by the method of least squares, against $E^{1/2}$ a periodic curve was obtained, whose period and amplitude increased with increasing field. That investigation was extended to fields of 2.6×10^5 volts cm^{-1}. Later work by the same authors[2] indicated that for clean tungsten the periodic behavior continued to fields as high as 9×10^5 volts cm^{-1}, and that the electron current from clean tantalum deviated in the same manner as tungsten, to the highest field investigated, namely, to 2.6×10^5 volts cm^{-1}. The purpose of the present investigation was to determine more accurately the values of $E^{1/2}$ at which the maximal and minimal values of the deviation occur and to make a more careful study of the phenomenon above fields of 2.6×10^5 volts cm^{-1}.

Experimental

The electrical circuit for measuring the electron current and for controlling the temperature of the filament was the same as described by Seifert and Phipps.[2] The Jones-Langmuir[3] temperature scale for tungsten was employed in the previous work.[2] In the present investigation a correction was made for the fact, observed by Seifert and

[1] T. E. Phipps and R. L. E. Seifert, Phys. Rev. **53**, 493 (1938).
[2] R. L. E. Seifert and T. E. Phipps, Phys. Rev., preceding paper.
[3] H. A. Jones and I. Langmuir, Gen. Elec. Rev. **30**, 310 (1927).

Phipps, that the resistance of the filament was 0.6 percent higher when it was mounted at the center of a reflecting metallic plate than when it was mounted in a clear glass tube. At 1700°K this correction was of the order 10°K.

The thermionic tube used in this work has been described by Clemens and Phipps.[4] Tension was supplied to the filament by means of a loop spring previously described.[2] For part of the investigation a tungsten filament (General Electric, Lot No. 218) 0.002534 cm in diameter was used. For the rest of the work tungsten of very high purity, 0.001288 cm in diameter, was used. This wire was furnished by the Incandescent Lamp Department of the General Electric Company, Cleveland Wire Works Division, through the courtesy of Messers B. L. Benbow and A. Poritsky. It was stated[5] that this sample contains 0.008 percent molybdenum and possibly also impurities of low work function in spectroscopic amounts. The method of determining the diameter has been described.[2]

Results

The deviation $\Delta \log_{10} i$ from a straight line was calculated by Seifert and Phipps[2] by using a reference line of which the slope and intercept were calculated by the method of least squares. The slopes so obtained depended, however, upon the values of $E^{\frac{1}{2}}$ at which the series was terminated and for this reason it was difficult to compare different sets of data which were terminated at different values of $E^{\frac{1}{2}}$. In the present investigation $\Delta \log_{10} i$ was calculated from the equation,

$$\Delta \log_{10} i = \log_{10} i - \log_{10} i_0 - ME^{\frac{1}{2}}.$$

The slope of the reference line, M, was the Schottky slope, calculated from the equation,

$$M = e^{\frac{3}{2}}/(kT).$$

Since varying the intercept of the Schottky line, $\log_{10} i_0$, resulted only in raising or lowering the Schottky line parallel to itself, arbitrary intercepts were used throughout this investigation. The characteristic Schottky distance, x_0, at which the maximal and minimal values of the deviation occur was calculated from the Schottky theory by the equation,

$$x_0 = e^{\frac{1}{2}}/(2E^{\frac{1}{2}}).$$

With the substitutions, $e = 4.8029 \times 10^{-10}$ e.s.u.,[6]

TABLE I. *The positions along the $E^{\frac{1}{2}}$ axis at which maximal and minimal values of the deviation from the Schottky line occur, with the corresponding Schottky distance, x_0, from the surface.*

	$E^{1/2}$	δ	x_0	Δx_0	$x_0^{3/4}$	$\Delta(x_0^{3/4})$
min.	110±2		172.5±3.0		47.6±0.8	
		0.01±0.01		26.5±5.2		5.6±1.4
max.	130±2		146.0±2.2		42.0±0.6	
		0.09±0.02		27.4±3.6		6.1±1.0
min.	160±2		118.6±1.4		35.9±0.4	
		0.15±0.03		23.7±2.3		4.8±0.7
max.	200±2		94.9±0.9		31.1±0.3	
		0.18±0.04		23.5±1.5		6.5±0.5
min.	266±1.5		71.4±0.6		24.6±0.2	
		0.23±0.03		16.9±1.0		4.6±0.4
max.	348±3		54.5±0.4		20.0±0.2	
		0.48±0.02		16.9±0.7		4.8±0.3
min.	505±4		37.6±0.3		15.2±0.1	
		0.77±0.02		12.4±0.5		4.0±0.2
max.	752±6		25.2±0.2		11.2±0.1	

[4] J. E. Clemens and T. E. Phipps, Rev. Sci. Inst. **8**, 133 (1937).

[5] Private communication from Mr. A. Poritsky to T. E. Phipps.

[6] R. T. Birge, Nature **137**, 187 (1936).

FIG. 1. Data for tungsten of diameter 0.002534 cm, showing detail at fields below $E^{\frac{1}{2}} = 155$ volts$^{\frac{1}{2}}$ cm$^{-\frac{1}{2}}$ for two temperatures. Ordinate: deviation of the logarithm of the specific emission from a line having the Schottky slope M and an arbitrary intercept I, as indicated. Abscissa: square root of the field at the surface of the emitter.

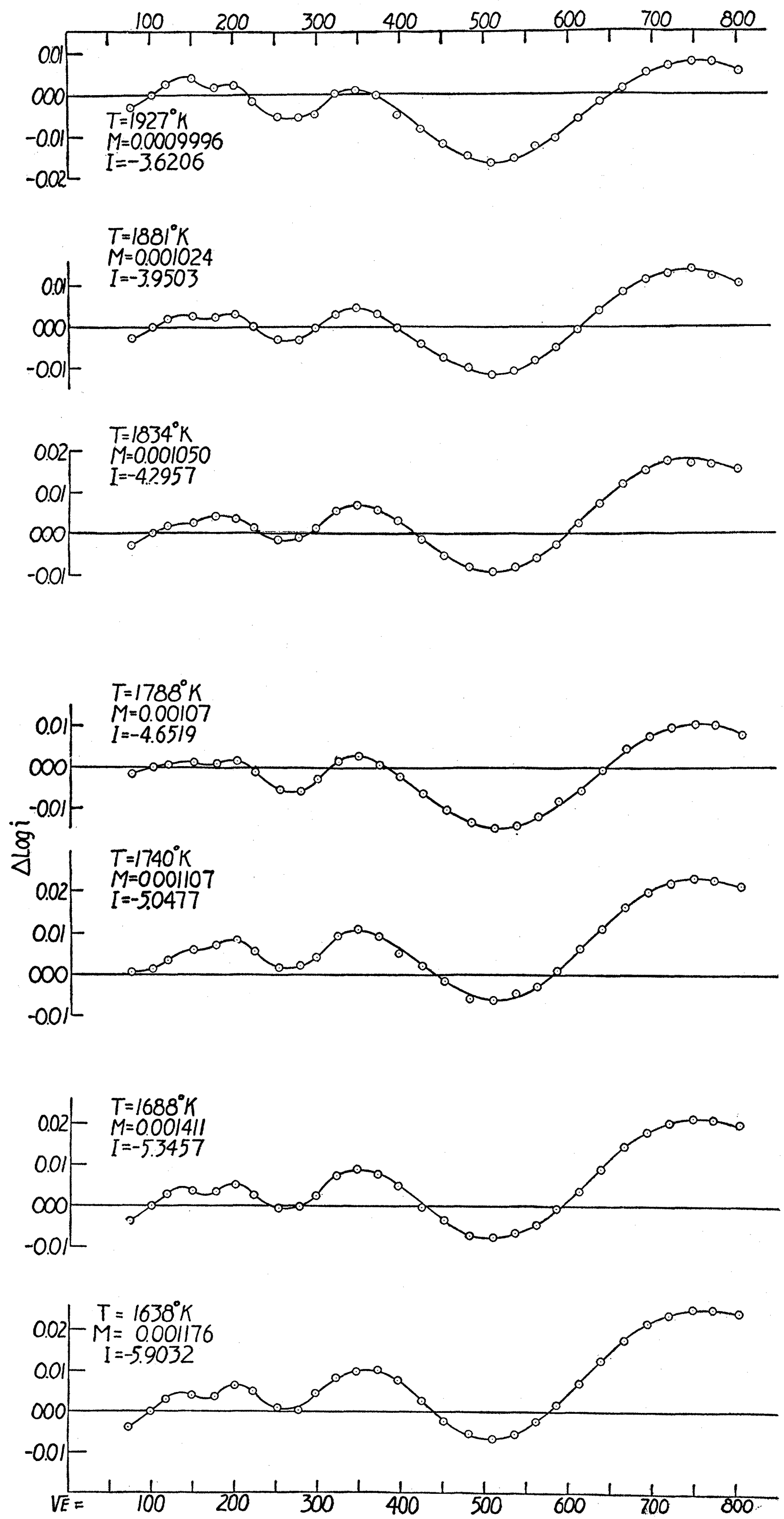

FIG. 2. Data for tungsten of diameter 0.001288 cm, up to fields of 6.5×10^5 volts cm^{-1} at seven temperatures. Ordinate: deviation of the logarithm of the specific emission from a line having the Schottky slope M and an arbitrary intercept I, as indicated. Abscissa: square root of the field at the surface of the emitter.

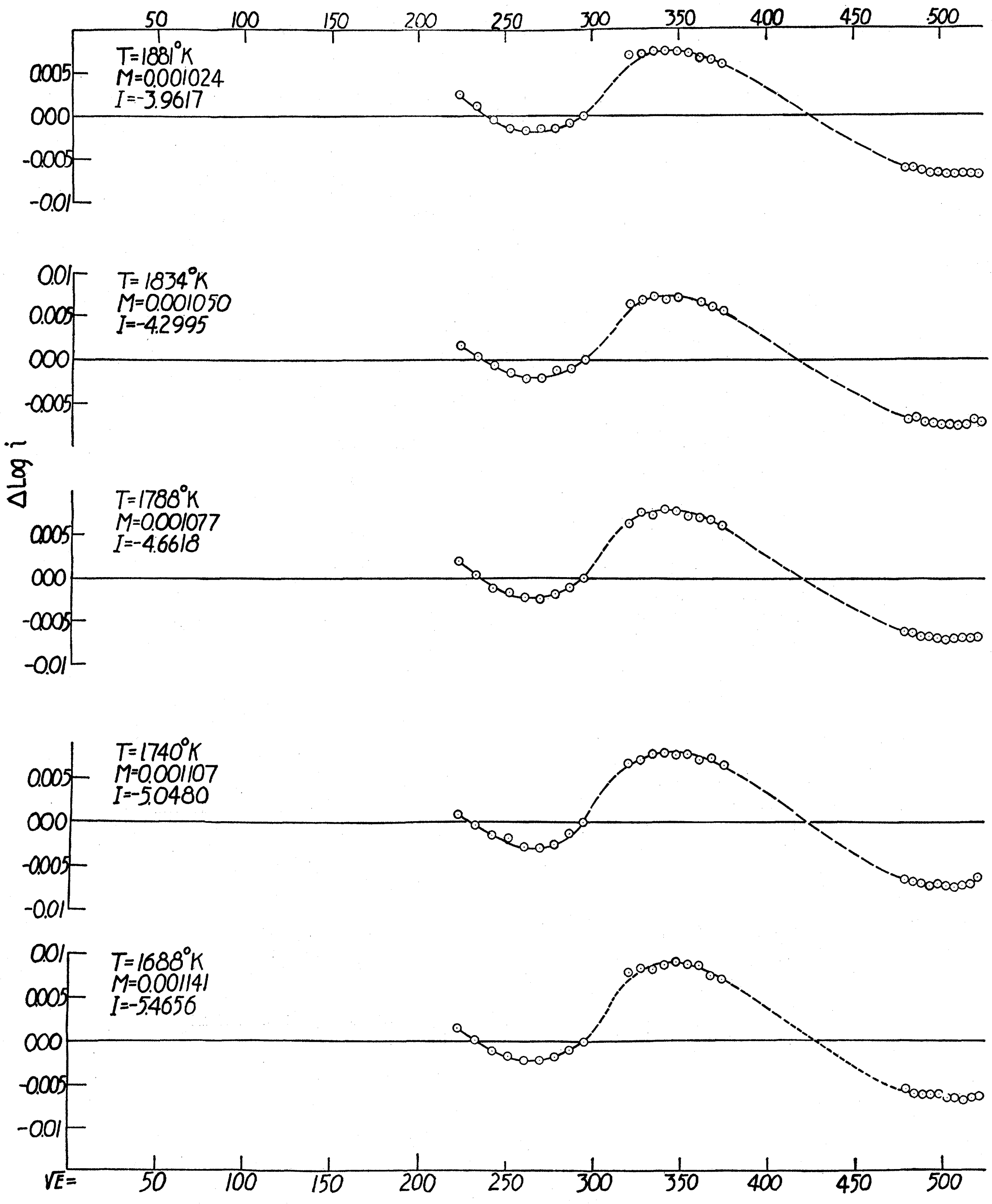

Fig. 3. Data for tungsten of diameter 0.001288 cm, showing greater detail in the region of the maximal and minimal values of the deviation, at five temperatures. Ordinate: deviation of the logarithm of the specific emission from a line having the Schottky slope M and an arbitrary intercept I, as indicated. Abscissa: square root of the field at the surface of the emitter.

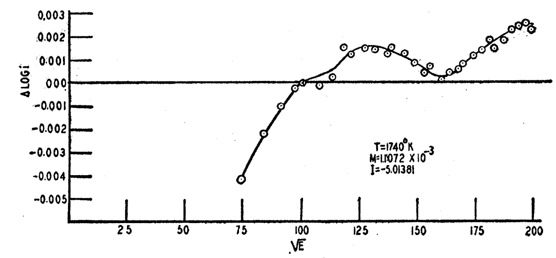

FIG. 4. Data for tungsten of diameter 0.001288 cm, showing greater details below $E^{\frac{1}{2}}=200$ volts$^{\frac{1}{2}}$ cm$^{-\frac{1}{2}}$ for one temperature. Ordinate: deviation of the logarithm of the specific emission from a line having the Schottky slope M and an arbitrary intercept I, as indicated. Abscissa: square root of the field at the surface of the emitter.

299.9 int. volt = 1 e.s.v., and 10^8A = 1 cm, x_0 is found in Angstrom units from the equation,

$$x_0=(1.898\times10^4)/E^{\frac{1}{2}},$$

in which E is given in int. volt cm^{-1}. The percent deviation from the Schottky line, δ (Table I, column 3), is defined as the absolute value of the quantity obtained by subtracting the percent deviation at a given maximum or minimum from the percent deviation of the minimum or maximum immediately preceding it. The error in δ (Table I, column 3), $d\delta$, was calculated from the formula,

$$d\delta=\pm\partial\log_{10}D_g\pm(Mq\partial V)/(2E^{\frac{1}{2}}),$$

in which D_g is the galvanometer deflection, q is the conversion factor from applied voltage to field at the surface of the emitter, and V is the applied voltage. There are errors in the determination of q and M, but since these quantities remain constant throughout the course of a given experiment their errors will not change the relative positions of the points on the deviation curve. In calculating $d\delta$, it was assumed that ∂V was ±0.0005 V and $\partial D_g=\pm0.2$ mm.

The results are shown graphically in Figs. 1, 2, 3 and 4 in which $\Delta\log_{10}i$ was plotted against $E^{\frac{1}{2}}$ and in which $\log_{10}i_0$ was replaced by the symbol I. Fig. 1 shows the results for tungsten 0.002534 cm in diameter, to fields of 5×10^5 volts cm^{-1}. The points are grouped very closely below $E^{\frac{1}{2}}=155$ in order to determine the detail of the curve more accurately than had been done previously in this region, where the deviation is small. Fig. 2 shows the results for tungsten wire of very high purity, 0.001288 cm in diameter, up to fields of 6.5×10^5 volts cm^{-1}, for seven temperatures between 1638°K and 1927°K. Fig. 3 shows the results for the same tungsten wire with the points grouped very closely in the region of the maximal and minimal values of the deviation, in order to determine more accurately the values of $E^{\frac{1}{2}}$ at which they occur. Fig. 4 shows the results for this wire with the points grouped closely below $E^{\frac{1}{2}}=200$.

Table I, column 2, shows the most probable values of $E^{\frac{1}{2}}$ at which the maximal and minimal values of the deviation occur, together with the probable errors in their determination. These values were obtained by assuming that there was no temperature dependence of the positions of the maximal and minimal values along the $E^{\frac{1}{2}}$ axis and by giving weight to the determinations in which the greatest number of experimental points were grouped in the region about the minimum or maximum value. The Δx_0 values (Table I, column 5) were found by subtracting from a given x_0 value the one just preceding it in the table. The significance of the Δx_0 values and $\Delta(x_0^{\frac{3}{2}})$ values are discussed in an accompanying note by H. M. Mott-Smith.[7] The percent deviation, δ (Table I, column 3), for $E^{\frac{1}{2}}\leqq160$ was calculated from data taken at 1918°K (Fig. 1); for $E^{\frac{1}{2}}\geqq200$ it was calculated from data taken at 1881°K (Fig. 2). These data show that the amplitude of the deviation and its period increase with increasing field up to 5.65×10^5 volts cm^{-1}. The difference between successive x_0 values are of the order of 24A between $E^{\frac{1}{2}}=160$ and $E^{\frac{1}{2}}=266$; of the order of 17A between $E^{\frac{1}{2}}=266$ and $E^{\frac{1}{2}}=505$; and of the order of 12A between $E^{\frac{1}{2}}=505$ and $E^{\frac{1}{2}}=752$. No marked difference was observed in the thermionic behavior of well-aged commercial tungsten wire and tungsten wire of very high purity.

The magnitude of the percent deviation, δ, increases with temperature. At 1638°K, the lowest temperature employed, the value of δ between $E^{\frac{1}{2}}_{\min}=505$ and $E^{\frac{1}{2}}_{\max}=752$ is 0.63 ± 0.02 percent; and at 1927°K, the highest temperature employed, it is 0.81 ± 0.02 percent. The position of the minimum occurring at $E^{\frac{1}{2}}=266$ was reproduced within ±1.5 volts$^{\frac{1}{2}}$ cm$^{-\frac{1}{2}}$ for temperatures from 1688°K to 1881°K. From this we may conclude that the temperature coefficient of the position of the maximum or minimum along the $E^{\frac{1}{2}}$ axis, defined as $dE_m^{\frac{1}{2}}/(E_m^{\frac{1}{2}}dT)$, is equal to or less than 6×10^{-5} deg.$^{-1}$.

[7] H. M. Mott-Smith, Phys. Rev., this issue.